Algebra on Rectangles

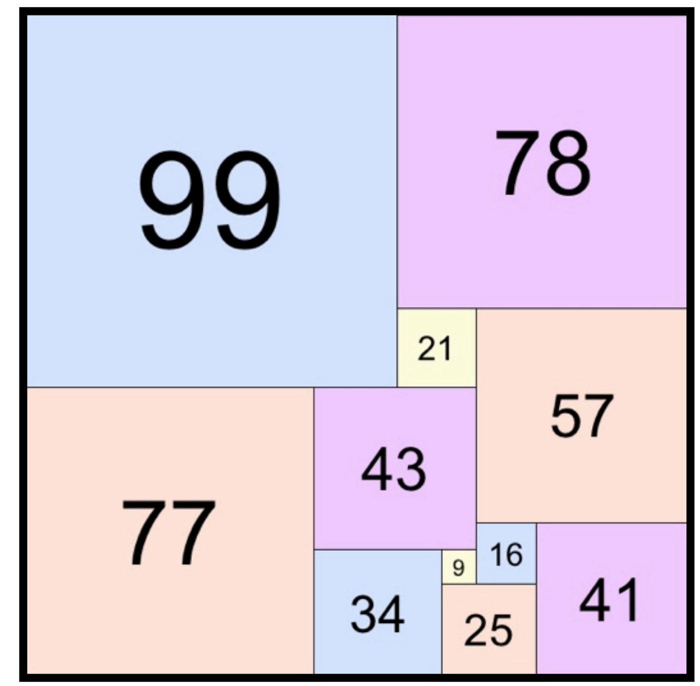
Roland Brooks

Arthur Stone

Cedric Smith

William Tutte

The quest for a square that could be tiled with smaller squares of all different sizes started with the discovery that it was possible to tile rectangles with squares.

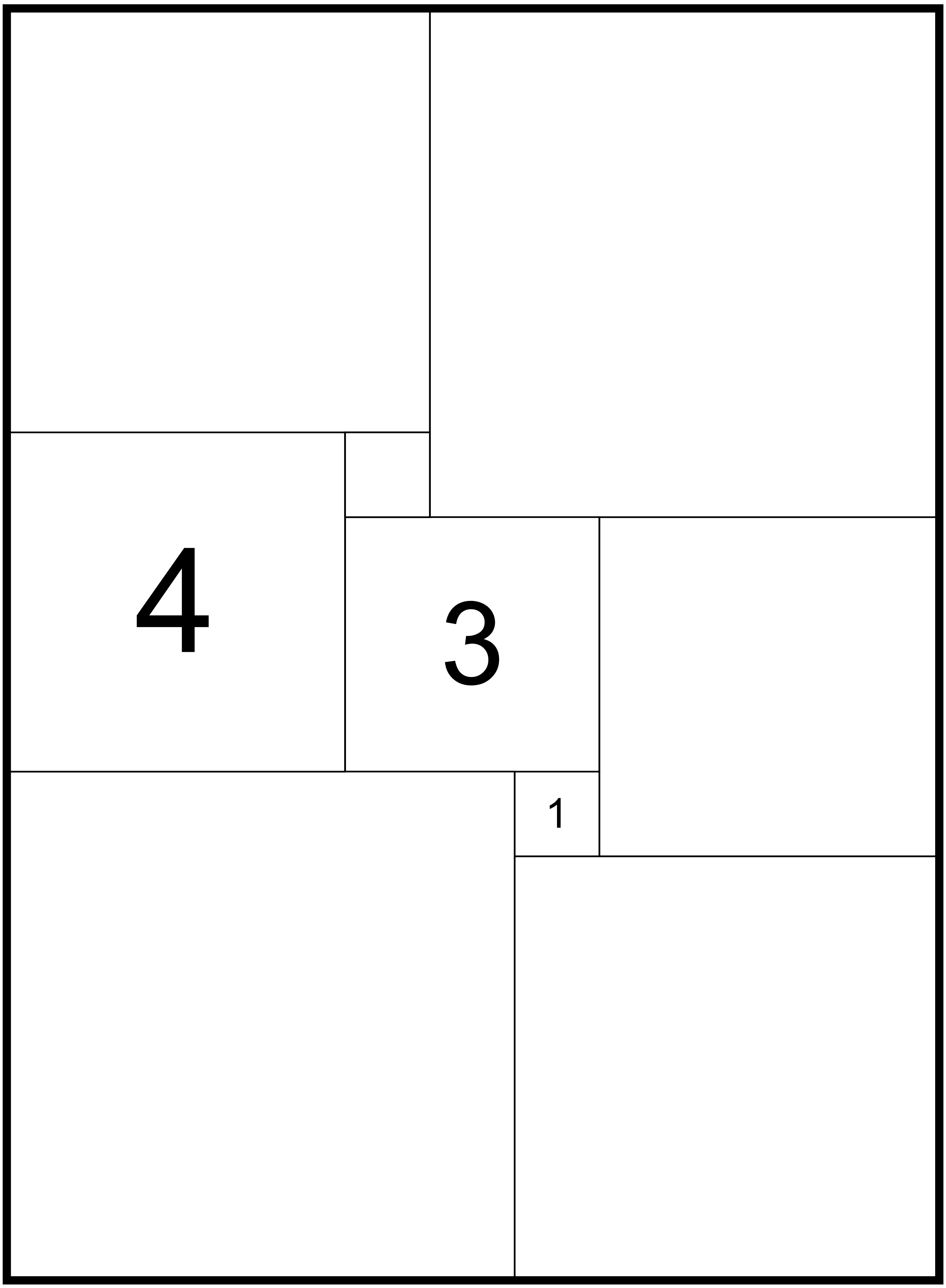


The square with a 21 in it means that this is a 21 by 21 square. This rectangle tiling is great, but imagine if a square could be found that could be tiled with little squares - all of different sizes.

That's what the Cambridge University gentlemen on the left were driven to discover in the 1930s. I'm keeping it a secret if they were successful or not, but during their hunt, they did discover many rectangular tilings. These will be the basis for the puzzles and problems at this station. squaring.net is a great place for the keen student to explore further.

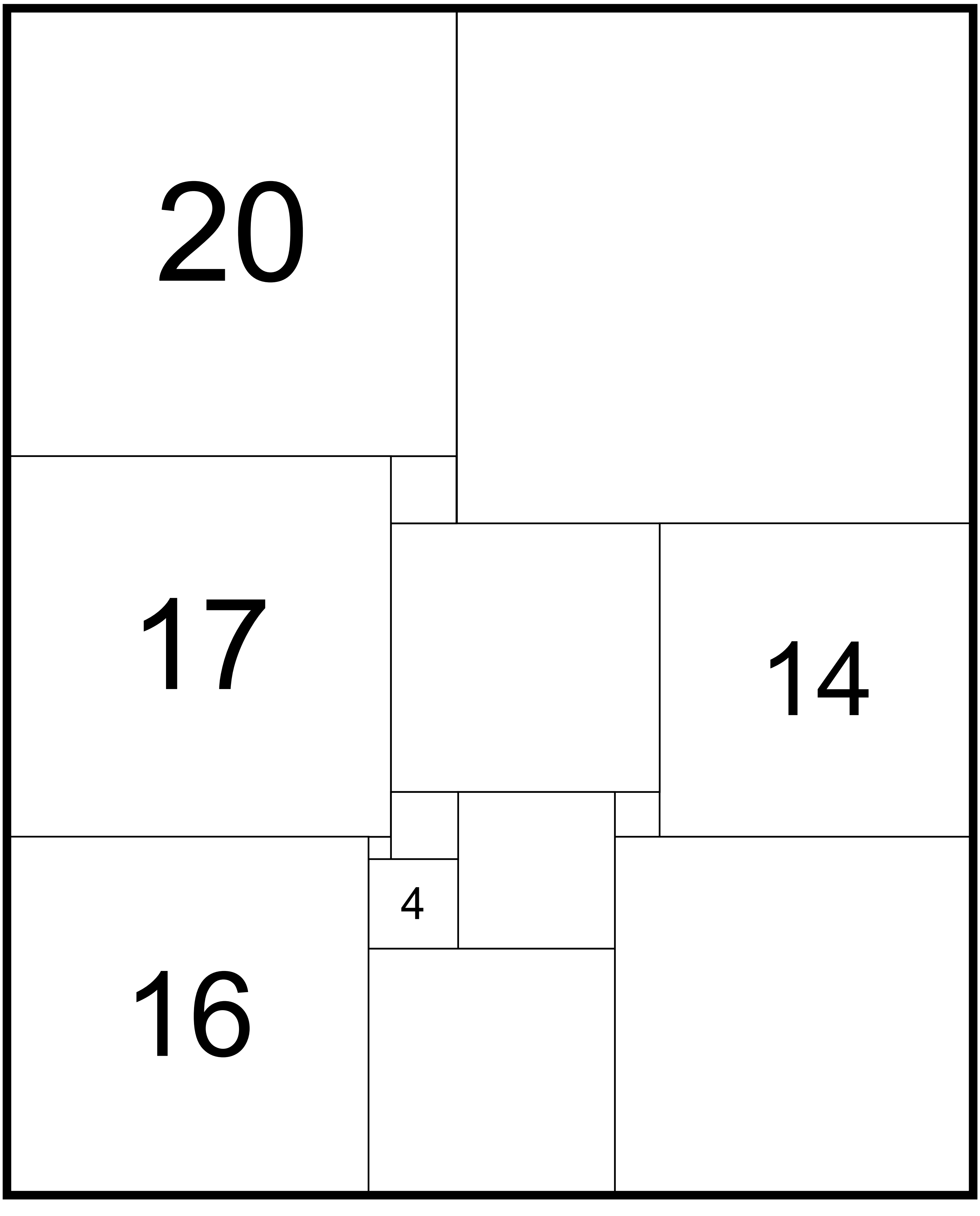
Challenge 1

This big rectangle has been tiled by squares. In the middle there is a 3 by 3 square, and there is also a 4 by 4 and little 1 by 1. What sizes are the other squares? If two squares have the same size, color them the same color. Hopefully, similar colors will not touch.



