

## Rainbow Squares

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Long, long ago, before the mischievous Leprechauns had inksloshed their first proper rainbow - they filled the rainy skies of Ulster with helter-skelter arcs of color, celebrating drizzle as if it were liquid silver.

The rules for how they played have just now become understood. They first tried placing numbers 1-10 on the horizon and then paired up the numbers with great arcs of color. They won if all the pairs added to a square number. Your students might like to try this as their first leprechaun challenge. They, like the leprechauns will find it impossible, but that doesn't mean they shouldn't try!

$$
\begin{aligned}
1 \times 1 & =1 \\
2 \times 2 & =4 \\
3 \times 3 & =9 \\
4 \times 4 & =16 \\
5 \times 5 & =25 \\
6 \times 6 & =36 \\
7 \times 7 & =49 \\
8 \times 8 & =64 \\
9 \times 9 & =81 \\
10 \times 10 & =100
\end{aligned}
$$



You can see that all the arcs drawn thus far are correct $(1+8=9 ; 4+5=9 ; 6+10=16 ; 7+9=16$.) Unfortunately, the final pair ( 2 and 3 ) do not add up to a square number.

Next, the Leprechauns tried pairing up 1-11 which of course failed because that's an odd number of numbers. In fact everything they tried failed until they came to 1-14. That worked beautifully and should be solved by the whole class. The Leprechauns rejoiced with this discovery until they got bored and started to wonder about other possibilities.

If the class is not confident - try solving 1-6 (impossible), 1-8 (possible), 1-16 (possible), 1-18 (possible), and 1-20 (impossible).

If the class is confident hand out the 1-26 leprechaun puzzles on the next pages (they are ordered from easiest to hardest)...


After your students have solved the first worksheet set them loose in two directions.

1) Some of your students may like to challenge themselves to find ALL numbers $n$ for which 1-n can be solved. This was Henri Picciotto's original problem and it is great.
2) Continue with the puzzle sheets.

It is good to have two different directions for students to go so it doesn't become a race for those students who don't thrive on competition.






Much thanks to the French illustrator Godo for permission to use the magical little guy who brings life to this puzzle.
https://godoillustrateur.wordpress.com/

I'm a father of two elementary school children, a mathematician, and designer of puzzles and board games. Students call me Dr. Pickle. There is nothing I enjoy more than stumping students and having them stump me.

I founded MathPickle.com in 2010 to inject new ideas into the classroom. MathPickle's primary objective is to get thirteen curricular unsolved problems into classrooms worldwide - one for each grade K-12. A conference in November 2013 established the thirteen unsolved problems. To aid with the dissemination of these awesome problems, MathPickle is looking at setting up a $\$ 1,000,000$ reward for each - the prize money to be split between the person who solves the problem and their most inspirational K -12 educator.

MathPickle is also developing a range of curricular puzzles like the ones you'll find at TpT. These help teachers them with their number one challenge:
"How to engage the spectrum of student ability?"
Whenever an elementary school teacher wants to teach addition, she will invariably face $20 \%$ of students who already know how to add and another 20\% who are struggling with last year's curriculum. How can she engage the top students without losing the bottom students? How can she engage the bottom students without boring the top students?

One solution: Parents of top students often ask that their child be allowed to accelerate through the curriculum. This exacerbates the problem for future teachers, and sets up a failure-impoverished education experience for the bright student.

A wiser approach is to use curricular puzzles, games and minicompetitions to simultaneously teach curriculum to the students who need it, and to deflect top students into tough problem solving activities. This is never time wasted, because problem solving is the primary reason we teach mathematics.

The experience of mathematics should be profound and beautiful Too much of the regular K-12 mathematics experience is trite and true. Children deserve tough, beautiful puzzles.

Gordon Hamilton MMath, PhD


All six solutions to the 1-26 puzzle.
\#1: $\{1,15\},\{2,14\},\{3,22\},\{4,21\},\{5,20\},\{6,19\},\{7,18\},\{8,17\},\{9,16\},\{10,26\},\{11,25\},\{12,24\},\{13,23\}$ \#2: $\{1,15\},\{2,23\},\{3,13\},\{4,21\},\{5,20\},\{6,19\},\{7,18\},\{8,17\},\{9,16\},\{10,26\},\{11,25\},\{12,24\},\{14,22\}$ \#3: $\{1,15\},\{2,23\},\{3,22\},\{4,21\},\{5,20\},\{6,19\},\{7,18\},\{8,17\},\{9,16\},\{10,26\},\{11,14\},\{12,13\},\{24,25\}$ \#4: $\{1,24\},\{2,14\},\{3,22\},\{4,12\},\{5,20\},\{6,19\},\{7,18\},\{8,17\},\{9,16\},\{10,26\},\{11,25\},\{13,23\},\{15,21\}$ \#5: $\{1,24\},\{2,14\},\{3,22\},\{4,21\},\{5,20\},\{6,19\},\{7,18\},\{8,17\},\{9,16\},\{10,15\},\{11,25\},\{12,13\},\{23,26\}$ \#6: $\{1,24\},\{2,23\},\{3,13\},\{4,12\},\{5,20\},\{6,19\},\{7,18\},\{8,17\},\{9,16\},\{10,26\},\{11,25\},\{14,22\},\{15,21\}$

It is unknown how many solutions exist for the 1-60 puzzle - but there are a LOT! That does not make it easy to solve. The next page took me an hour to find...


