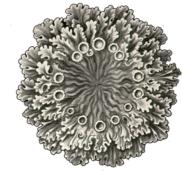


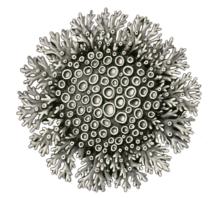
Lichen Puzzles



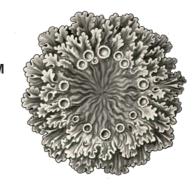
multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904)

Essential Video to Begin	http://youtu.be/D8wXJ0tZUx4
Written Rules for Lichen Puzzles	2
Problem Set 1	2
\$100 challenge	6
Problem Set 1 Answers	7
Problem Set 2	12
Problem Set 2 Answers	15
Problems for Projection to the Class	20
Common Core State Standards	48
Alternate starting puzzle set	49
About MathPickle	50

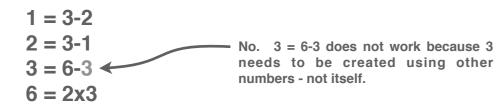




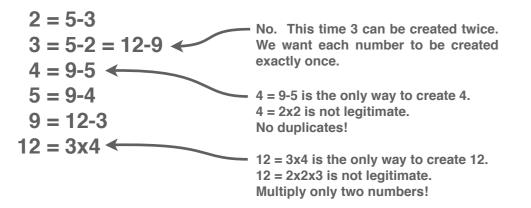
Lichen Puzzles



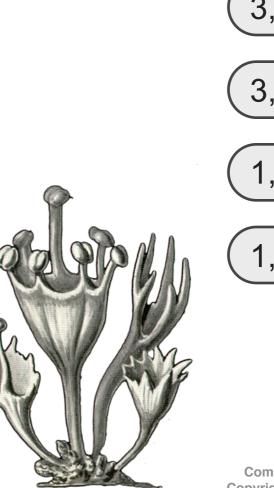
Fungus and algae grow together and help each other. It's called lichen - the algae provides energy and the fungus provides protection. When creatures help each other like the algae and fungus, we call it a symbiotic relationship. Symbiotic sets have the property that *every* number can be created exactly *once* using the *other* numbers in the set. Here we are going to work with multiplication and subtraction. Is $\{1,2,3,6\}$ symbiotic? Let's see...



Is {2,3,4,5,9,12} symbiotic? Let's see...



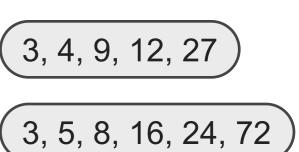
Half of the sets on the right are symbiotic and half are impostors. Find the 3 impostors.



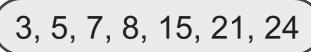
multiplication & subtraction

Lichen drawings by Ernst Haeckel

Art Forms of Nature (1904)



