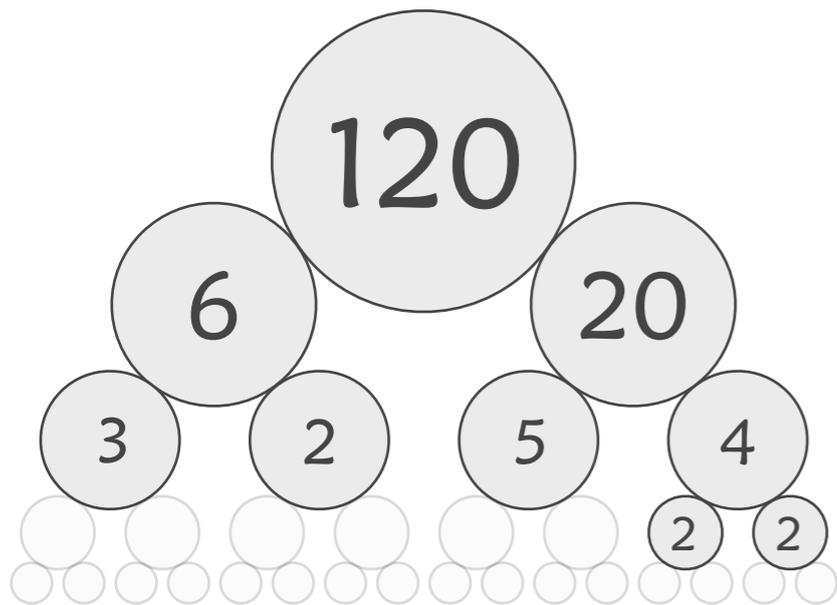
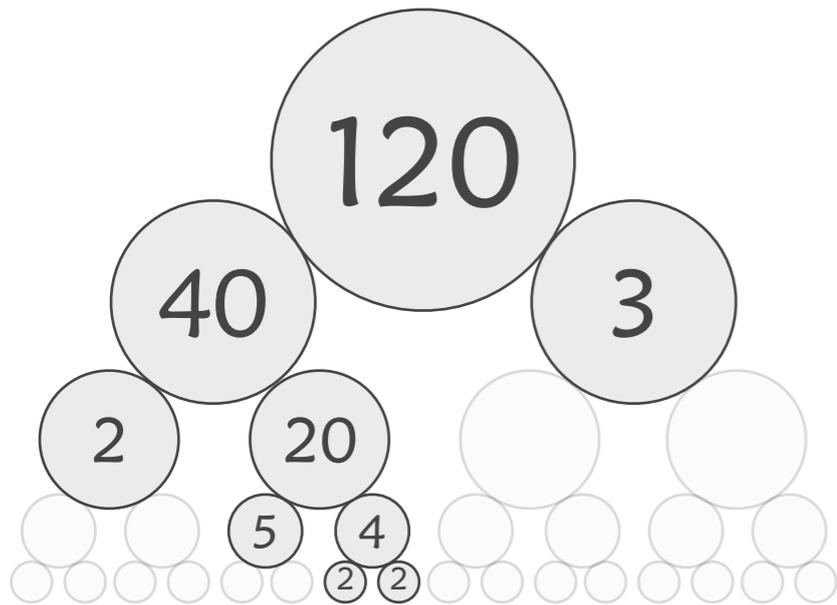


