

# Unfair Thrones





The easy part is conquering the world.  
The hard part is ensuring a stable handover of  
power to your bickering children.

This puzzle was my first contribution to the  
New York Times puzzle section - [Numberplay](#).

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## Standards for Mathematical Practice

All MathPickle puzzle designs, including **Unfair Thrones** are guaranteed to engage a wide spectrum of student abilities while targeting the following Standards for Mathematical Practice:

**MP1 Toughen up!**

This is problem solving where our students develop grit and resiliency in the face of nasty, thorny problems. It is the most sought after skill for our students.

**MP3 Work together!**

This is collaborative problem solving in which students discuss their strategies to solve a problem and identify missteps in a failed solution. MathPickle recommends pairing up students for all its puzzles.

**MP6 Be precise!**

This is where our students learn to communicate using precise terminology. MathPickle encourages students not only to use the precise terms of others, but to invent and rigorously define their own terms.

**MP7 Be observant!**

One of the things that the human brain does very well is identify pattern. We sometimes do this too well and identify patterns that don't really exist.

## Common Core State Standards

**Unfair Thrones** target Common Core State Standards for grades 5-7. It is a difficult enough problem to engage students at any higher grade.

[CCSS.MATH.CONTENT.5.NF.A.1](#)

Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example,  $2/3 + 5/4 = 8/12 + 15/12 = 23/12$ .

[CCSS.MATH.CONTENT.5.NBT.A.3](#)

Read, write, and compare decimals to thousandths.

[CCSS.MATH.CONTENT.5.NBT.A.4](#)

Use place value understanding to round decimals to any place.

[CCSS.MATH.CONTENT.5.NBT.B.6](#)

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division.

[CCSS.MATH.CONTENT.7.NS.A.2.D](#)

Convert a rational number to a decimal using long division.



Bow to a student - and address her as the empress. Introduce her two bickering children and ask them to bring their thrones to the front and have a seat facing the class.

Above each of the two bickering children, draw a line and immediately ask another student to put a number 1-4 above one of the lines. The next student chooses a unused number 1-4 for underneath. Keep going (asking different students) until all four numbers have been placed.

If one of the fractions is improper (numerator larger than denominator), tell the students that this throne is upside-down. Erase all numbers and start again - asking different students.

*Do not tell the students that all the fractions they make must be proper. Let them discover this rule through failure. Telling rules at the start will bore some students. Failure because of lack of information is interesting and engaging. You should erase all the numbers even though some students will just want to flip the fraction. Why? Because it is a more dramatic failure. It is more catastrophic to have to start from scratch. That's good for students who didn't understand.*

When all fractions are proper, ask the whole class which is the most comfortable throne. They will invariably choose the larger fraction (At least this is true for every class I've gone into ;-)

Ask the bickering child on the less comfortable throne how that makes him or her feel. Hint at the fact that they could consider civil war to right this injustice.

Now announce to the class that the chance of civil war is equal to the most comfortable throne minus the least comfortable throne. In the example below the calculation is  $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$  or 25%