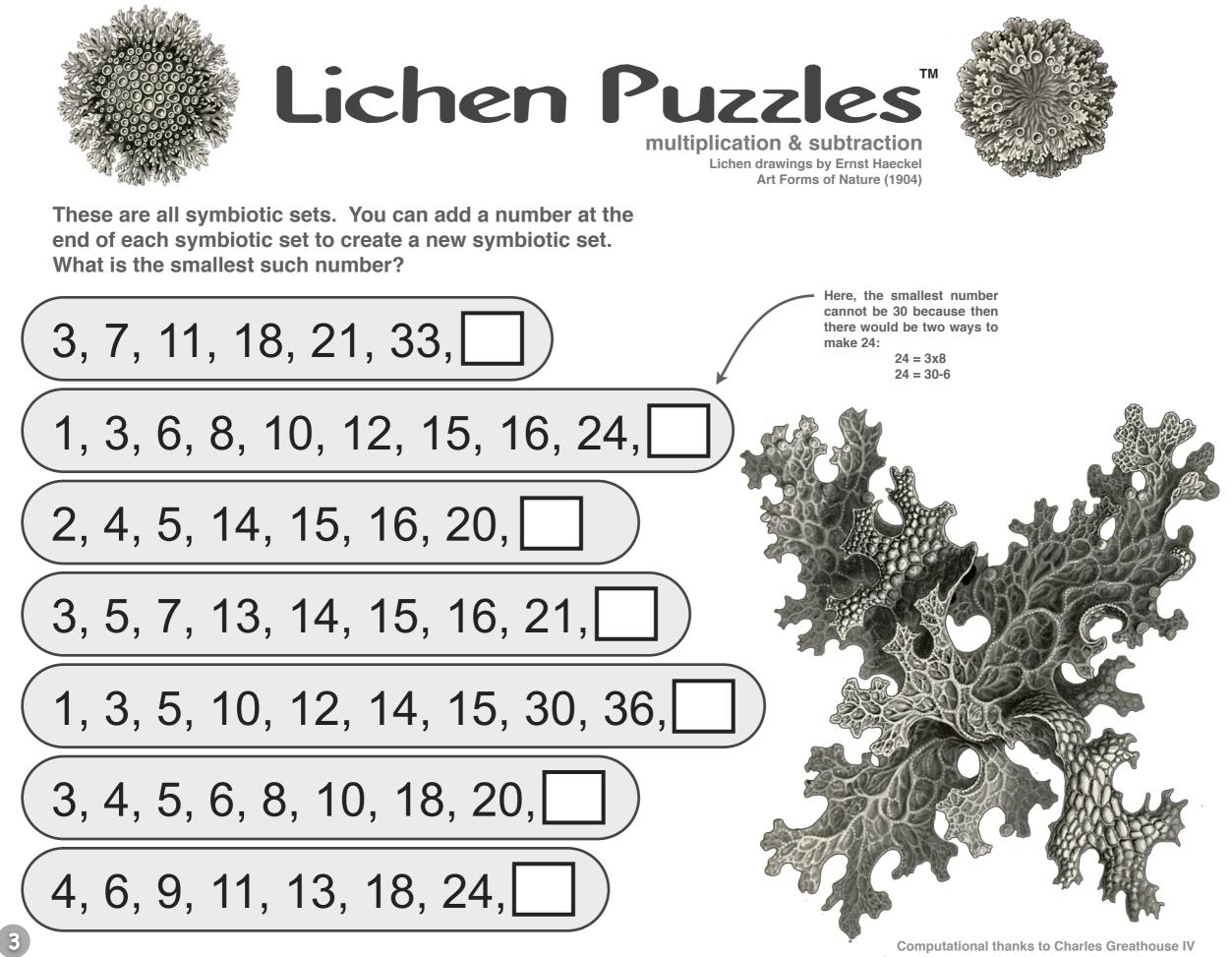
7, 9, 16



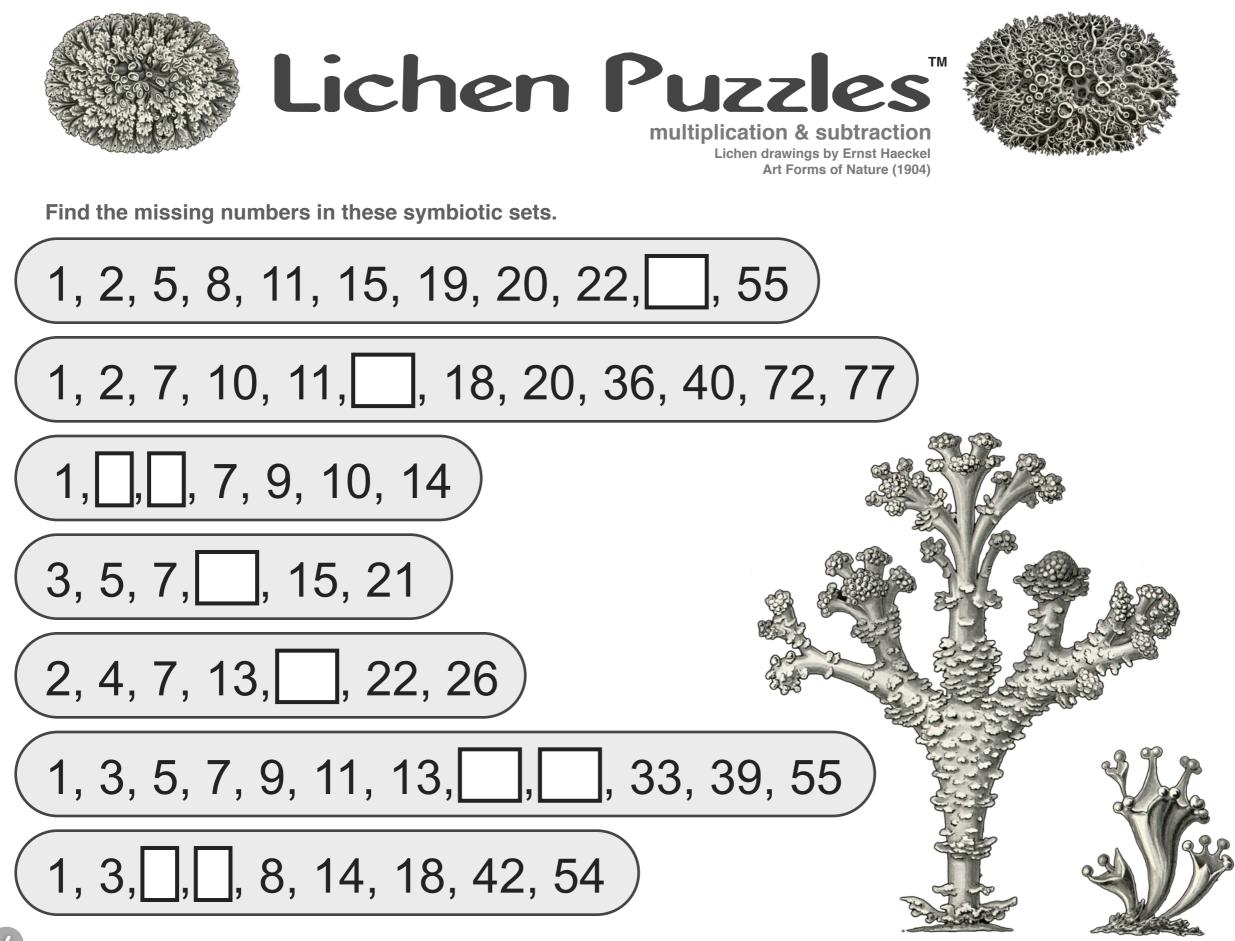




Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved



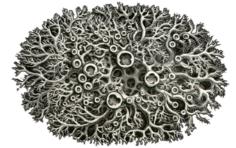
Copyright © 2014 MathPickle - All Rights Reserved



Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved



Lichen Puzzles



multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904)

These sets are not symbiotic, but choose a single number to squish and they will be.

4, 5, 7, 17, 24, 28 4, 5, 6, 7, 19, 20, 21, 22, 23, 28, 42 2, 3, 4, 7, 9, 10, 15, 16, 21, 22, 25, 27 3, 9, 10, 11, 15, 16, 17, 18, 27, 30 2, 5, 6, 10, 13, 18, 20, 26, 36 1, 2, 10, 12, 13, 15, 20, 25, 26 4, 6, 18, 20, 24, 44, 80 1, 2, 3, 6, 10, 12, 17, 20, 24 5 Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved



Lichen Puzzles multiplication & subtraction

Lichen drawings by Ernst Haeckel

Art Forms of Nature (1904)



Find a symbiotic set with the fewest numbers.

Find the symbiotic set with the smallest biggest number.

Symbiotic sets can actually allow more than one way to create a number. What's important is that all numbers can be created in the same number of ways. Below I was trying to make a symbiotic set in which each number is created in three ways. Did I succeed?

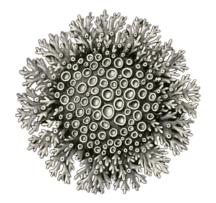
1, 2, 3, 5, 6, 9, 10, 12, 20, 24, 36, 40, 45, 50, 60

\$100 reward for the first person to find a symbiotic set (using multiplication and subtraction) with each number being created twice. Charles Greathouse IV has not been able to find such a set. He has searched all possible sets in which the largest number is less than 40. Email me (gord@mathpickle.com) to claim the reward or commiserate.