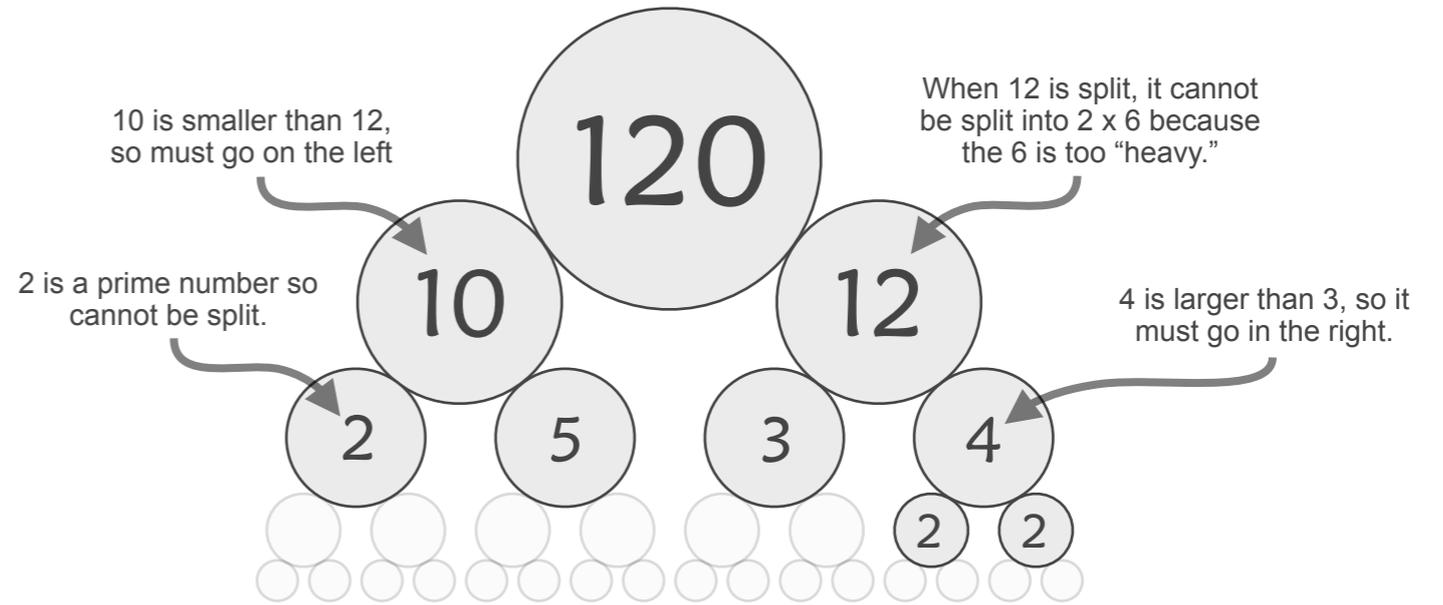
# INTEGRAL FISSION

In the 20th century, integers peacefully dreamed of their prime factorization trees. A prime factorization tree is created by splitting an integer into factors  $\geq 2$  and repeating the splitting on these fractions etc.. Most integers have many prime factorization trees. Here are some for the number 120:

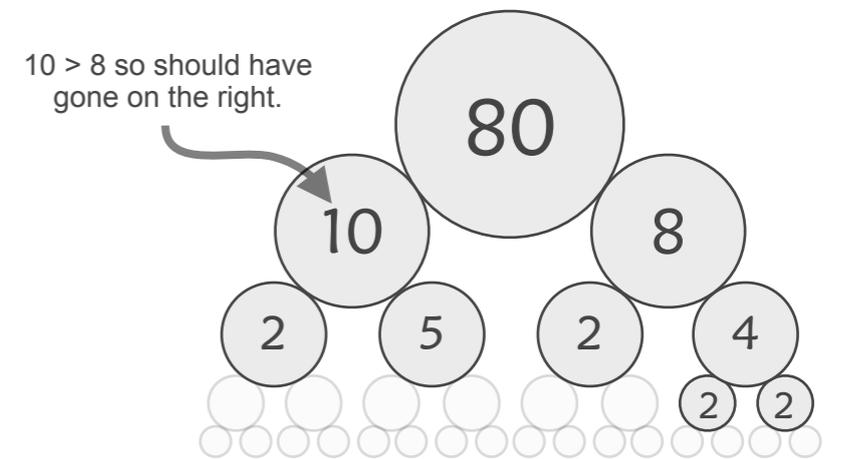
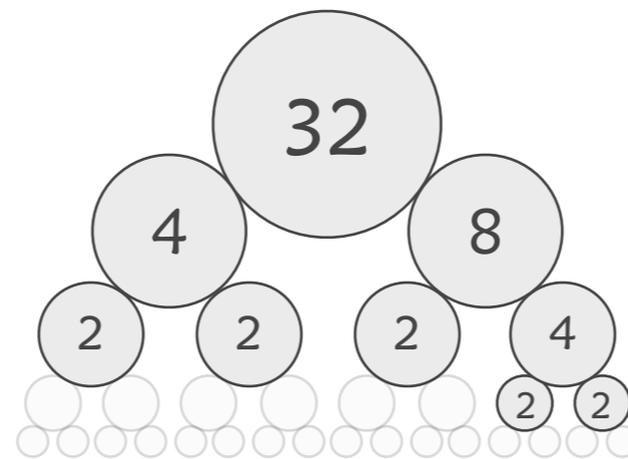


In the 21st century, numbers now dream of exploding in integral fission. There are two additional rules for integral fission:

- 1) Splits must be as equal as possible.
- 2) If one of the splits is bigger than the other, it must go on the right.