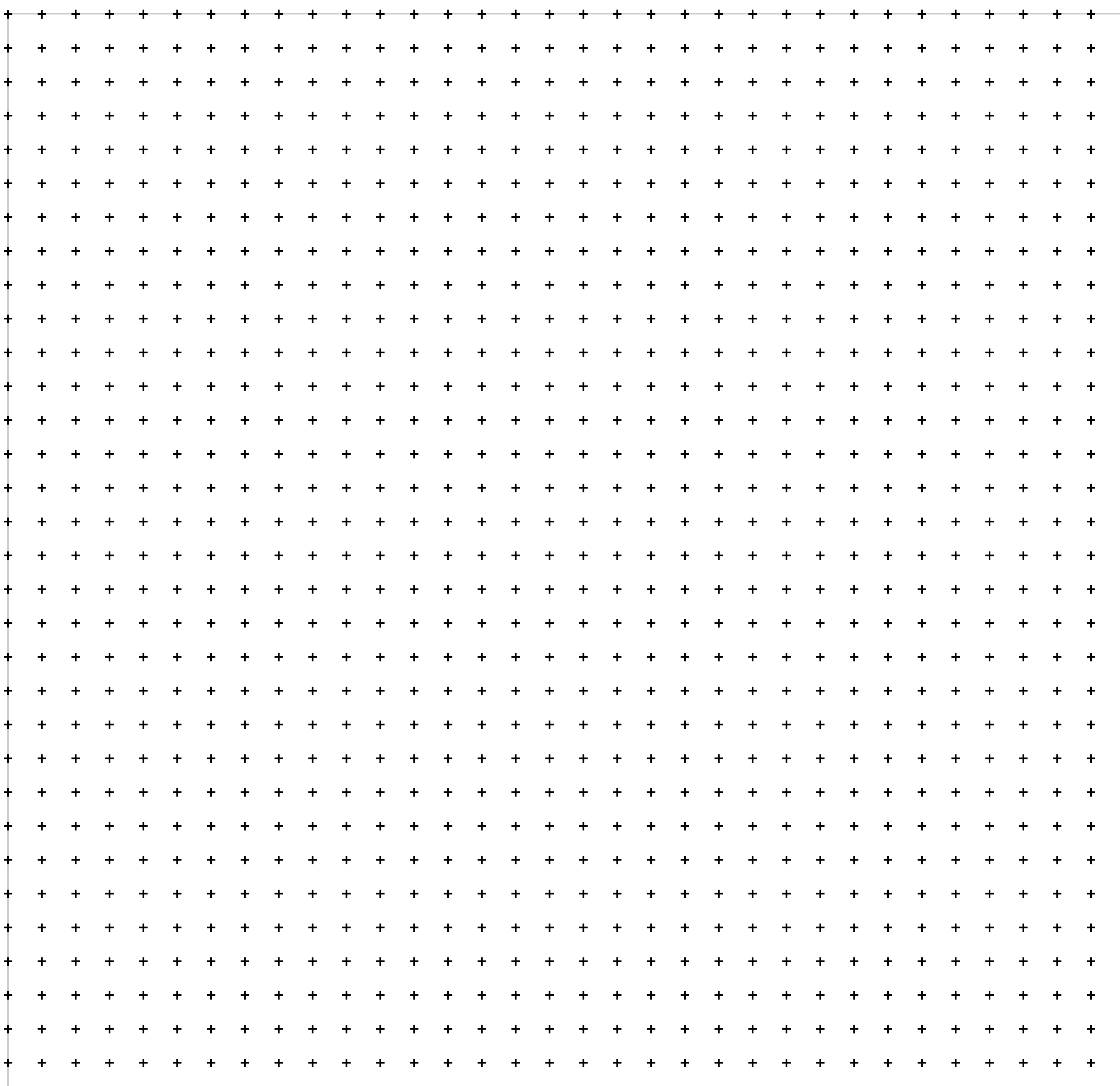
Challenge 2

Just like the last rectangle, this one has been tiled by squares. Hopefully this time, no two will be the same size. If two are the same size, we'll just give up and go on to the next challenge.



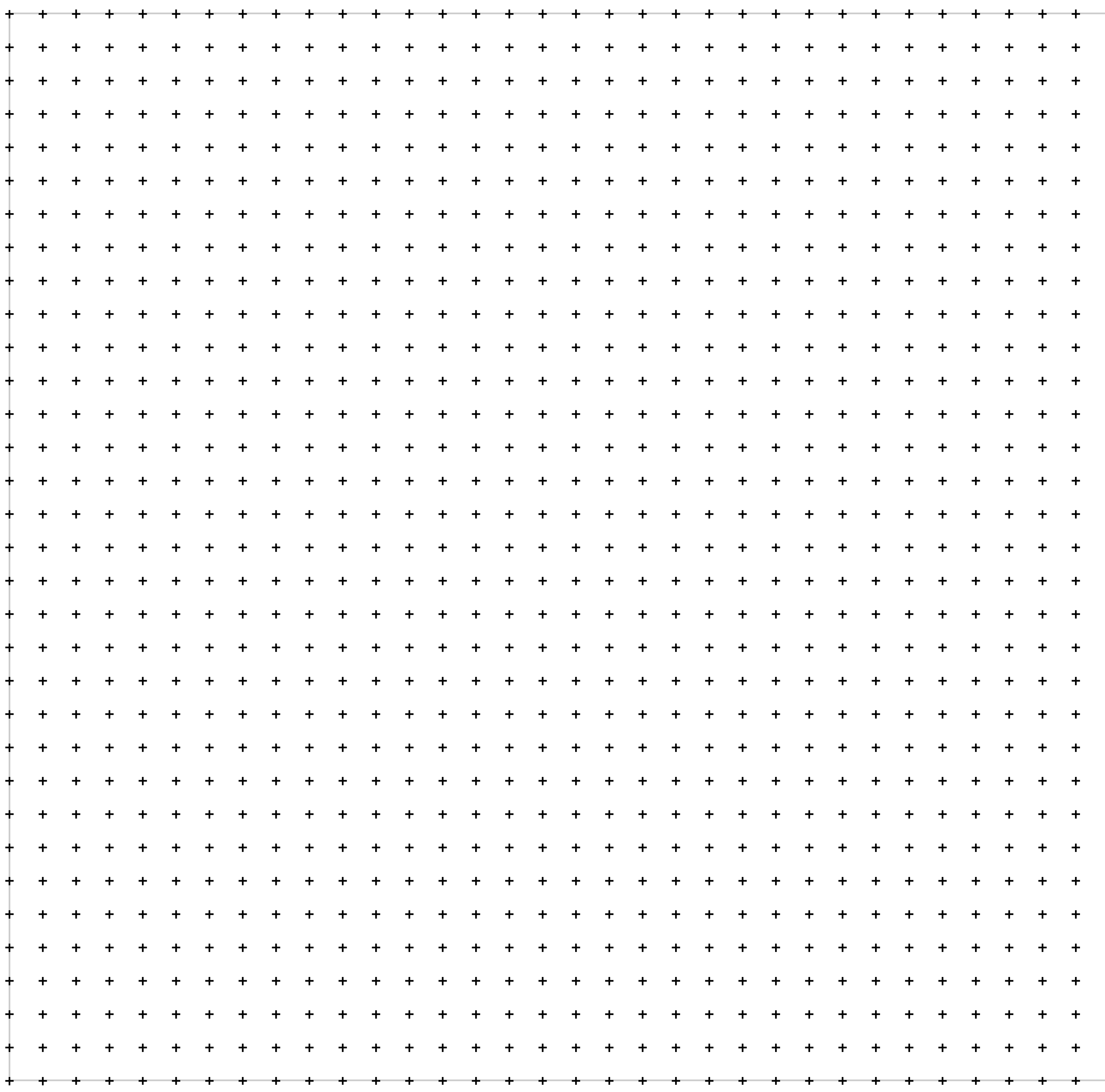
Challenge 3

That was nice, but we really want to tile using different sizes of squares. Try to tile a rectangle using all different sized squares. This 32 by 33 rectangle can be, but it is difficult. If you fail at this task, don't worry, we'll find a way to solve it in a later challenge.