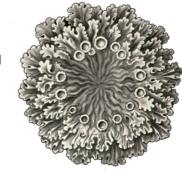






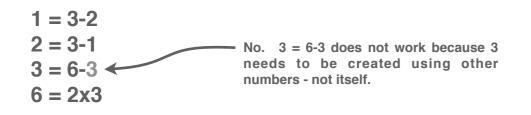




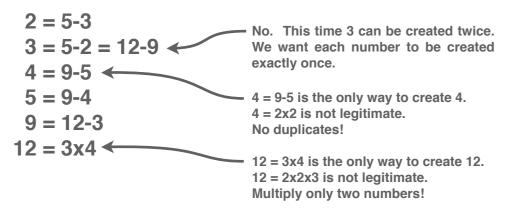


http://youtu.be/D8wXJ0tZUx4

Fungus and algae grow together and help each other. It's called lichen - the algae provides energy and the fungus provides protection. When creatures help each other like the algae and fungus, we call it a symbiotic relationship. Symbiotic sets have the property that every number can be created *once* using the *other* numbers in the set. Here we are going to work with multiplication and subtraction. Is $\{1,2,3,6\}$ symbiotic? Let's see...







Half of the sets on the right are symbiotic and half are impostors. Find the 3 impostors.



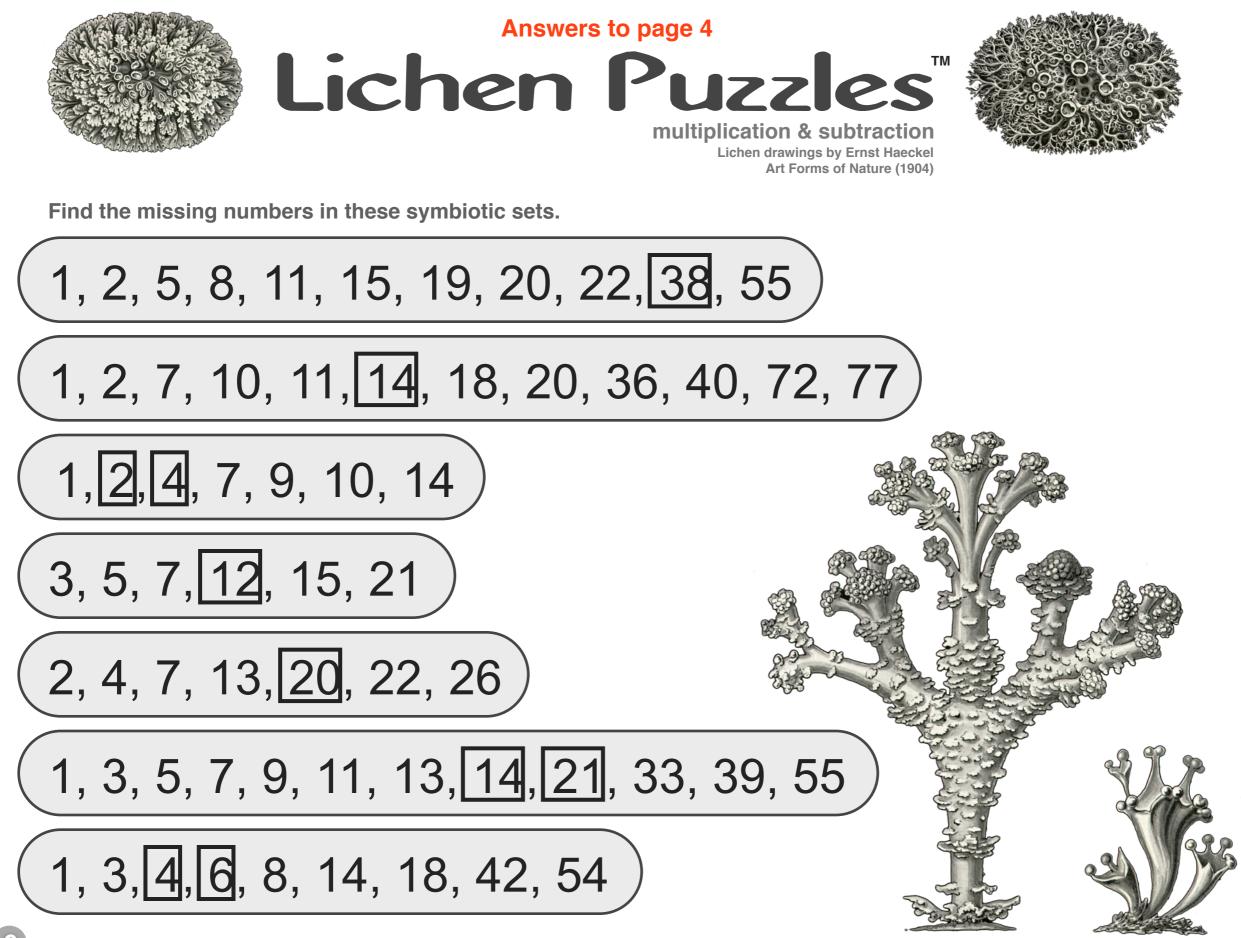
Lichen drawings by Ernst Haeckel

Art Forms of Nature (1904)



Answers to page 3 Lichen Puzzles multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904) These are all symbiotic sets. You can add a number at the end of each symbiotic set to create a new symbiotic set. What is the smallest such number? Here, the smallest number cannot be 30 because then 3, 7, 11, 18, 21, 33, 54 there would be two ways to make 24: 24 = 3x824 = 30-61, 3, 6, 8, 10, 12, 15, 16, 24, 45 2, 4, 5, 14, 15, 16, 20, 28 3, 5, 7, 13, 14, 15, 16, 21, 39 1, 3, 5, 10, 12, 14, 15, 30, 36, 60 3, 4, 5, 6, 8, 10, 18, 20, 32 4, 6, 9, 11, 13, 18, 24, 36 Computational thanks to Charles Greathouse IV

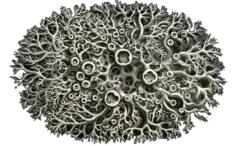
Copyright © 2014 MathPickle - All Rights Reserved



Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved







multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904)

These sets are not symbiotic, but choose a single number to squish and they will be.