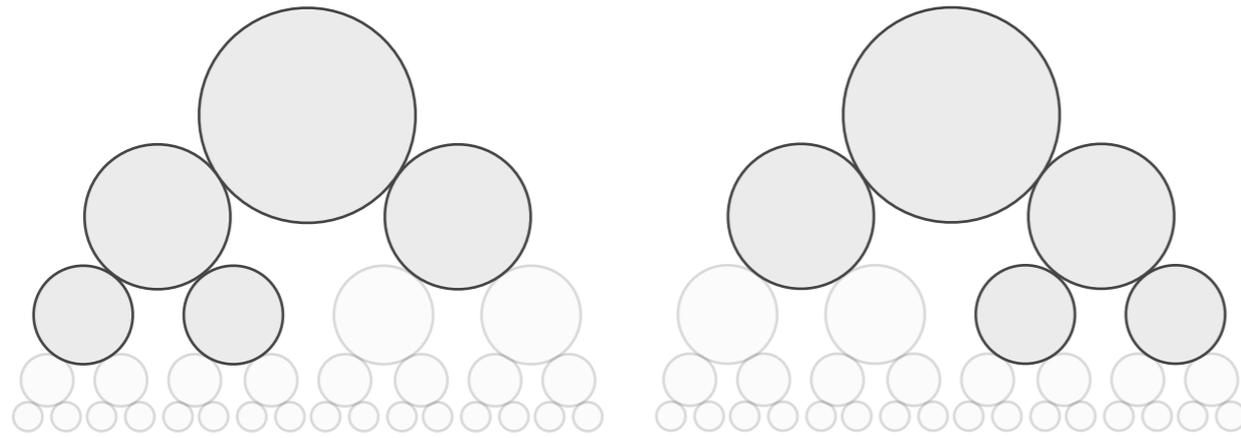


Unlike prime factorization trees, integral fission is unique for each positive integer. The “shape” of an integer is the pattern remaining when the numbers are removed. 32 is the smallest integer which shares the same shape as 120. Does 80 have the same shape? No.



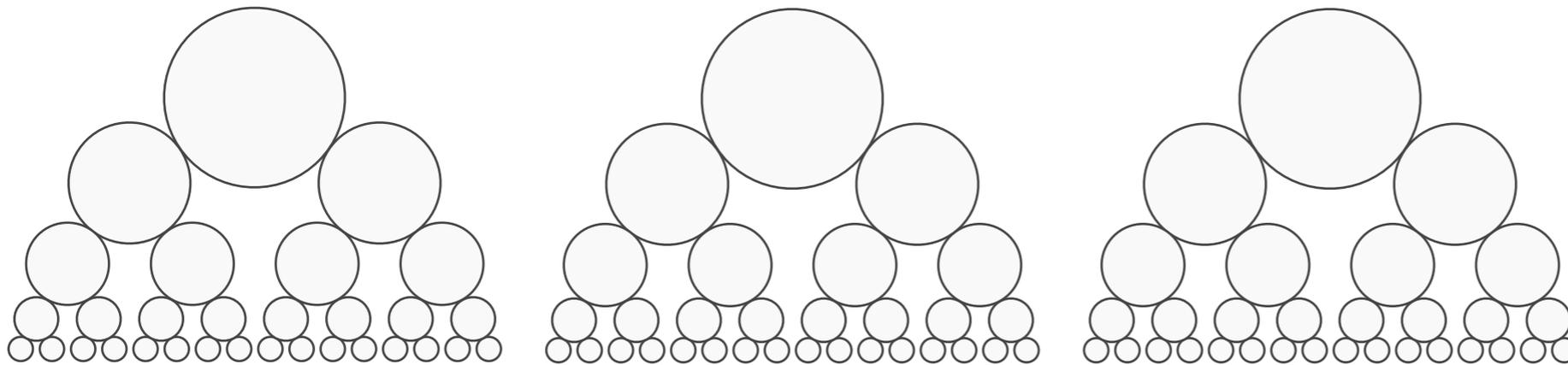
Some grade 5 / 6 students in River Valley School challenge you to find:

- 1) When the shape of the left first appears?  
 (The shape on the right first appears at 8.)