Algebra on Rectangles

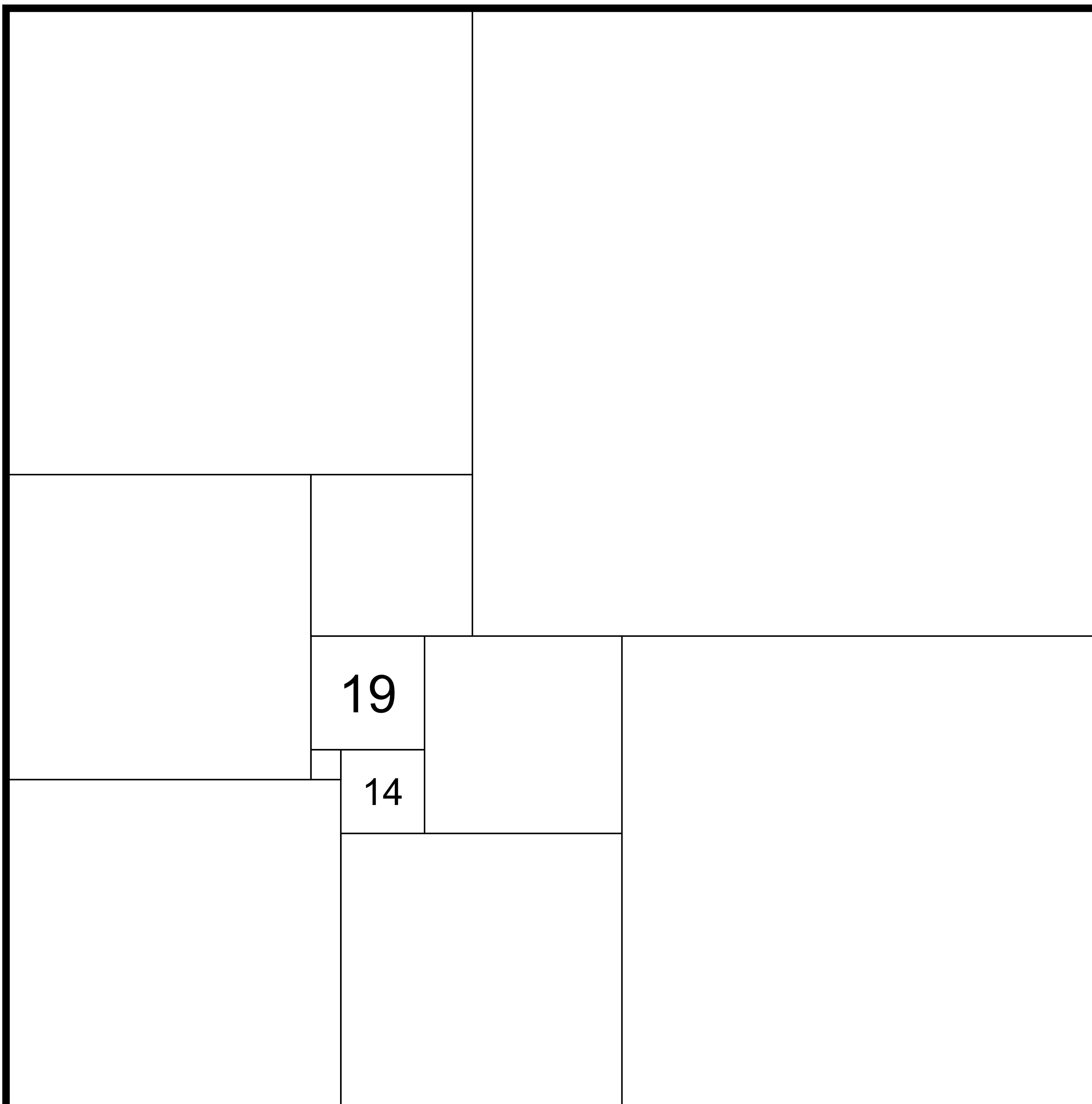
Cover this 32 x 33 rectangle using different sizes of non-overlapping squares.



Challenge 4

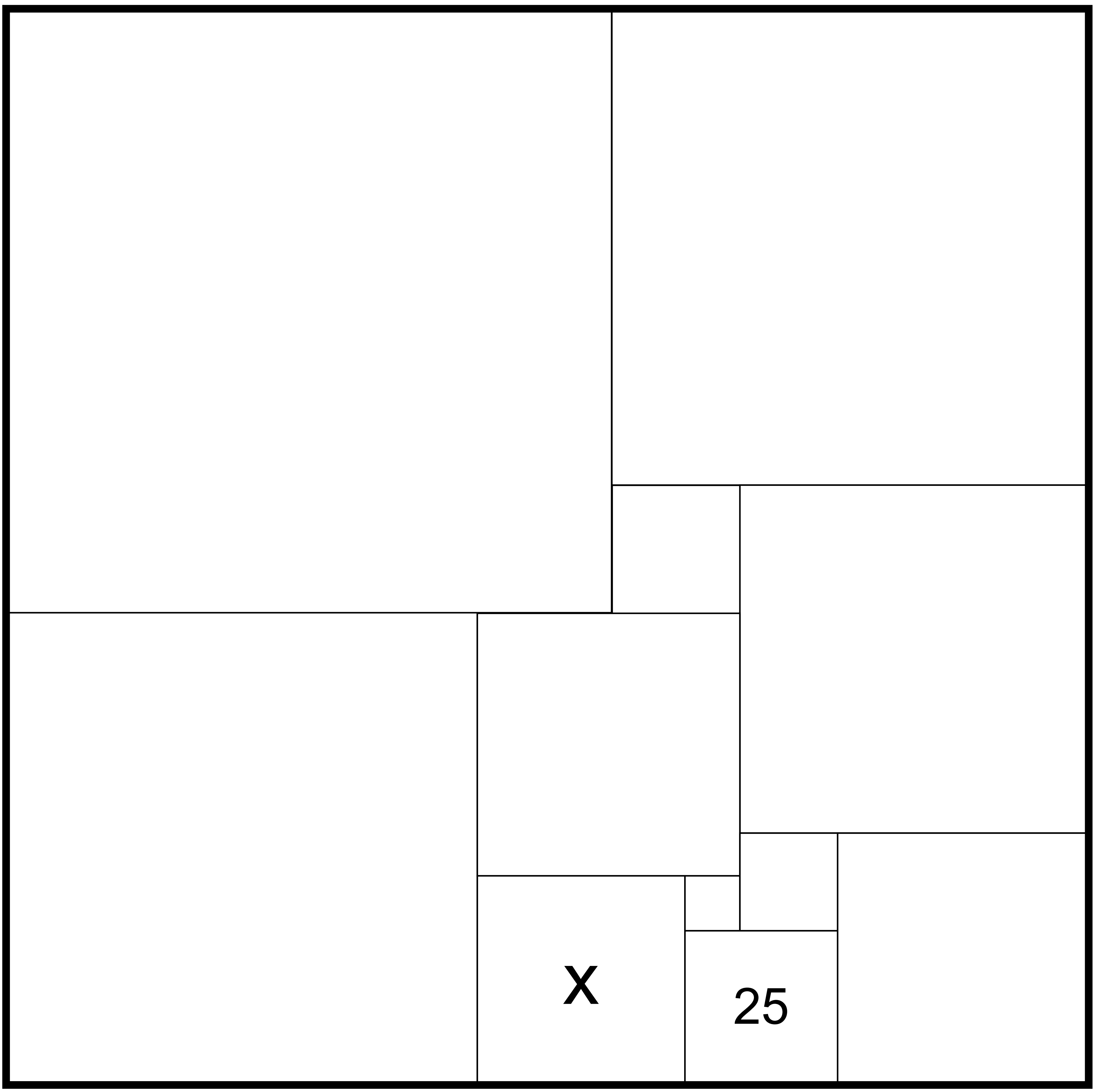
This big rectangle is nearly a square, but not quite. It looks like it has been successfully tiled with eleven squares of different sizes including a 14 by 14 and a 19 by 19 square.

Find the sizes of all the other squares.