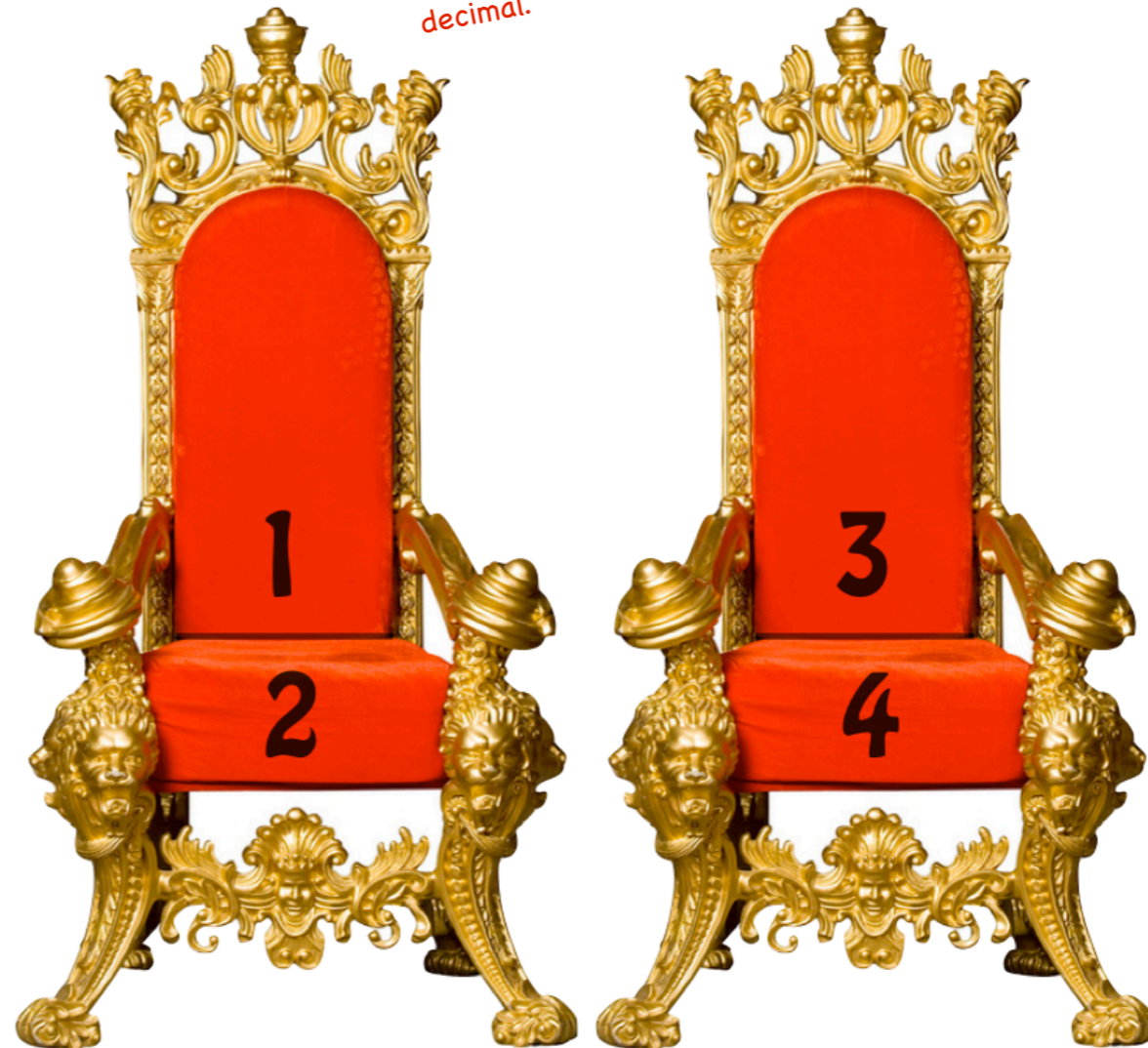
*Depending on your curricular goal, you can choose not to make the conversion to a decimal.*

Ask the empress and her class to find a way to minimize the chance of civil war. The best possible is  $\frac{2}{4} - \frac{1}{3} = \frac{1}{6}$  or about 17%.

Whenever the class solves this (in pairs or together as a whole class)... congratulate the empress on a NEW BABY BOY!!!

People cheer and a new throne and student is taken to the front of the class.

Now the task is to place the numbers 1-6 to form three thrones (three fractions) so that the biggest minus the least is as small as possible to reduce the chance of civil war.



Again, go around the class asking different students to choose a number 1-6 (I do not have hands raised because I want to engage all students through contributing.)

Below is a typical solution. Which is the most comfortable throne? Katrina answers: 4/6. You ask if it is not 3/5?