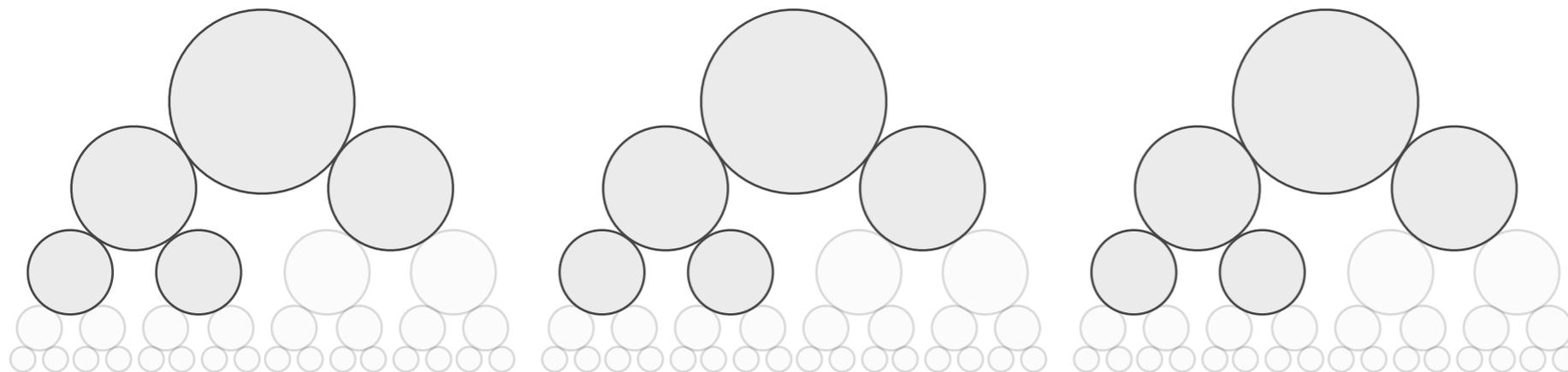


Bonus: Max wants you to find the first time an integer  $k$  has the shape on the left and  $k+1$  has the shape on the right.

- 2) Three consecutive integers between 2 and 50 that have the same shape.  
 (They're not telling you what the shape is)

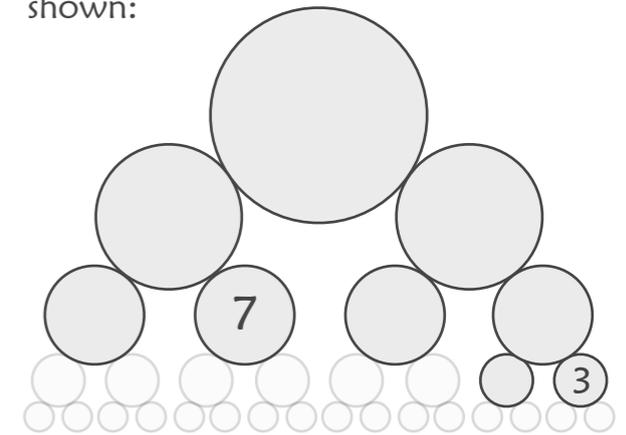


- 3) The first time a certain shape arises as a pair of consecutive integers is 116 and 117. Find the first time this shape arises in three consecutive integers (hint: it's between 160 and 180):

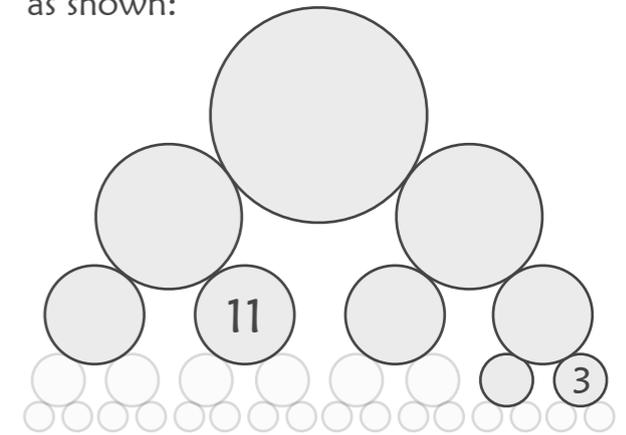


Mr. Pickle asks you to find:

- 1) The first 25 shapes to appear.  
 2) The five integers which fission in this pattern with the integers 7 & 3 as shown:



- 3) The five integers which fission in this pattern with the integers 11 & 3 as shown:



- 4) What shape do the integers 1431 and 1432 share?

- 5) What shape do the integers 1885, 1886 and 1887 share?

- 6) Is there a shape that appears for the first time associated with an odd integer? (I don't know - \$100 reward)

- 7) Is there an integer  $>1$  which is a cube and has a shape that has mirror symmetry? (I don't know - \$100 reward)