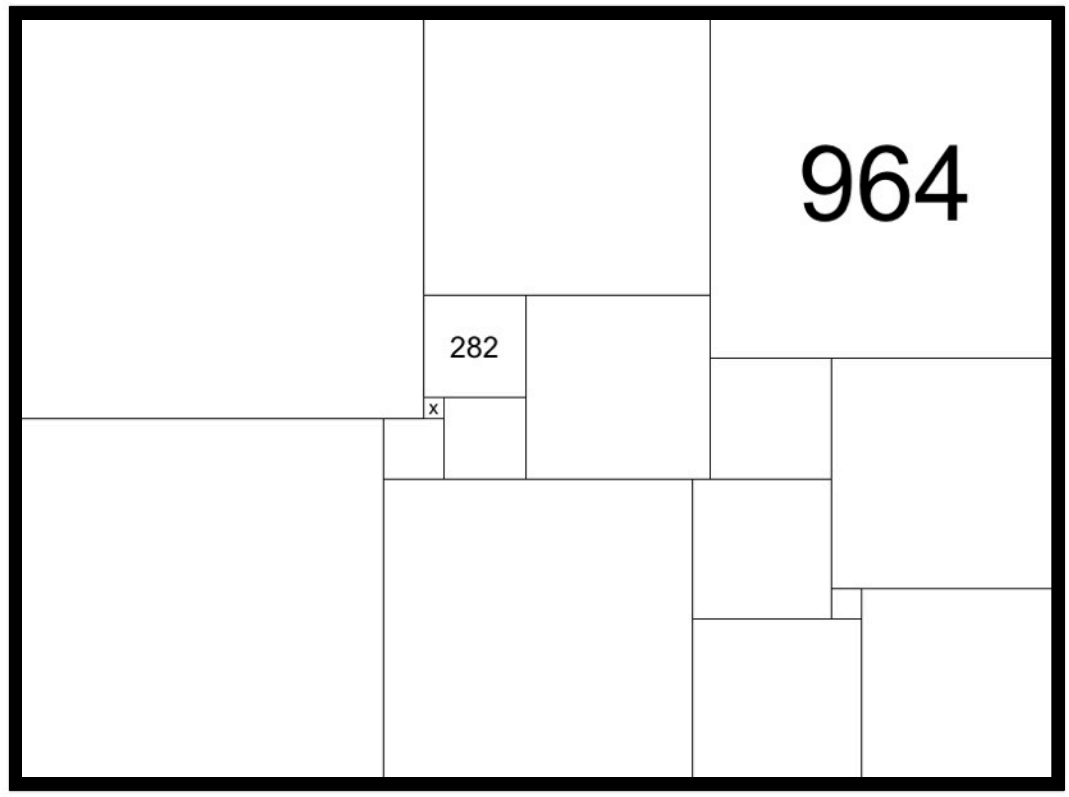
Challenge 5

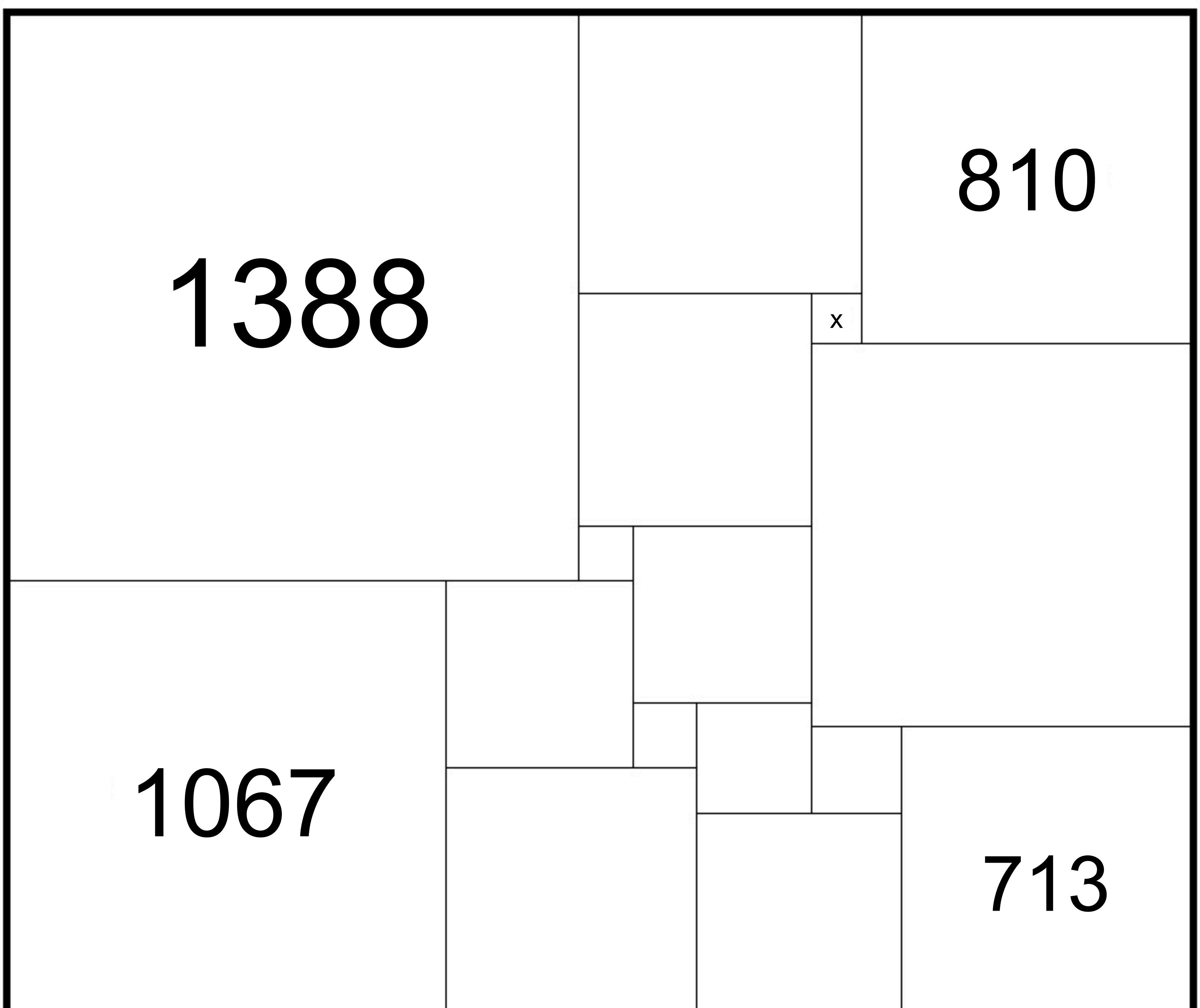
This is another rectangular tiling, but this time we are given fewer clues. Use the 25 by 25 square to figure out the dimensions of all the other squares - and the value of x in particular.



Challenge 6

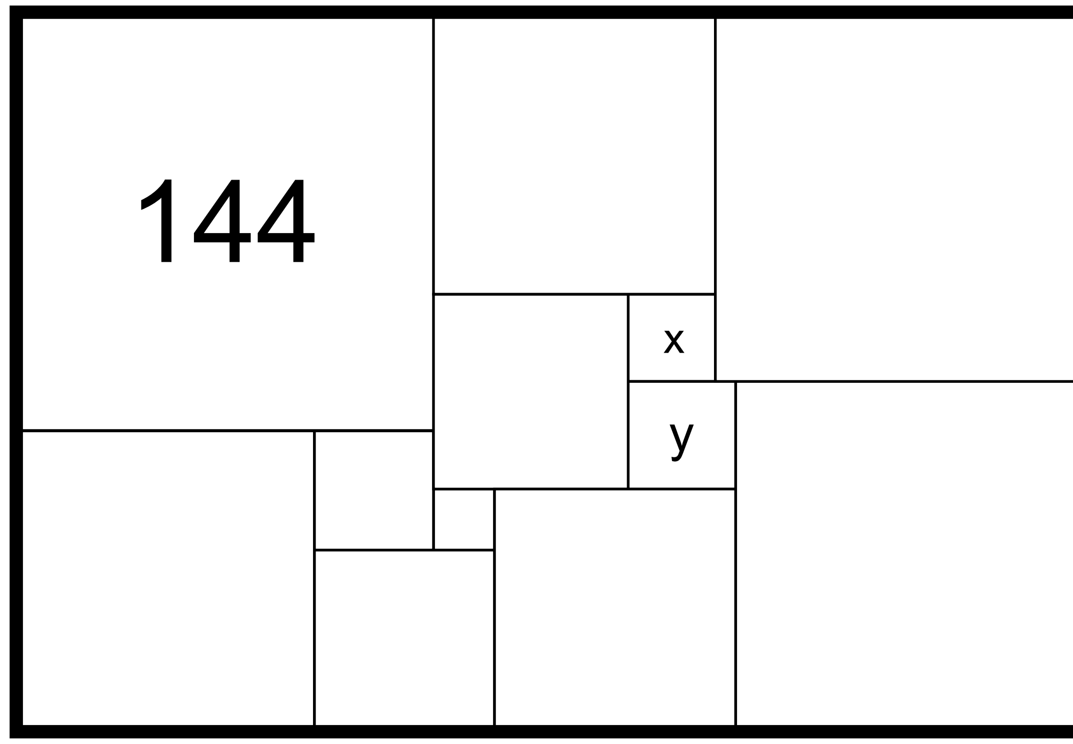
These are bigger numbers, but the challenge remains the same - find the dimensions of all squares.