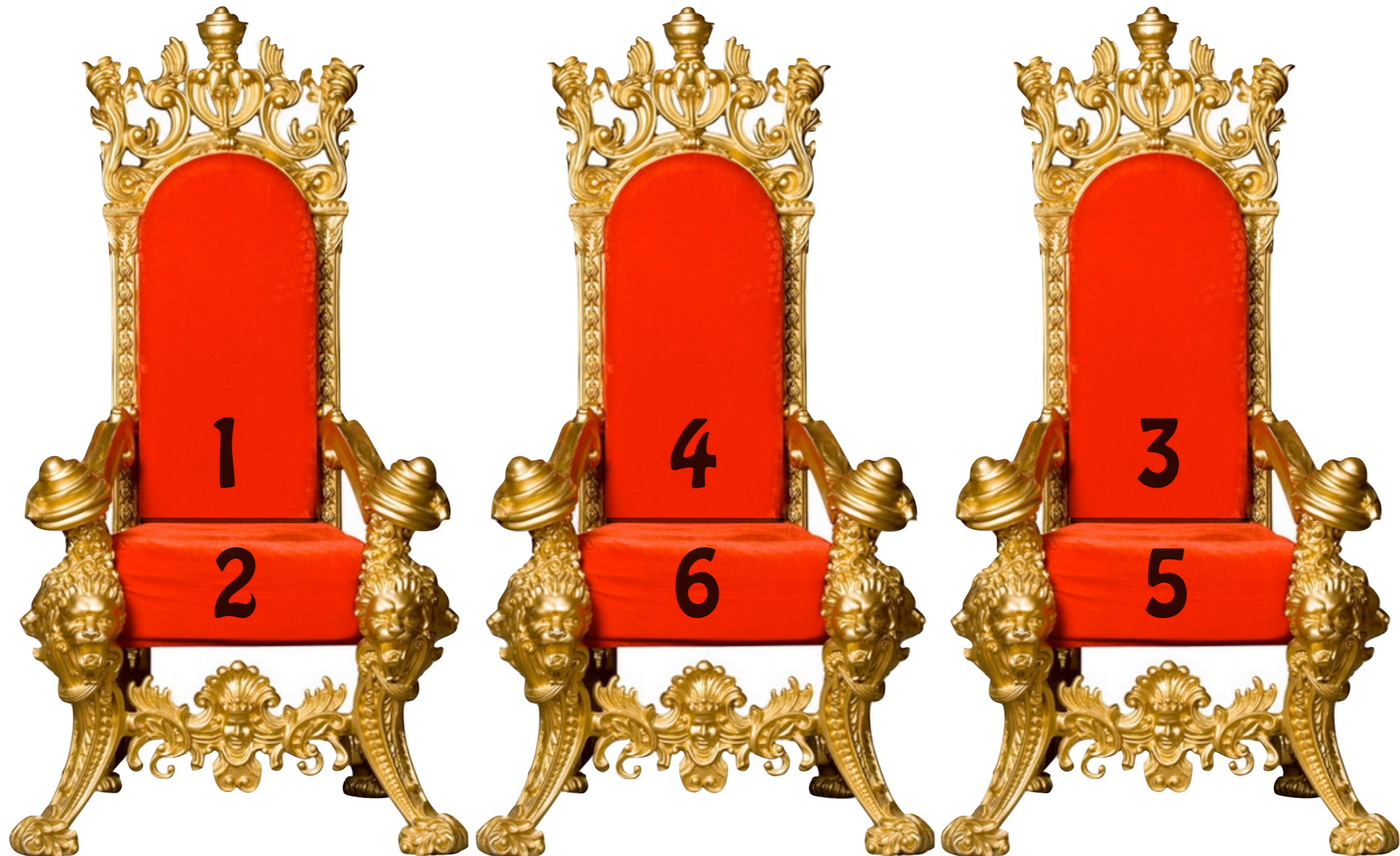
To figure out which is bigger... show the class that they can multiply  $3/5$  by  $6/6$  (you can multiply anything by 1 without changing its value) to give  $18/30$ .

Multiplying  $4/6$  by  $5/5$  (again you can multiply anything by 1) to give  $20/30$ .

$3/5 = 18/30 < 20/30 = 4/6$  so yes Katrina, you were right.

The chance of civil war is therefore  $4/6 - 1/2 = 1/6$  which is optimal. (most classes will not find this quickly)

If the students have not solved the puzzle optimally, let them try as a class or release them into smaller groups who can compete against each other for the smallest chance of civil war.