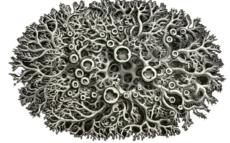
4, 5, 7, 17, 24, 28 4, 5, 6, 7, 19, 20, 21, 22, 23, 28, 42 2, 3, 4, 7, 9, 10, 15, 16, 21, 22, 25, 27 3, 9, 10, 11, 15, 16, 17, 18, 27, 30 2, 5, 6, 10, 13, 18, 20, 26, 36 1, 2, 10, 12, 13, 15, 20, 25, 26 4, 6, 18, 20, 24, 44, 80 1, 2, 3, 6, 10, 12, 17, 20, 24 10 **Computational thanks to Charles Greathouse IV** Copyright © 2014 MathPickle - All Rights Reserved





Lichen drawings by Ernst Haeckel

Art Forms of Nature (1904)



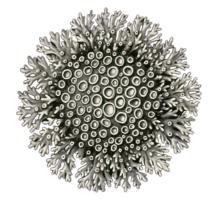
Find a symbiotic set with the fewest numbers. Five is the fewest (see below).

Find the symbiotic set with the smallest biggest number. The smallest biggest number is 12. There are six solutions:

Symbiotic sets can actually allow more than one way to create a number. What's important is that all numbers can be created in the same number of ways. Below I was trying to make a symbiotic set in which each number is created in three ways. Did I succeed? No - there are many numbers that fall short...

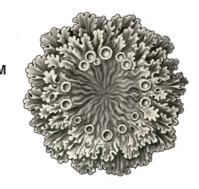
\$100 reward for the first person to find a symbiotic set (using multiplication and subtraction) with each number being made twice. Charles Greathouse IV has not been able to find such a set. He has searched all possible sets in which the largest number is less than 40. Email me (gord@mathpickle.com) to claim the reward or commiserate.



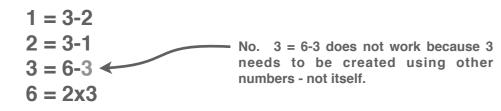


12

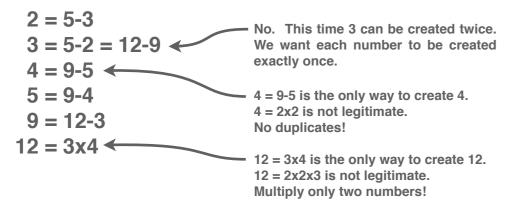
Lichen Puzzles



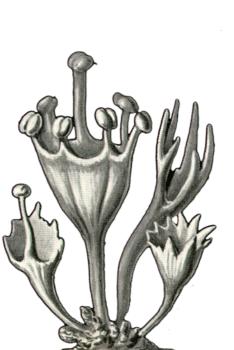
Fungus and algae grow together and help each other. It's called lichen - the algae provides energy and the fungus provides protection. When creatures help each other like the algae and fungus, we call it a symbiotic relationship. Symbiotic sets have the property that *every* number can be created exactly *once* using the *other* numbers in the set. Here we are going to work with multiplication and subtraction. Is $\{1,2,3,6\}$ symbiotic? Let's see...



Is {2,3,4,5,9,12} symbiotic? Let's see...



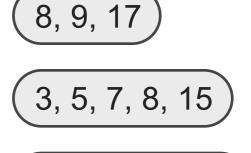
Half of the sets on the right are symbiotic and half are impostors. Find the 3 impostors.



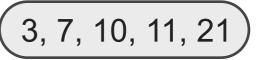
multiplication & subtraction

Lichen drawings by Ernst Haeckel

Art Forms of Nature (1904)