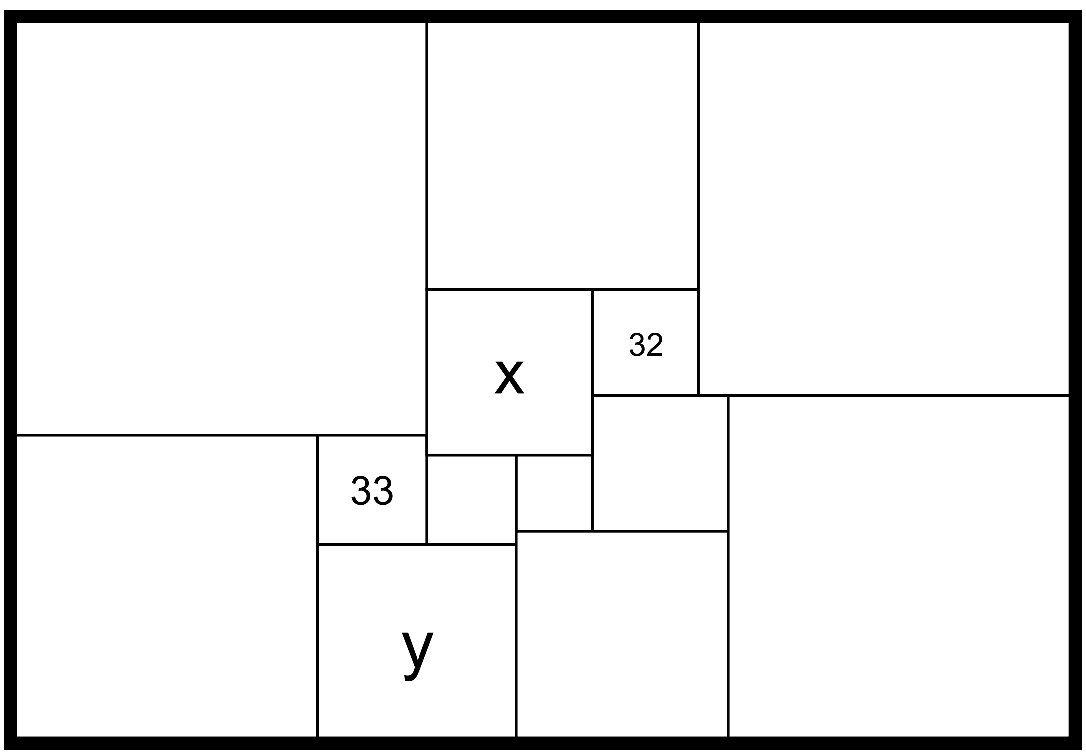


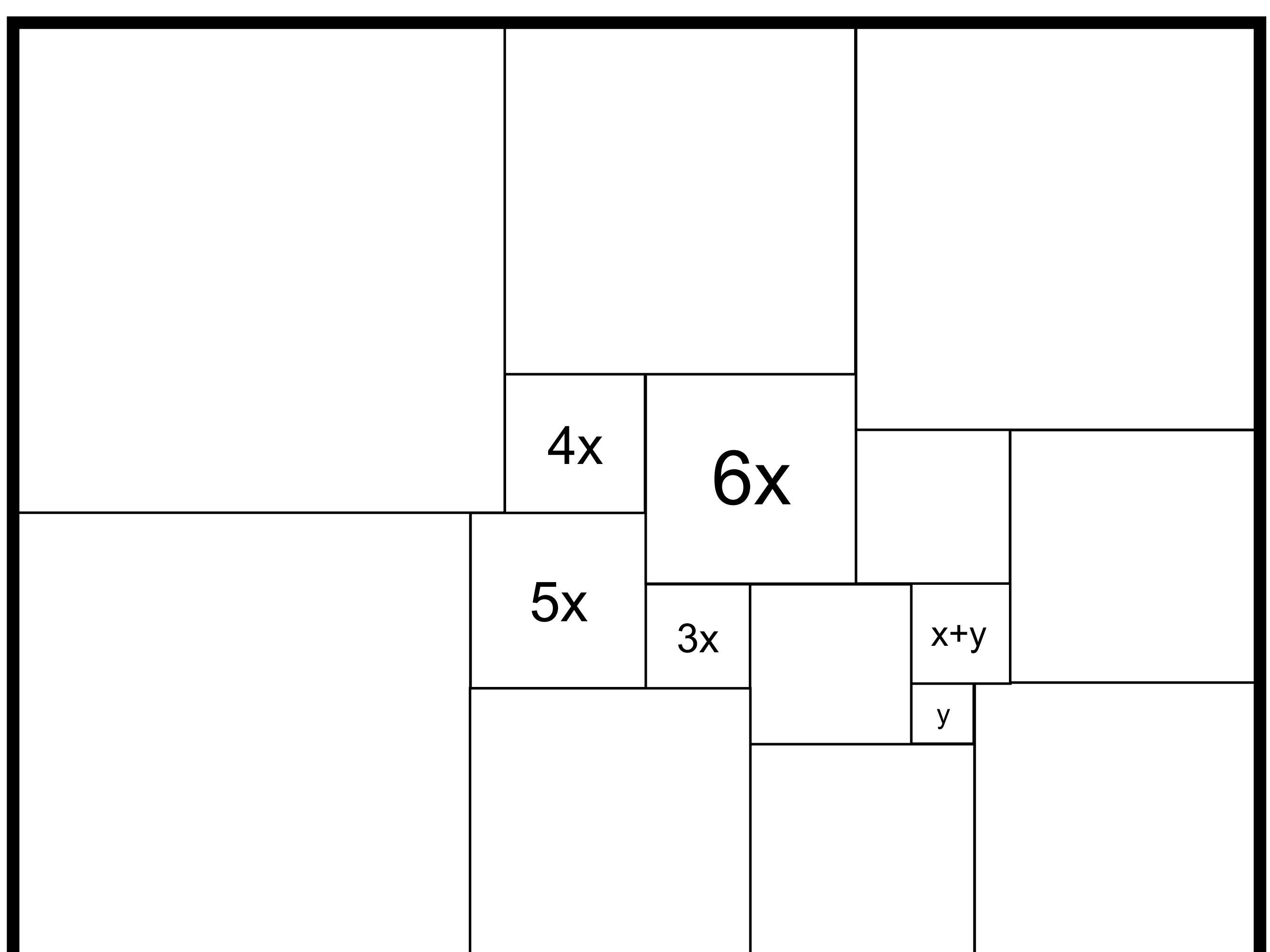


Challenge 7

Here we have two variables. Again - find the dimensions of all squares. Do these three puzzles have unique answers?