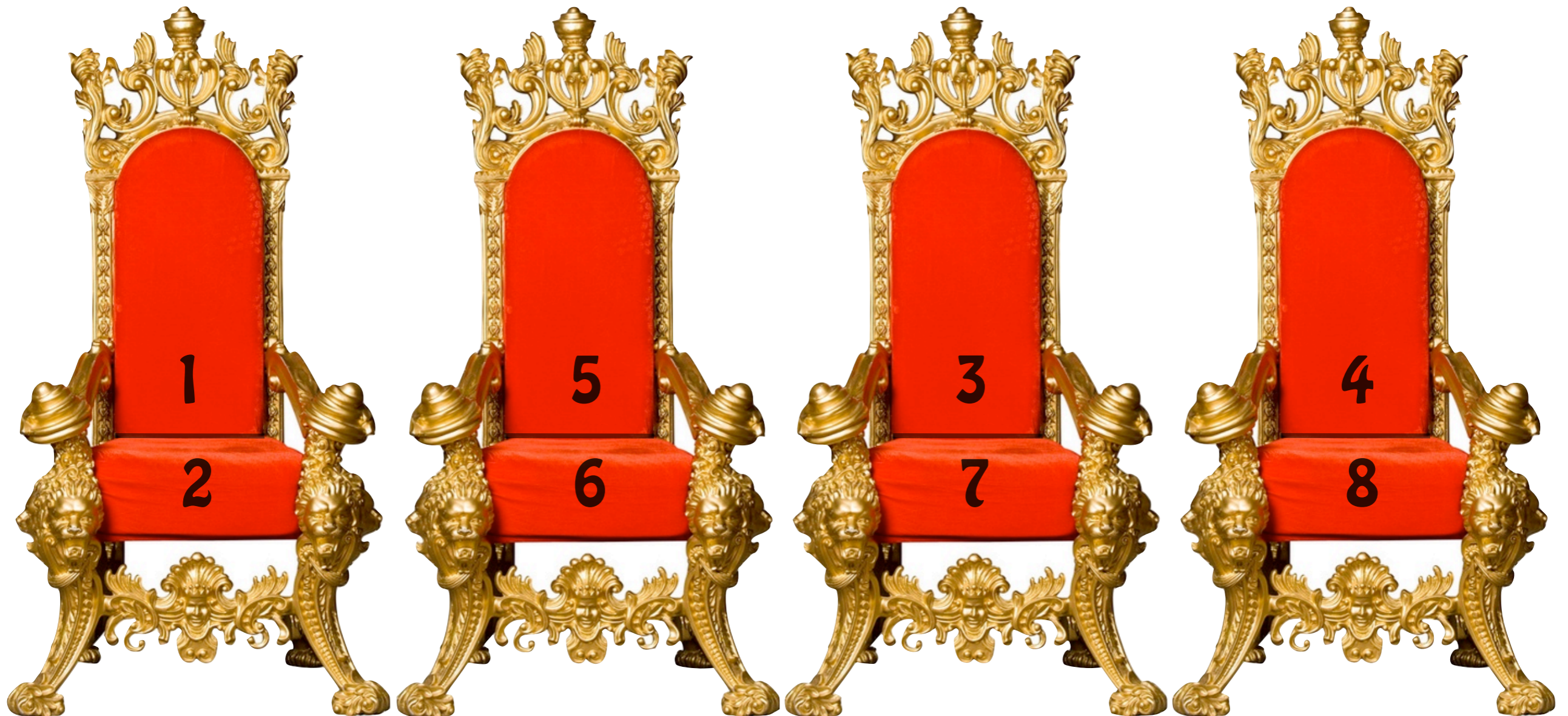


Any time a group solves it, you can do one of the following:

- 1) call the class together and collectively move on to tackle more bickering children.
- 2) quietly add another bickering child for that group alone.
- 3) Ask them if they are SURE (subtly hinting that their might be a better answer even though there is not.)
- 4) Ask them to experiment with a different 6 positive integers... what positive integers  $\leq 20$  could you choose to get a zero chance of civil war? ... what positive integers  $\leq 20$  could you choose to sabotage the country?

Each new birth makes the problem more difficult. With  $n$  births you must create  $n$  fractions using the integers  $\{1, 2, 3... 2n\}$ . After I designed this puzzle while sitting in my car one day, I could not intuit whether increasing the number of thrones would tend to increase or decrease the chance of civil war. Interested students can see the interesting submissions to the [New York Times](#) which show results for a large number of thrones.

Hope you like using Unfair Thrones in your classroom. Please contact me with any comments or stories. My email is [gord@mathpickle.com](mailto:gord@mathpickle.com).





## **\$100 Unfair Thrones Challenge**

Perhaps your students have found a pattern, or a student who is not usually excited about math has become engaged. Perhaps a pair of students discovered a general rule that allows them to predict what happens for a large number of thrones... Whatever your inspirational experience with this gem of a puzzle, I'd like to know.

I'll offer \$100 for a photograph and/or story highlighting Unfair Thrones in the classroom.

All students featured must have appropriate consent. All photographs and stories submitted may be used in an updated version of this pdf file and to promote this puzzle elsewhere.

Send submissions to [gord@mathpickle.com](mailto:gord@mathpickle.com). Use "\$100 Klingon challenge" as the subject of the email. The winning classroom will be announced the first March 14th that I have at least 10 submissions from different classrooms. I hope this will be March 14th, 2016.