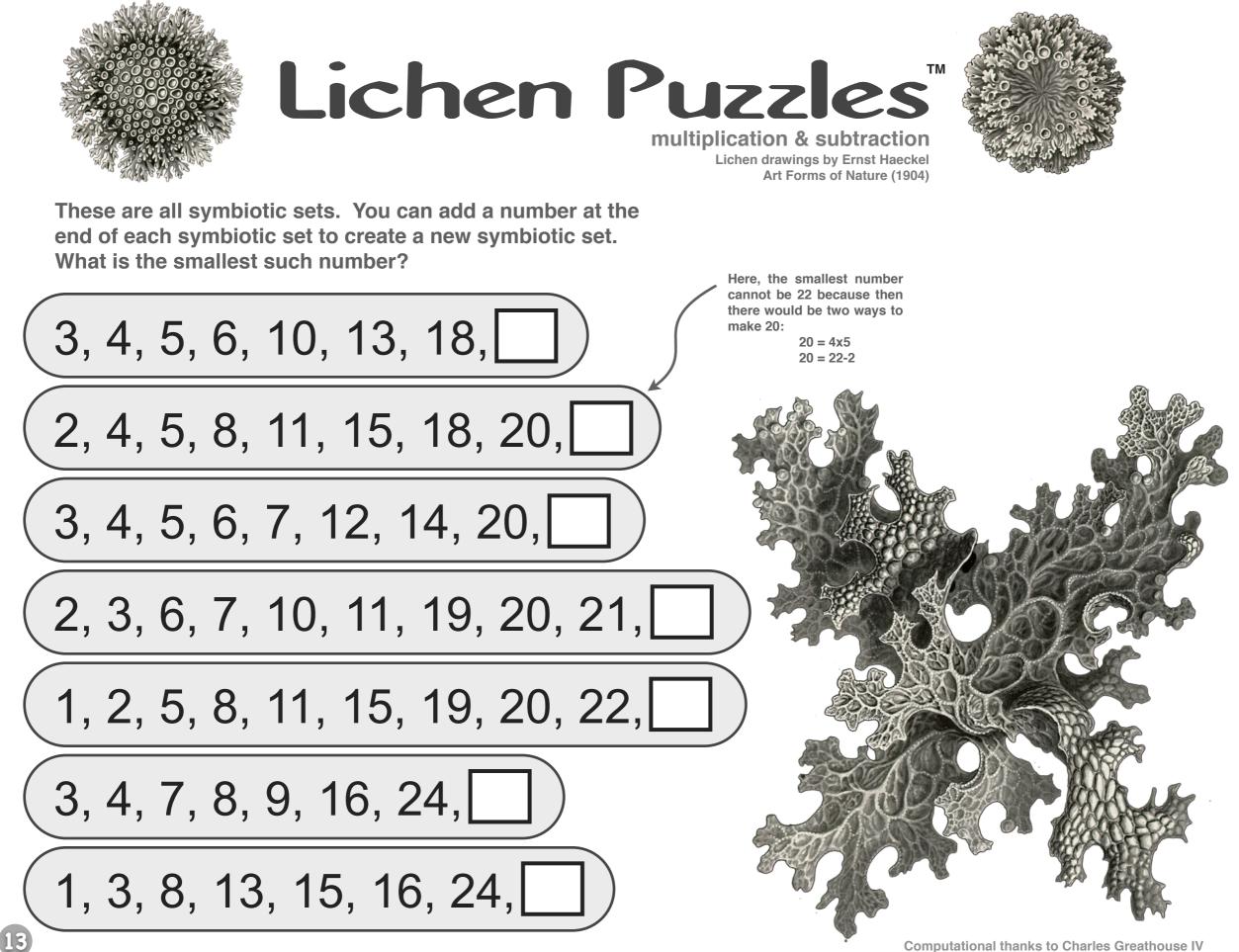




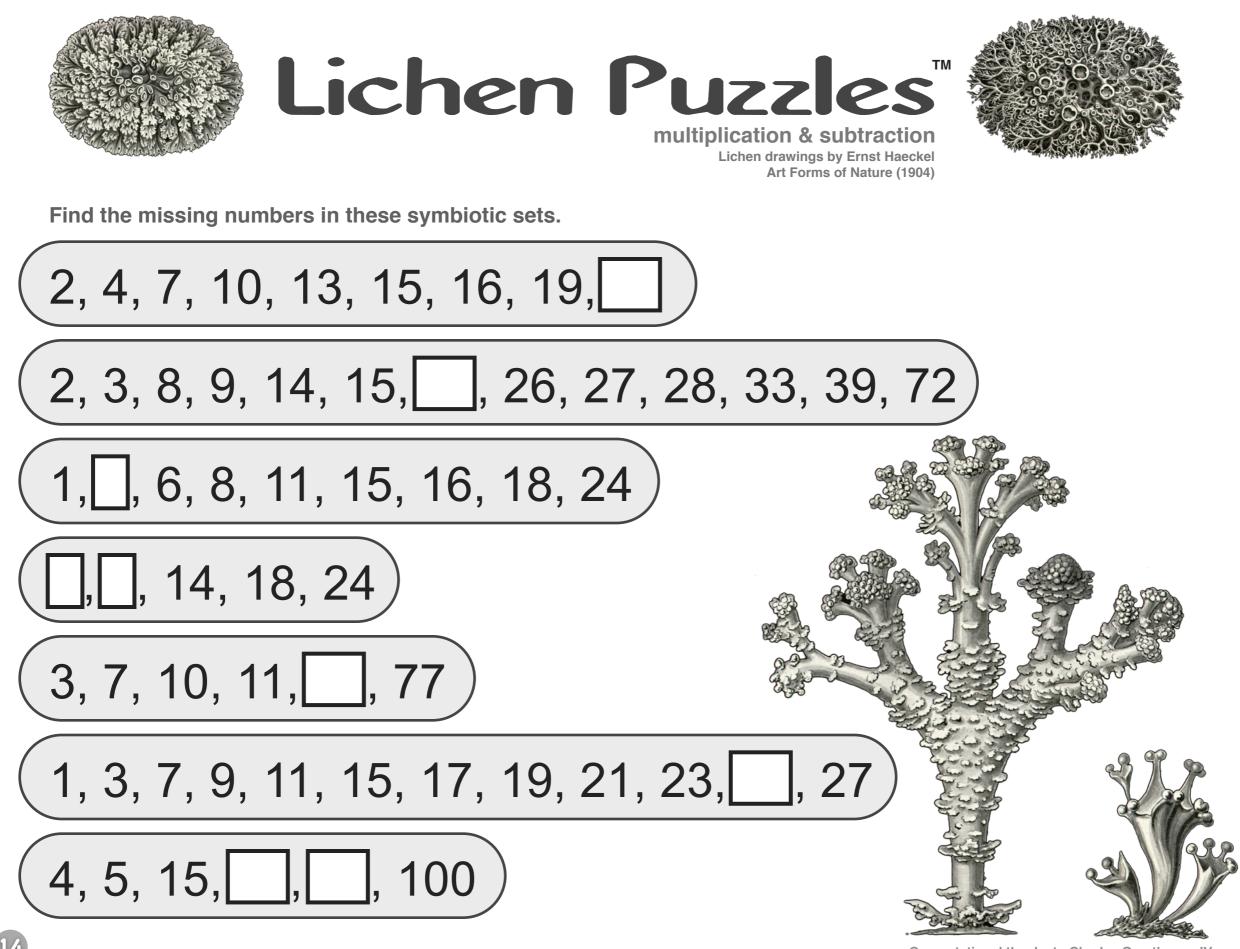




Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved



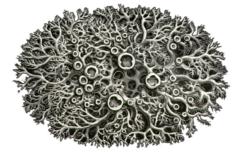
Copyright © 2014 MathPickle - All Rights Reserved



Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved

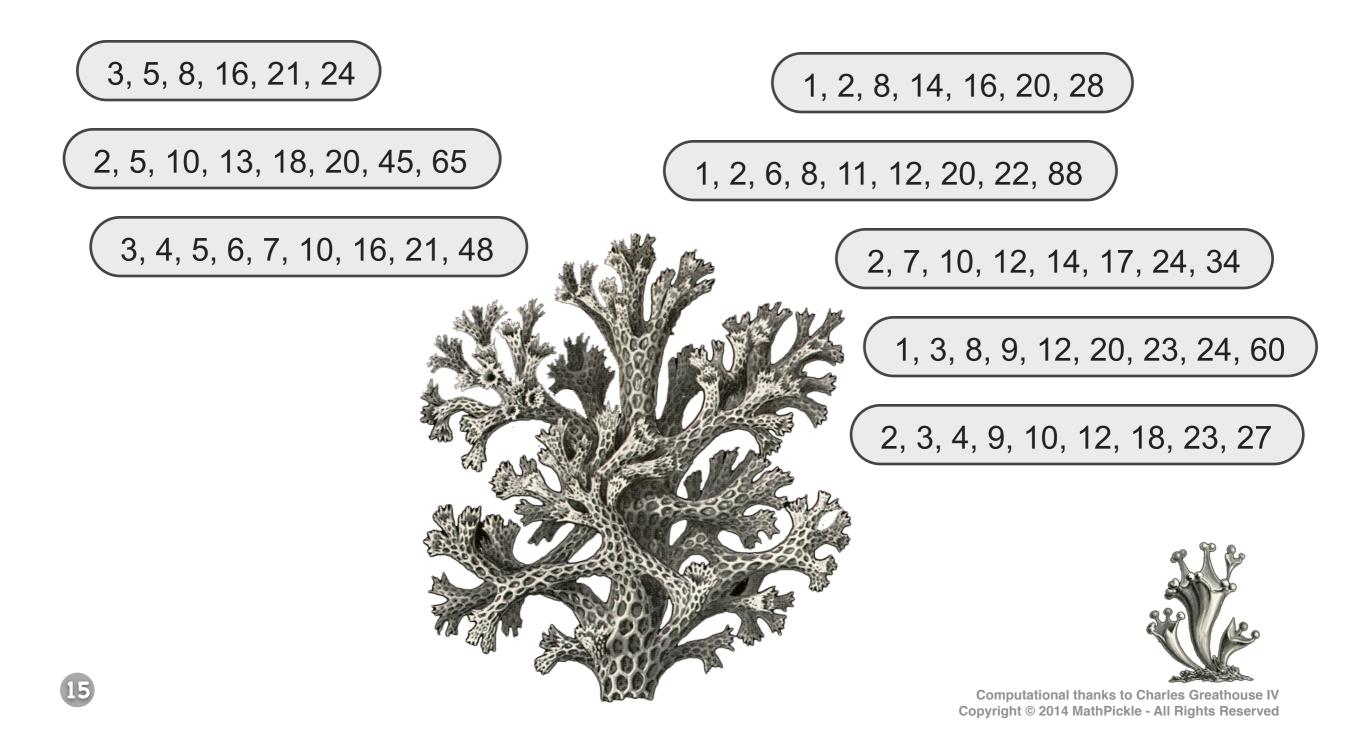


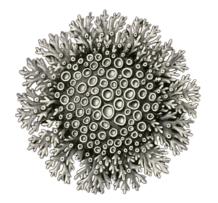
Lichen Puzzles



multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904)

These sets are not symbiotic, but squish the right number and they will be.







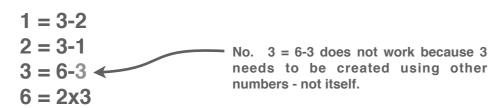
Lichen drawings by Ernst Haeckel

Art Forms of Nature (1904)

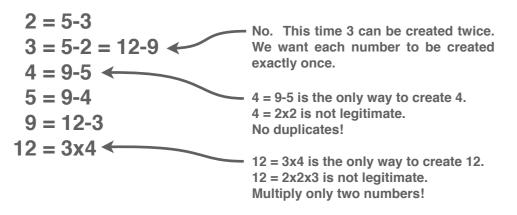


http://youtu.be/D8wXJ0tZUx4

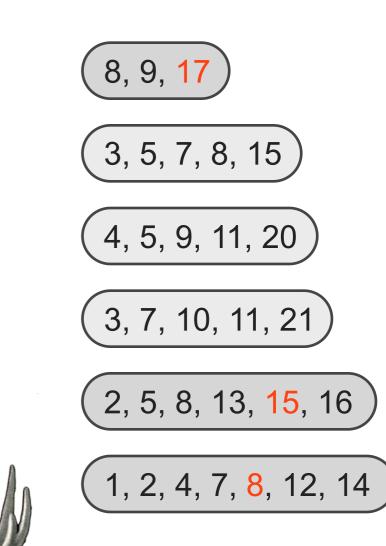
Symbiotic sets have the property that every number can be created *once* using the *other* numbers in the set. Here we are going to work with multiplication and subtraction. Is $\{1,2,3,6\}$ symbiotic? Let's see...



Is {2,3,4,5,9,12} symbiotic? Let's see...



Half of the sets on the right are symbiotic and half are impostors. Find the 3 impostors.