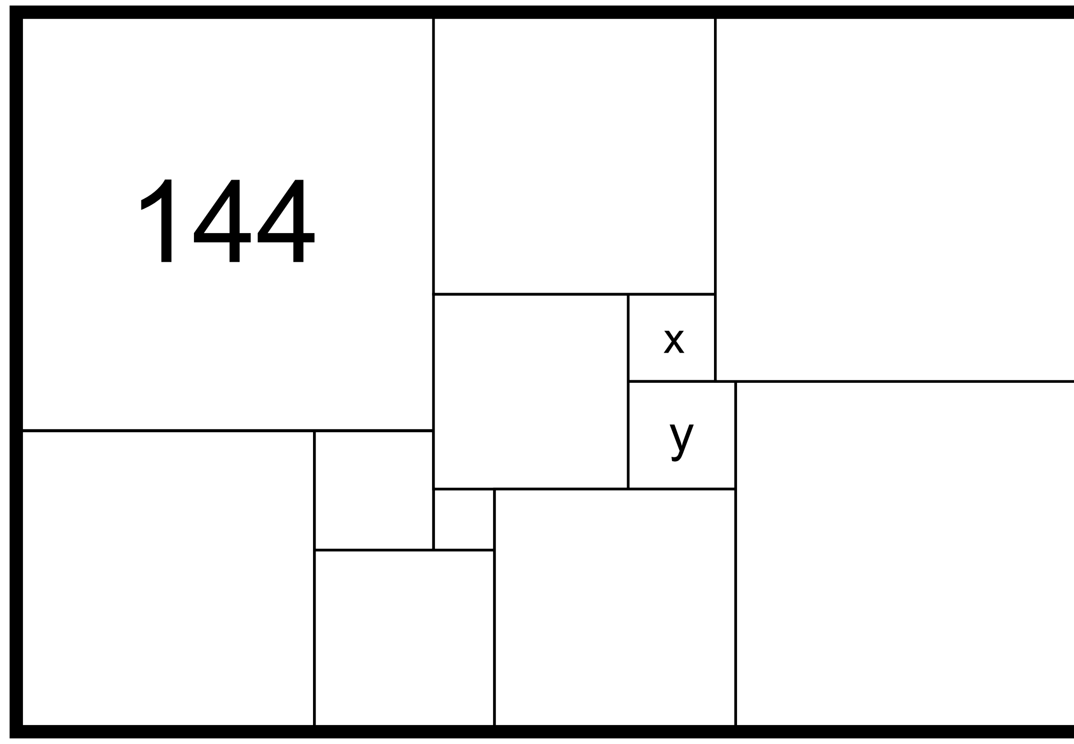


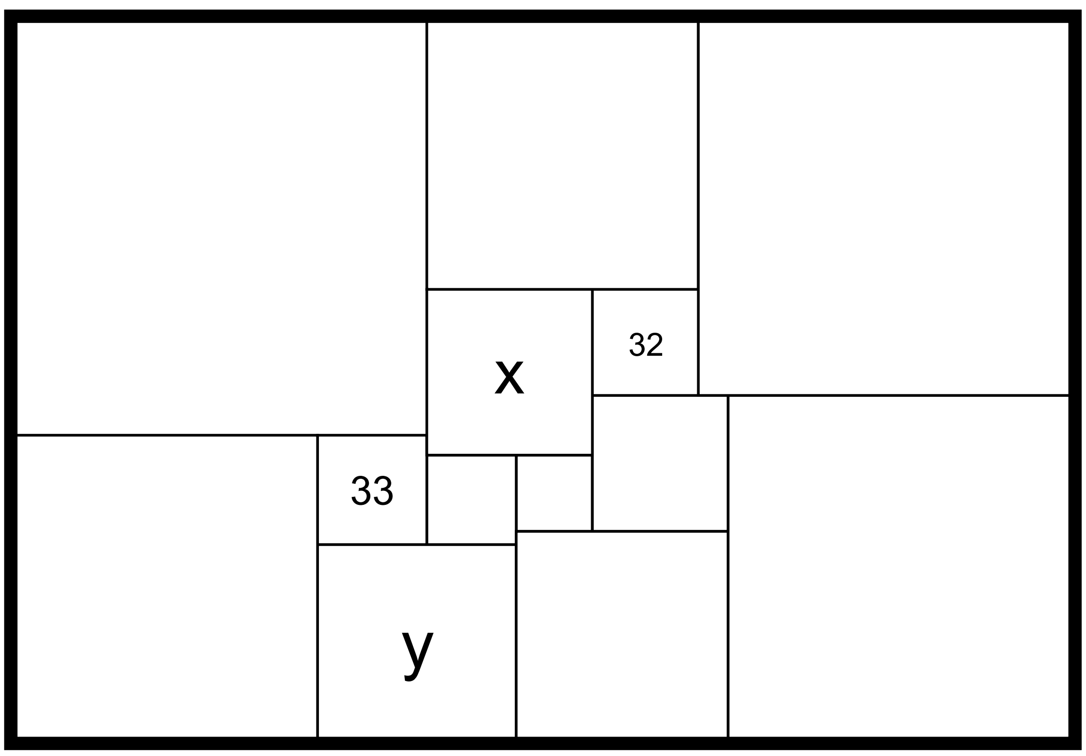


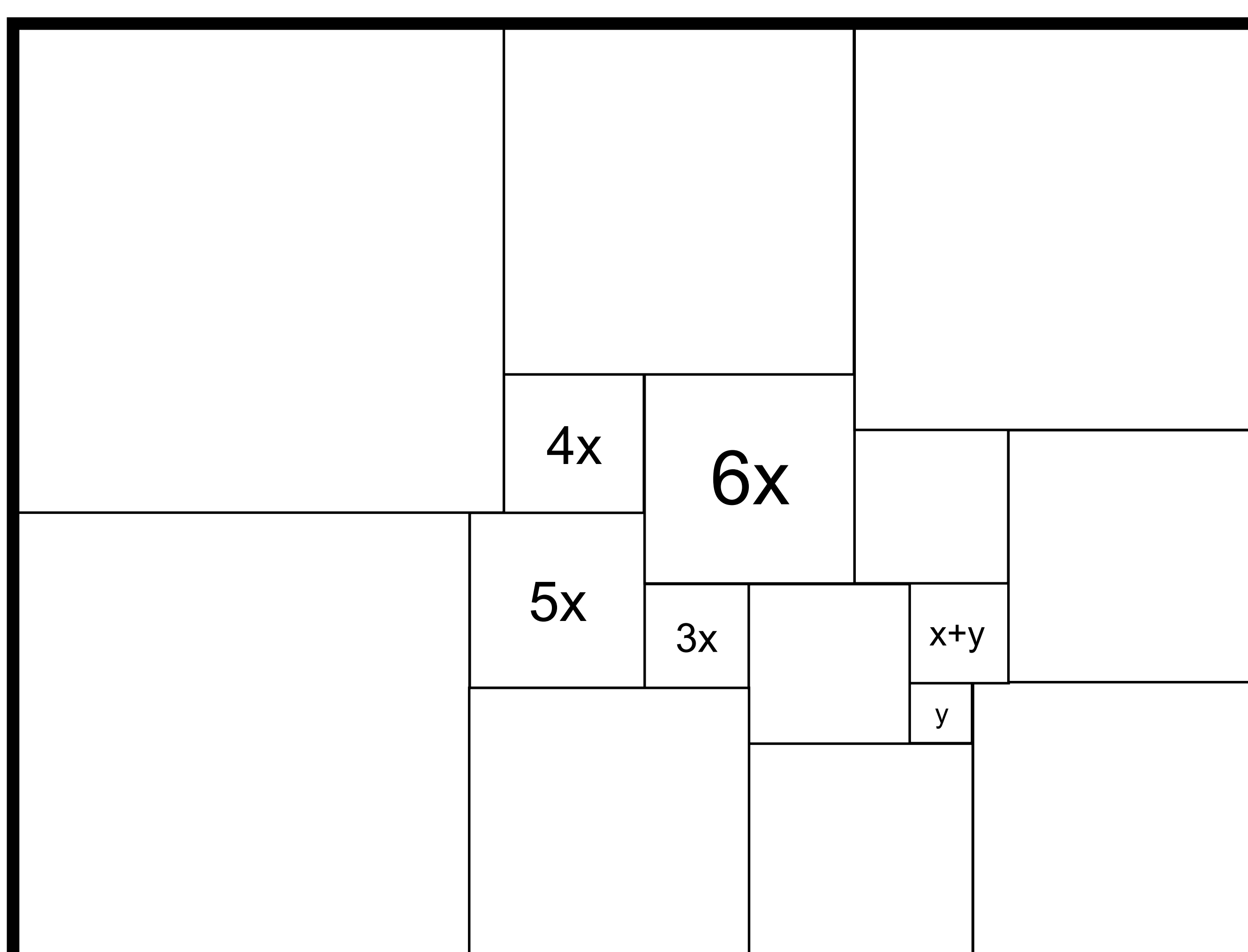


Algebra on Rectangles

Here we have two variables. Again - find the dimensions of all squares. Do these three puzzles have unique answers?

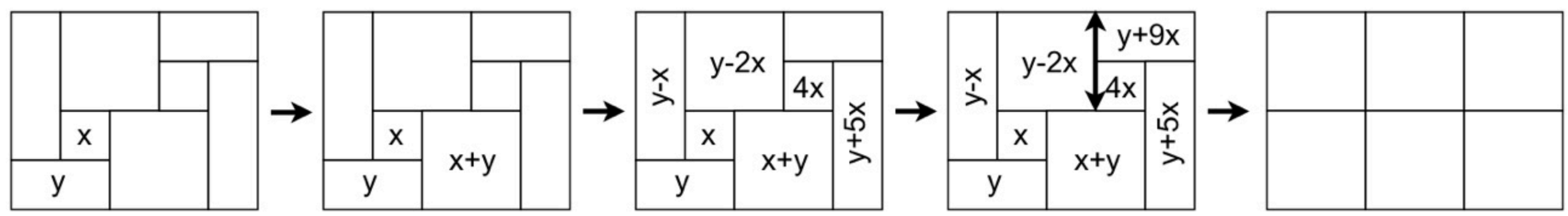




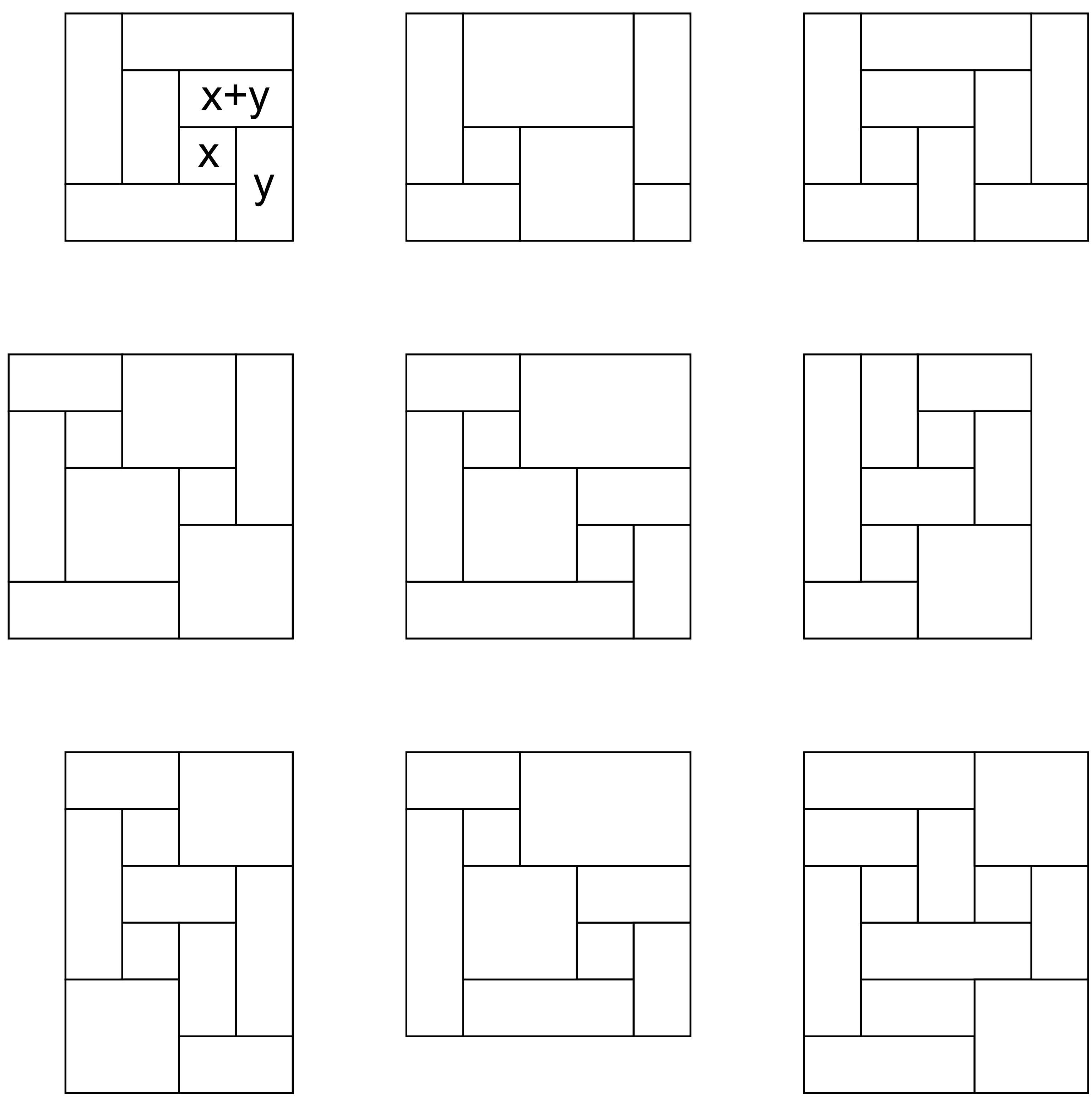


Challenge 8

Remember back in challenge 3, you tried the very difficult task of tiling a rectangle with squares of different sizes. You were even helped by being given the dimensions of the rectangle. It was still tough! In contrast, the figures on this page are easy to create. These appear to be rectangular tilings of squares and rectangles, but looks can be deceiving! We are going to imagine that all of the little tiling rectangles are actually squares. As before, we will put a number or variable in each to represent edge length. It looks funny, but after we solve these puzzles we will be able to stretch each square so that it's the right size. Here is an example:

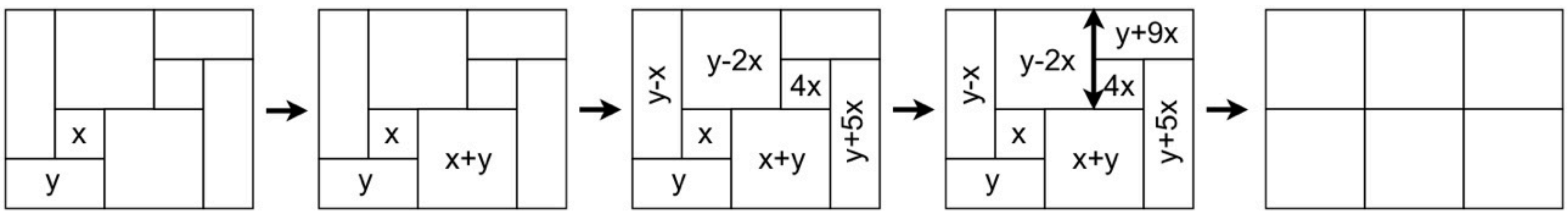


The second last diagram gives us $y-2x = y+13x$ = from which we conclude that $x = 0$ and can simplify the diagram by stretching and contorting the other rectangles so they become the six squares in the last diagram. Here, all the squares are not different sizes, but two of the tilings below do give answers in which all the squares will be of different sizes. One of these will be the solution to challenge 3. The other seven will run into difficulties.

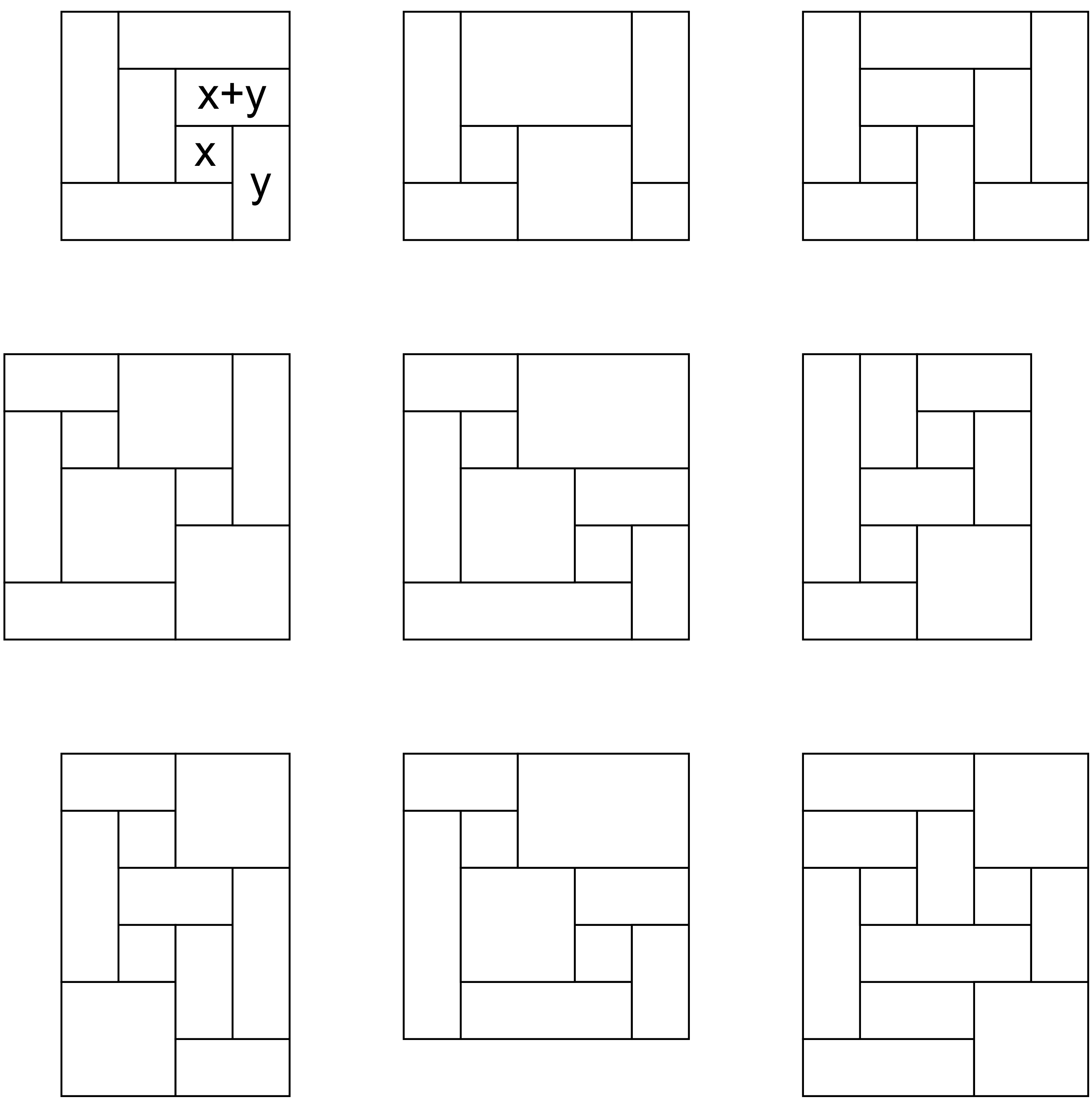


Algebra on Rectangles

Remember in the 1st challenge, you tried the very difficult task of tiling a rectangle with squares of different sizes. You were even helped by being given the dimensions of the rectangle. It was still tough! In contrast, the figures on this page are easy to create. These appear to be rectangular tilings of squares and rectangles, but looks can be deceiving! We are going to imagine that all of the little tiling rectangles are actually squares. As before, we will put a number or variable in each to represent edge length. It looks funny, but after we solve these puzzles we will be able to stretch each square so that it's the right size. Here is an example:

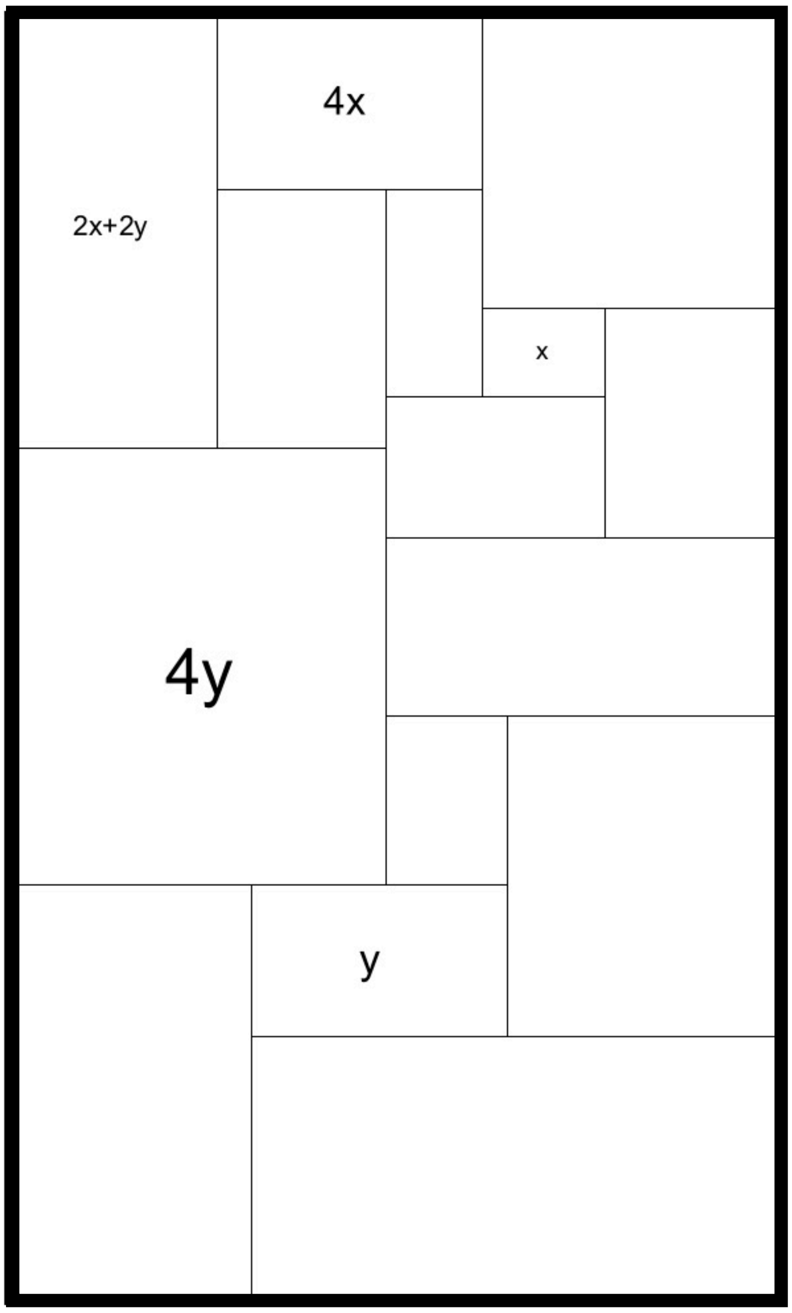


The second last diagram gives us $y-2x = y+13x$ = from which we conclude that $x = 0$ and can simplify the diagram by stretching and contorting the other rectangles so they become the six squares in the last diagram. Here, all the squares are not different sizes, but two of the tilings below do give answers in which all the squares will be of different sizes. One of these will be the solution to challenge 3. The other seven will run into difficulties.



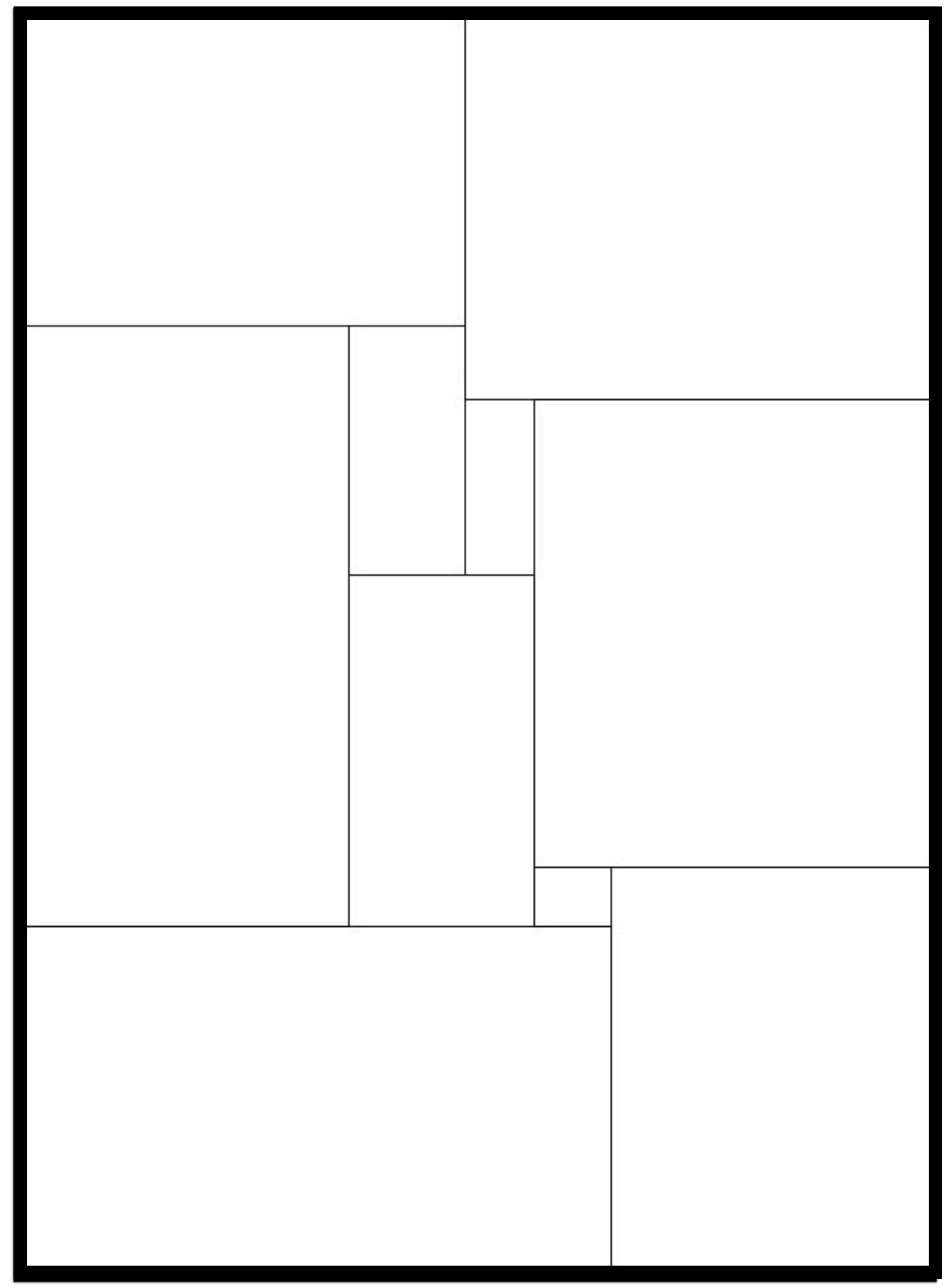
Challenge 9

Algebraically assume these are squares and solve.

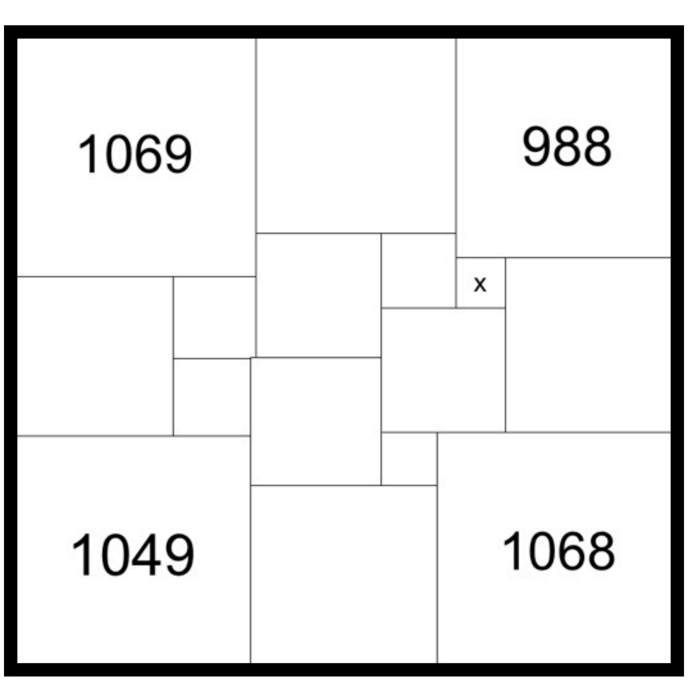


Challenge 10

Solve algebraically. Think and then make a sketch on graph paper of the geometric solution that this implies.



Challenge 11



The tiling on the left is very special. It is the current record holder for the lowest ratio of Largest Square to Smallest Square. Could some tiling exist with a 2:1 ratio? What about 3:1 or 4:1? What is the lowest ratio possible? MathPickle.com will pay $100 for an example of a 4:1 tiling or a published proof that it is impossible. Contact gord@mathpickle.com for your reward or to commiserate and discuss this difficult problem.

Standards for Mathematical Practice

All MathPickle puzzle designs, including Algebra on Rectangles, are guaranteed to engage a wide spectrum of student abilities while targeting the following Standards for Mathematical Practice:

MP1 Toughen up!
This is problem solving where our students develop grit and resiliency in the face of nasty, thorny problems. It is the most sought after skill for our students.

MP3 Work together!
This is collaborative problem solving in which students discuss their strategies to solve a problem and identify missteps in a failed solution. MathPickle recommends pairing up students for all its puzzles.

MP6 Be precise!
This is where our students learn to communicate using precise terminology. MathPickle encourages students not only to use the precise terms of others, but to invent and rigorously define their own terms.

MP7 Be observant!
One of the things that the human brain does very well is identify pattern. We sometimes do this too well and identify patterns that don't really exist.

(http://www.corestandards.org/Math/Practice/)

Common Core State Standards

Algebra on Rectangles targets the following Common Core State Standards from K-12:

CCSS.MATH.CONTENT.1.OA.B.4
Understand subtraction as an unknown-addend problem. For example, subtract 10 - 8 by finding the number that makes 10 when added to 8.

CCSS.MATH.CONTENT.2.NBT.B.5
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

CCSS.MATH.CONTENT.3.NBT.A.2
Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

CCSS.MATH.CONTENT.6.EE.A.2
Write, read, and evaluate expressions in which letters stand for numbers.

CCSS.MATH.CONTENT.7.EE.B.4
Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

CCSS.MATH.CONTENT.8.EE.C.7
Solve linear equations in one variable.

CCSS.MATH.CONTENT.HSA.CED.A.1
Create equations and inequalities in one variable and use them to solve problems.Include equations arising from linear and quadratic functions, and simple rational and exponential functions.

CCSS.MATH.CONTENT.HSA.CED.A.2
Create equations in two or more variables to represent relationships between quantities