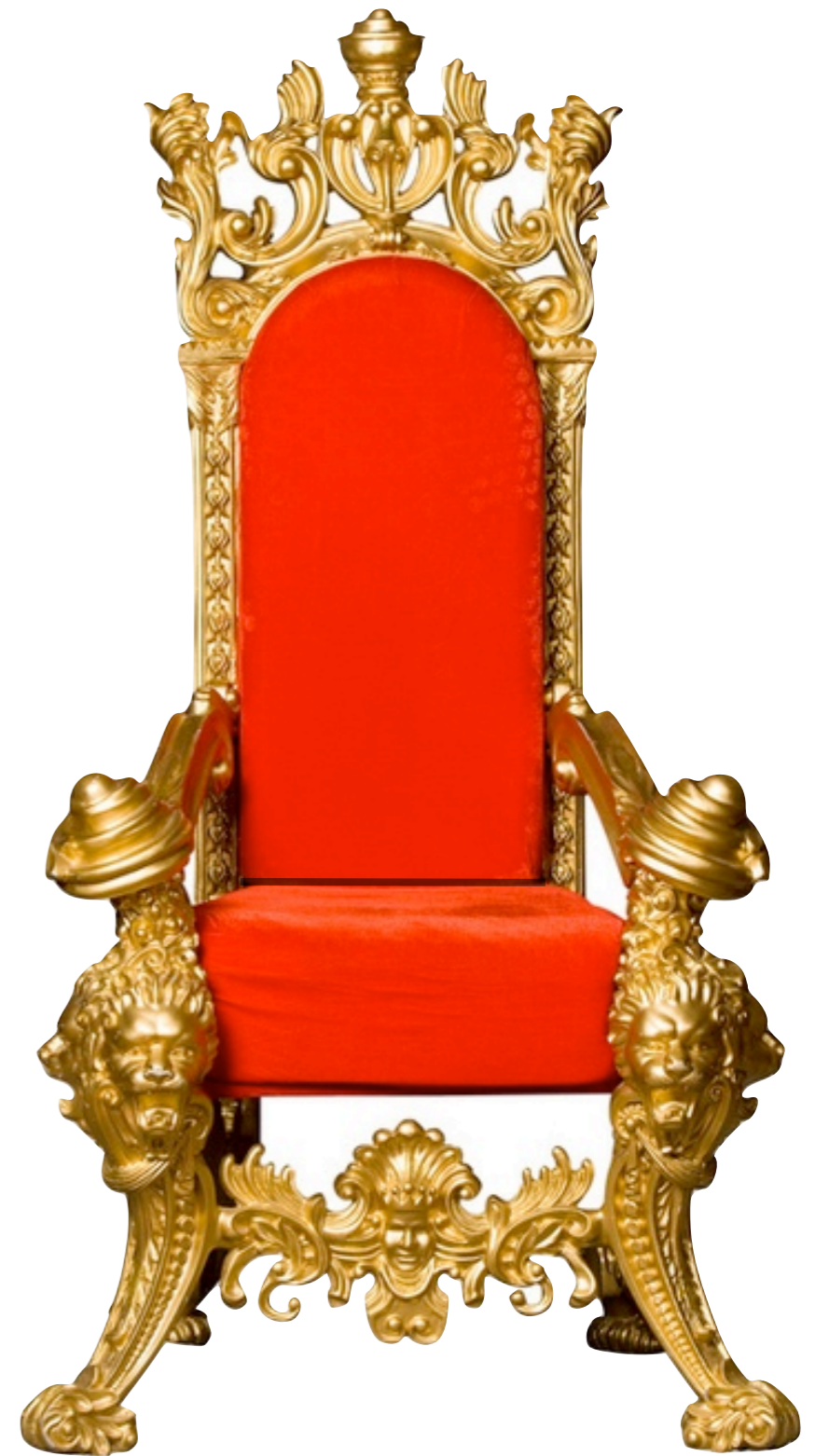