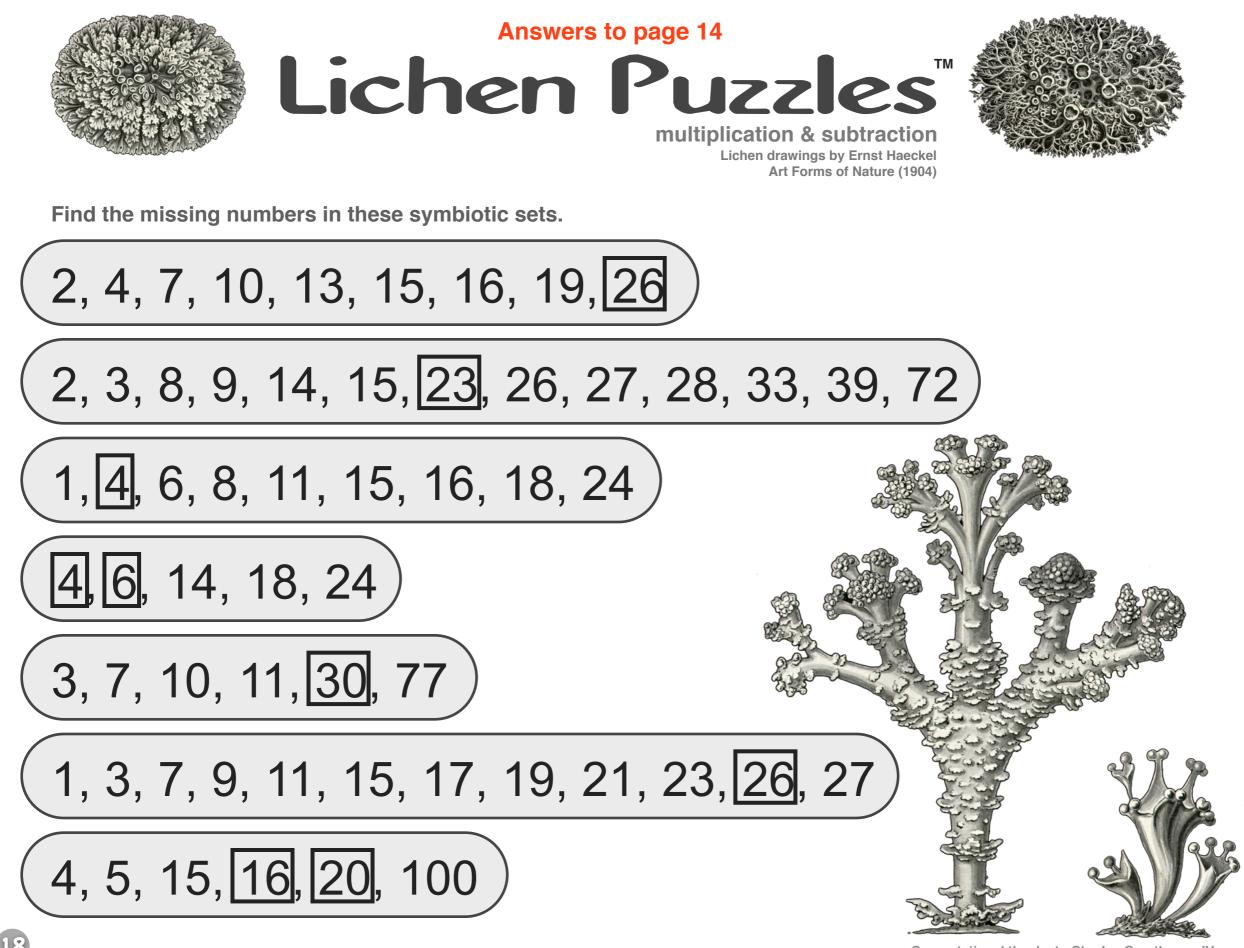


Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved



Answers to page 13 Lichen Puzzles multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904) These are all symbiotic sets. You can add a number at the end of each symbiotic set to create a new symbiotic set. What is the smallest such number? Here, the smallest number cannot be 22 because then there would be two ways to 3, 4, 5, 6, 10, 13, 18, 20 make 20: 20 = 4x520 = 22 - 22, 4, 5, 8, 11, 15, 18, 20, 30 3, 4, 5, 6, 7, 12, 14, 20, 30 2, 3, 6, 7, 10, 11, 19, 20, 21, 38 1, 2, 5, 8, 11, 15, 19, 20, 22, 38 3, 4, 7, 8, 9, 16, 24, 36 1, 3, 8, 13, 15, 16, 24, 39 17 Computational thanks to Charles Greathouse IV

Copyright © 2014 MathPickle - All Rights Reserved



Computational thanks to Charles Greathouse IV Copyright © 2014 MathPickle - All Rights Reserved

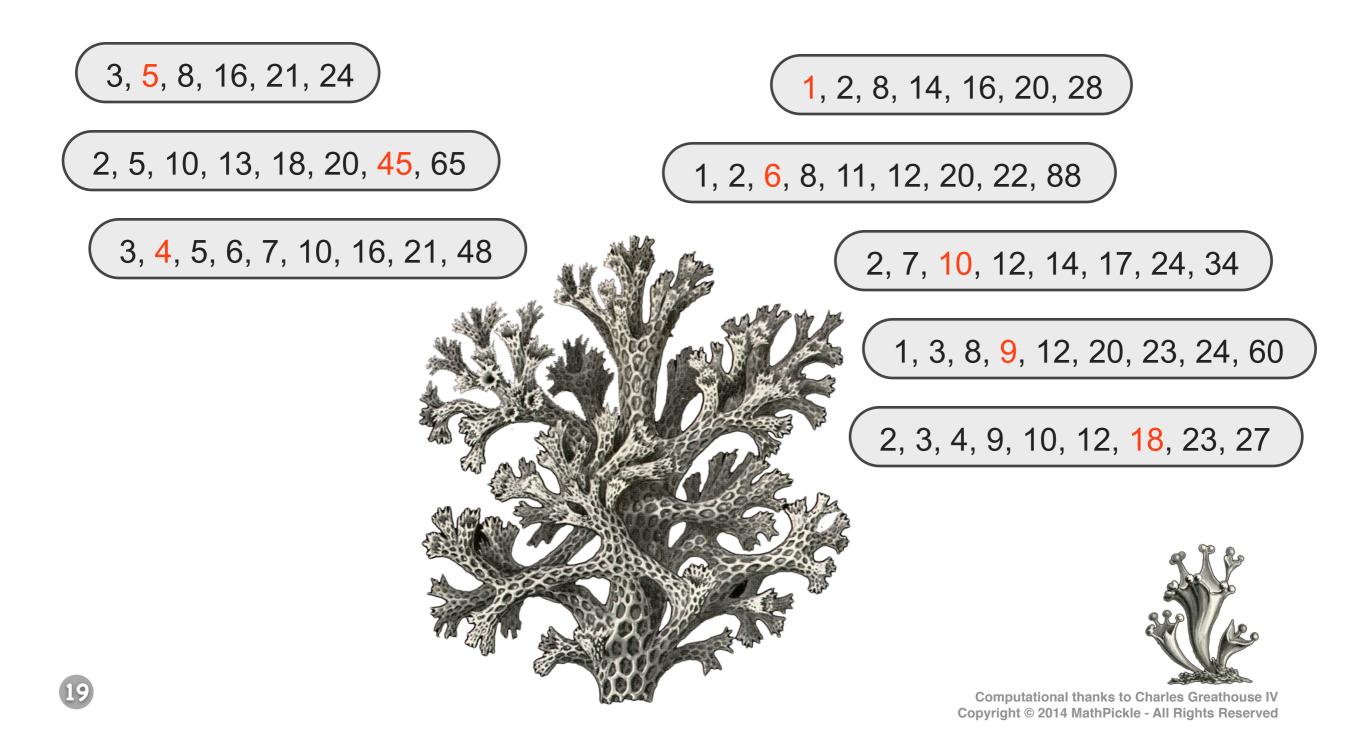


Answers to page 15



multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904)

These sets are not symbiotic, but squish the right number and they will be.





3, 4, 5, 7, 12, 20, 60

3, 4, 5, 7, 12, 20, 60

3, 4, 5, 7, 12, 20, 60

60 = 5x12 = 3x20

2, 10, 13, 15, 25, 26

2, 10, 13, 15, 25, 26

2, 10, 13, 15, 25, 26

25 = ?