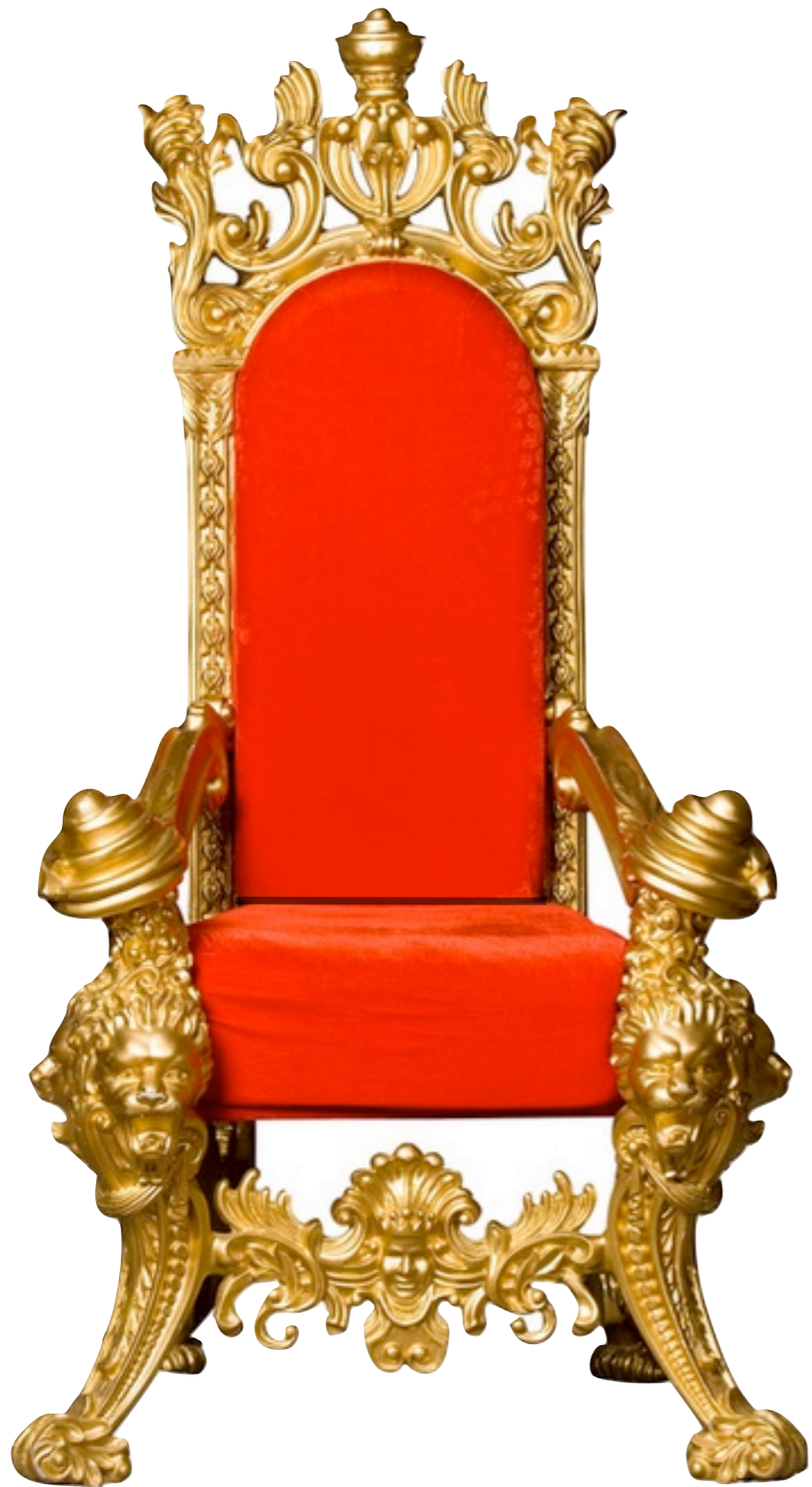


**Alexandra of Denmark 1844 - 1925**





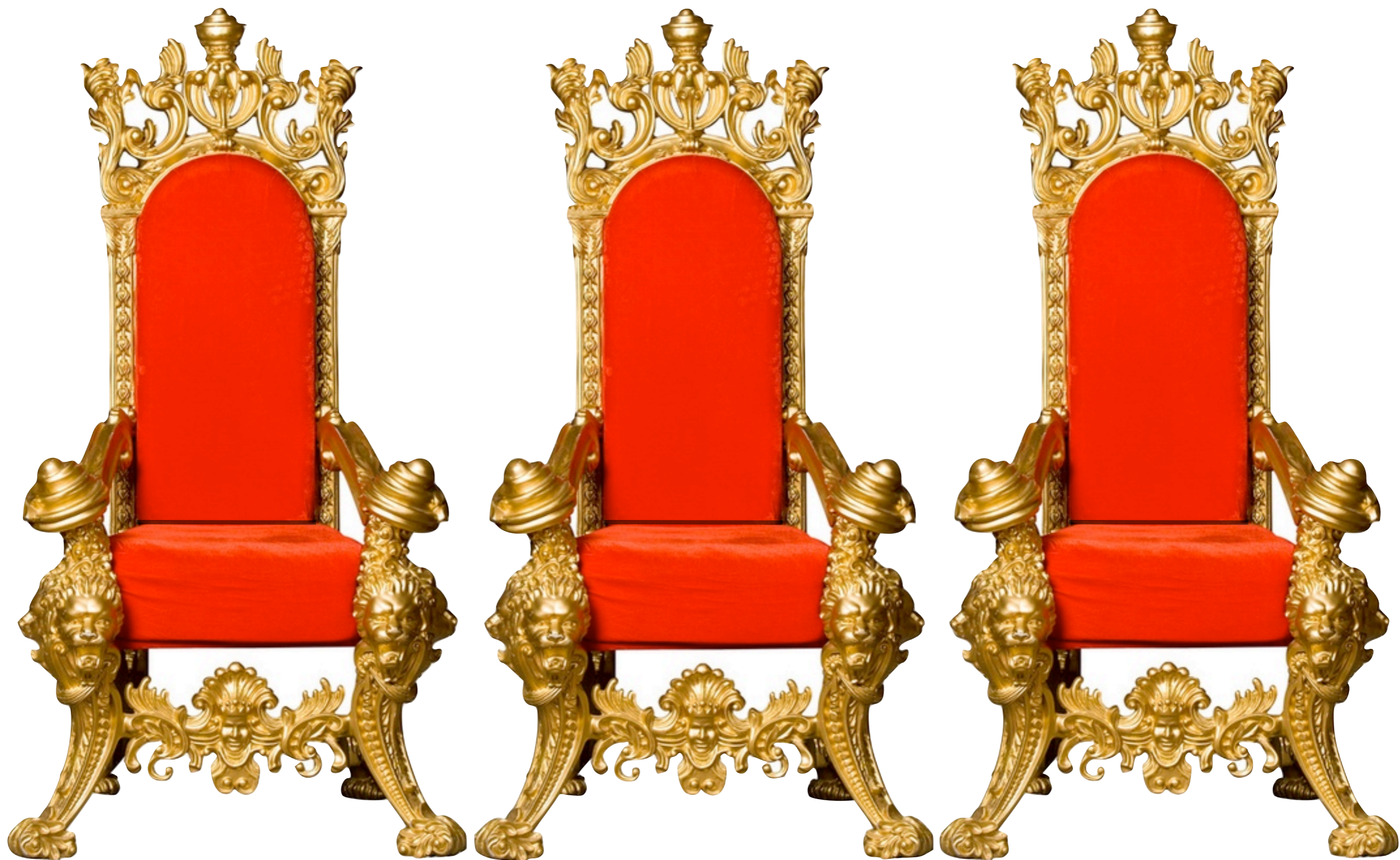




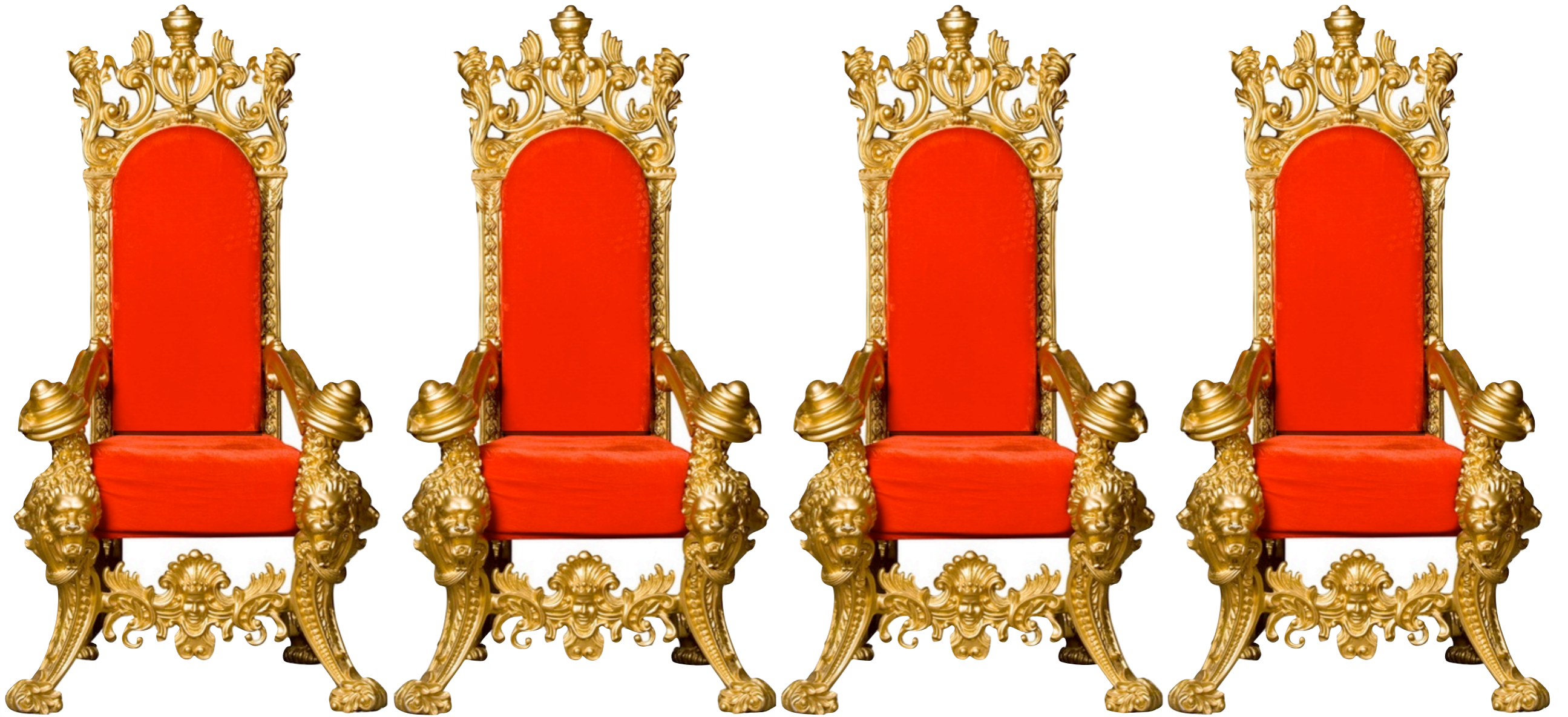


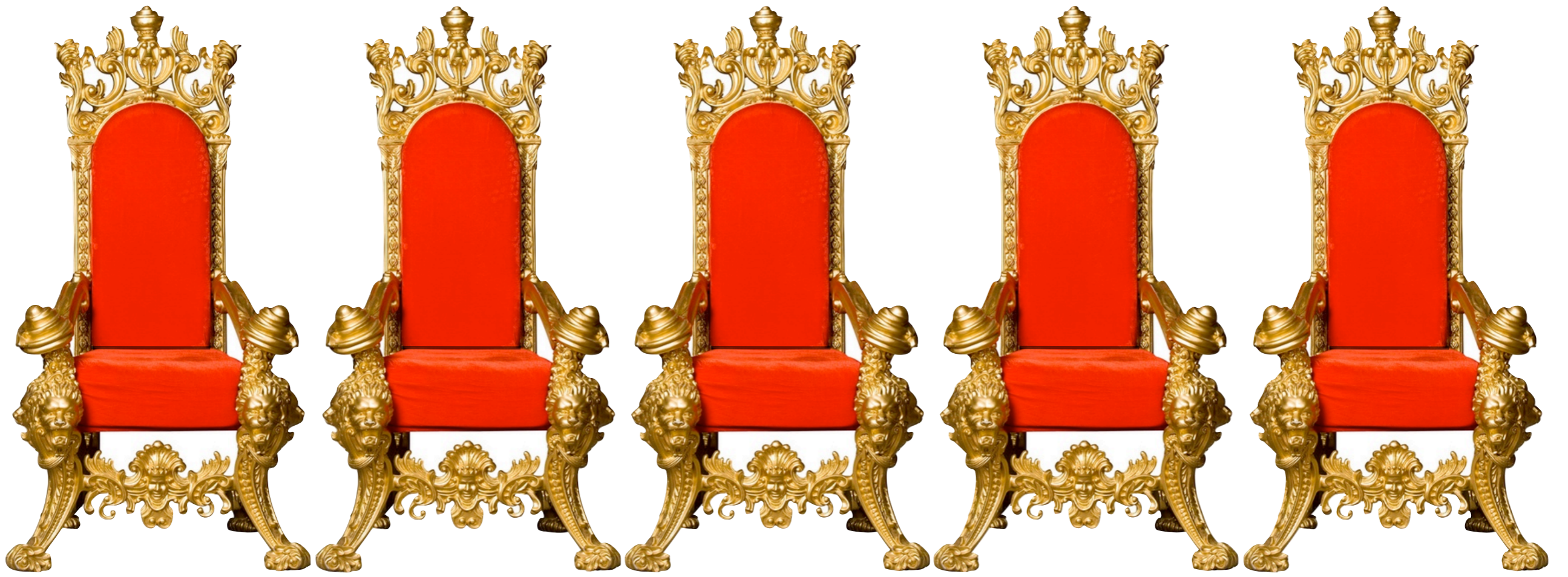






















answers

# Lowest chance of civil war

	$1/6$	16.6%
	$1/6$	16.6%
	$1/8$	12.5%
	$6/35$	17.1%
	$5/24$	21.5%
	$3/14$	21.4%
	$3/16$	18.8%
	$1/6$	16.6%



## Put Your Students in a Pickle!

I'm a father of two elementary school children, a mathematician, and designer of puzzles and board games. Students call me Dr. Pickle. There is nothing I enjoy more than stumping students and having them stump me.

I founded MathPickle.com in 2010 to inject new ideas into the classroom. MathPickle's primary objective is to get thirteen curricular unsolved problems into classrooms worldwide - one for each grade K-12. A conference in November 2013 established the thirteen unsolved problems. To aid with the dissemination of these awesome problems, MathPickle is looking at setting up a \$1,000,000 reward for each - the prize money to be split between the person who solves the problem and their most inspirational K-12 educator.

MathPickle is also developing a range of curricular puzzles like the ones you'll find at TpT. These help teachers them with their number one challenge:

“How to engage the spectrum of student ability?”

Whenever an elementary school teacher wants to teach addition, she will invariably face 20% of students who already know how to add and another 20% who are struggling with last year's curriculum. How can she engage the top students without losing the bottom students? How can she engage the bottom students without boring the top students?

One solution: Parents of top students often ask that their child be allowed to accelerate through the curriculum. This exacerbates the problem for future teachers, and sets up a failure-impooverished education experience for the bright student.

A wiser approach is to use curricular puzzles, games and mini-competitions to simultaneously teach curriculum to the students who need it, and to deflect top students into tough problem solving activities. This is never time wasted, because problem solving is the primary reason we teach mathematics.

The experience of mathematics should be profound and beautiful. Too much of the regular K-12 mathematics experience is trite and true. Children deserve tough, beautiful puzzles.

Gordon Hamilton  
MMath, PhD