3, 5, 6, 7, 8, 15, 21

3, 5, 6, 7, 8, 15, 21

3, 5, 6, 7, 8, 15, 21

15 = 21 - 6 = 3x5

4, 6, 14, 20, 24, 60, 80

4, 6, 14, 20, 24, 60, 80

4, 6, 14, 20, 24, 60, 80

20 = 24-4 = 80-60

AST ALK



3, 5, 7, 8, 15, 25, 40

3, 5, 7, 8, 15, 25, 40

3, 5, 7, 8, 15, 25, 40

15 = 3x5 = 40 - 25

4, 7, 10, 17, 21, 28

4, 7, 10, 17, 21, 28

4, 7, 10, 17, 21, 28

AN ALAL

7 = 17-10 = 28-21

2, 5, 13, 18, 24, 26

2, 5, 13, 18, 24, 26

2, 5, 13, 18, 24, 26

18 =

Standards for Mathematical Practice

All MathPickle recommendations, including **Lichen Puzzles**, are guaranteed to engage a wide spectrum of student abilities while targeting the following Standards for Mathematical Practice:

MP1 Toughen up!

This is problem solving where our students develop grit and resiliency in the face of nasty, thorny problems. It is the most sought after skill for our students.

MP3 Work together!

This is collaborative problem solving in which students discuss their strategies to solve a problem and identify missteps in a failed solution. MathPickle recommends pairing up students for all its puzzles.

MP6 Be precise!

This is where our students learn to communicate using precise terminology. MathPickle encourages students not only to use the precise terms of others, but to invent and rigorously define their own terms.

MP7 Be observant!

One of the things that the human brain does very well is identify pattern. We sometimes do this too well and identify patterns that don't really exist.

Common Core State Standards

Lichen Puzzles target Common Core State Standards for students practicing multiplication. Although it covers curricular topics in grade 3, I'm only recommending it for good grade 3 classes and forward because the puzzles become quite challenging.

Grades 3-4

CCSS.MATH.CONTENT.3.OA.A.1

Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each.

CCSS.MATH.CONTENT.3.OA.A.4

Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48, 5 =$ $\div 3, 6 \times 6 = ?$

CCSS.MATH.CONTENT.3.OA.C.7

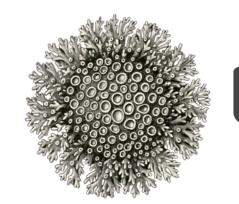
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

CCSS.MATH.CONTENT.4.OA.A.1

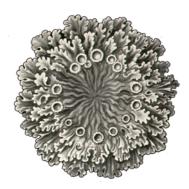
Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

CCSS.MATH.CONTENT.4.NBT.B.5

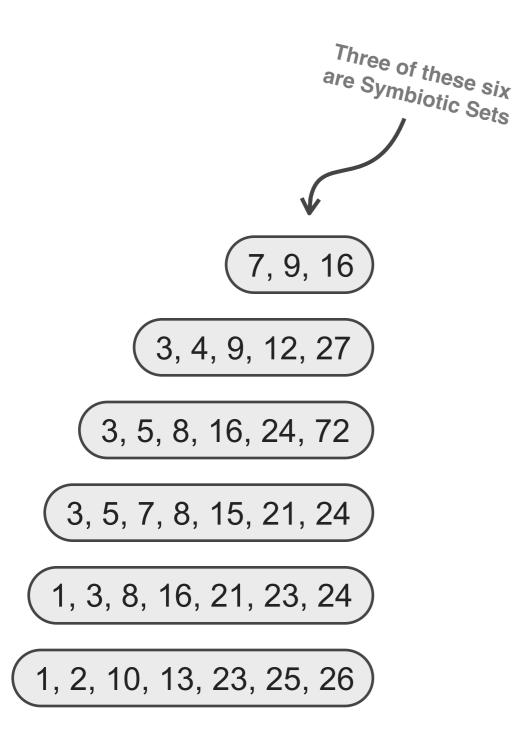
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations.

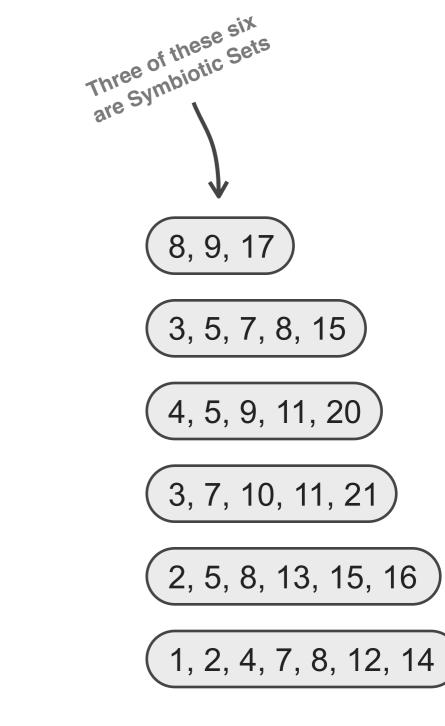


Lichen Puzzles



multiplication & subtraction Lichen drawings by Ernst Haeckel Art Forms of Nature (1904)





Put Your Students in a Pickle!

I'm a father of two elementary school children, a mathematician, and designer of puzzles and board games. Students call me Dr. Pickle. There is nothing I enjoy more than stumping students and having them stump me.

I founded MathPickle.com in 2010 to inject new ideas into the classroom. MathPickle's primary objective is to get thirteen curricular unsolved problems into classrooms worldwide - one for each grade K-12. A conference in November 2013 established the thirteen unsolved problems. To aid with the dissemination of these awesome problems, MathPickle is looking at setting up a \$1,000,000 reward for each - the prize money to be split between the person who solves the problem and their most inspirational K-12 educator.

MathPickle is also developing a range of curricular puzzles like the ones you'll find at TpT. These help teachers them with their number one challenge:

"How to engage the spectrum of student ability?"

Whenever an elementary school teacher wants to teach addition, she will invariably face 20% of students who already know how to add and another 20% who are struggling with last year's curriculum. How can she engage the top students without losing the bottom students? How can she engage the bottom students without boring the top students?

One solution: Parents of top students often ask that their child be allowed to accelerate through the curriculum. This exacerbates the problem for future teachers, and sets up a failure-impoverished education experience for the bright student.

A wiser approach is to use curricular puzzles, games and minicompetitions to simultaneously teach curriculum to the students who need it, and to deflect top students into tough problem solving activities. This is never time wasted, because problem solving is the primary reason we teach mathematics.

The experience of mathematics should be profound and beautiful. Too much of the regular K-12 mathematics experience is trite and true. Children deserve tough, beautiful puzzles.

> Gordon Hamilton MMath, PhD