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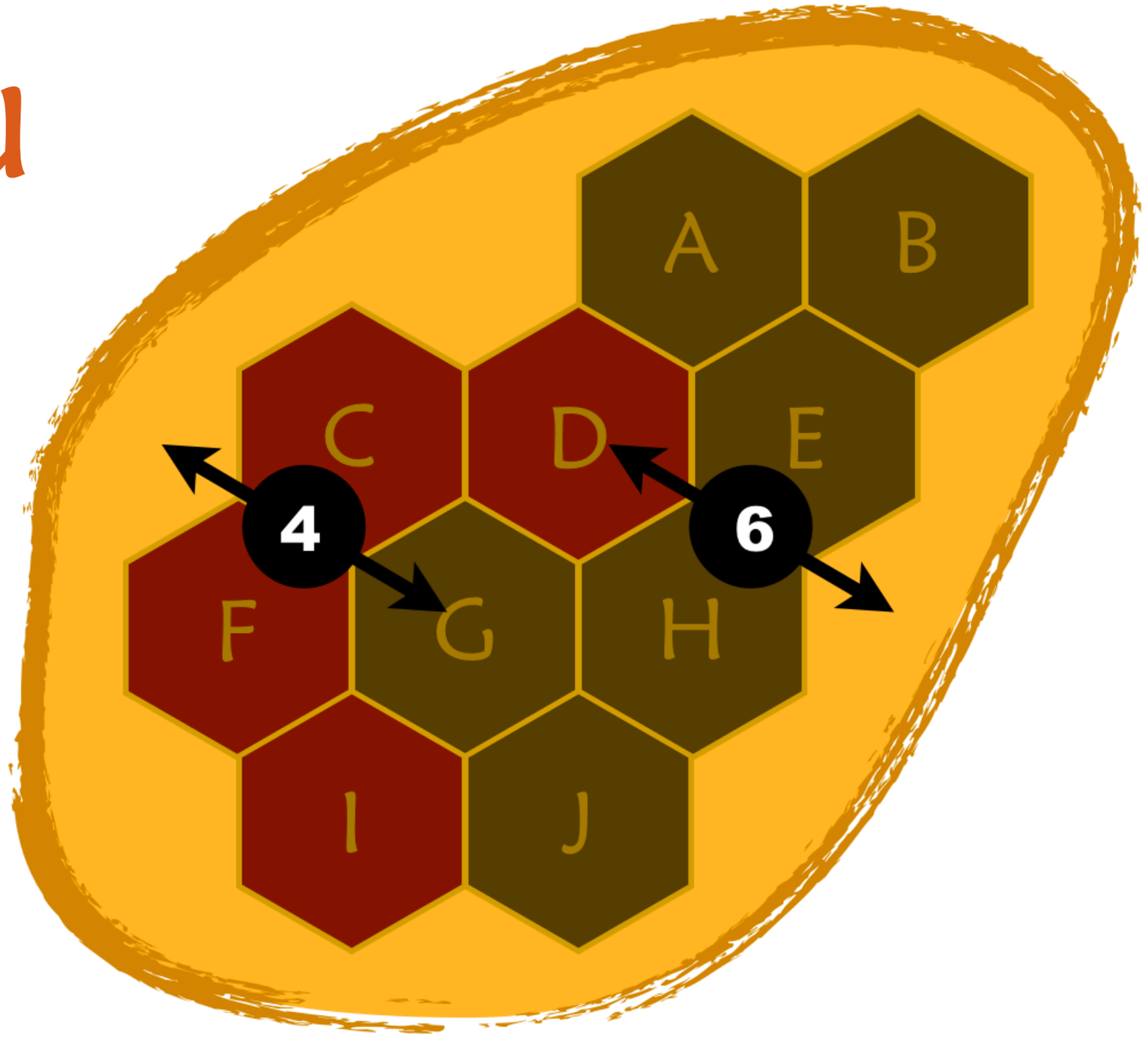
kajitsu



Contents

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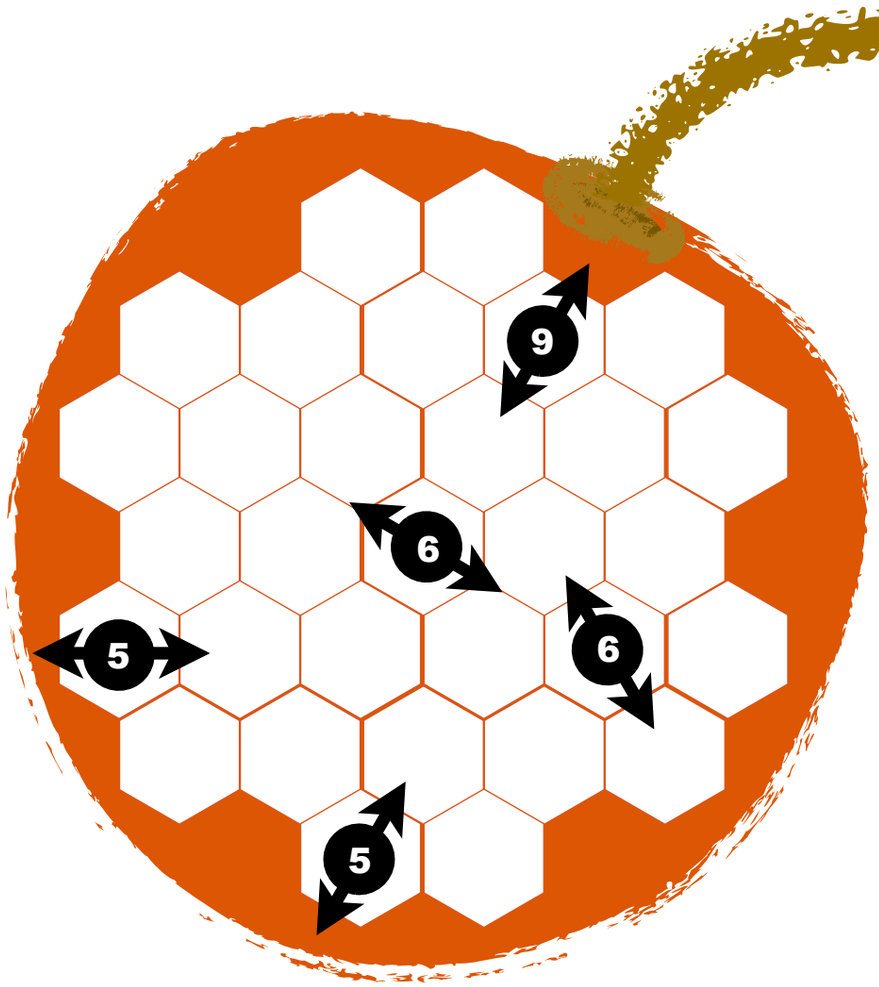
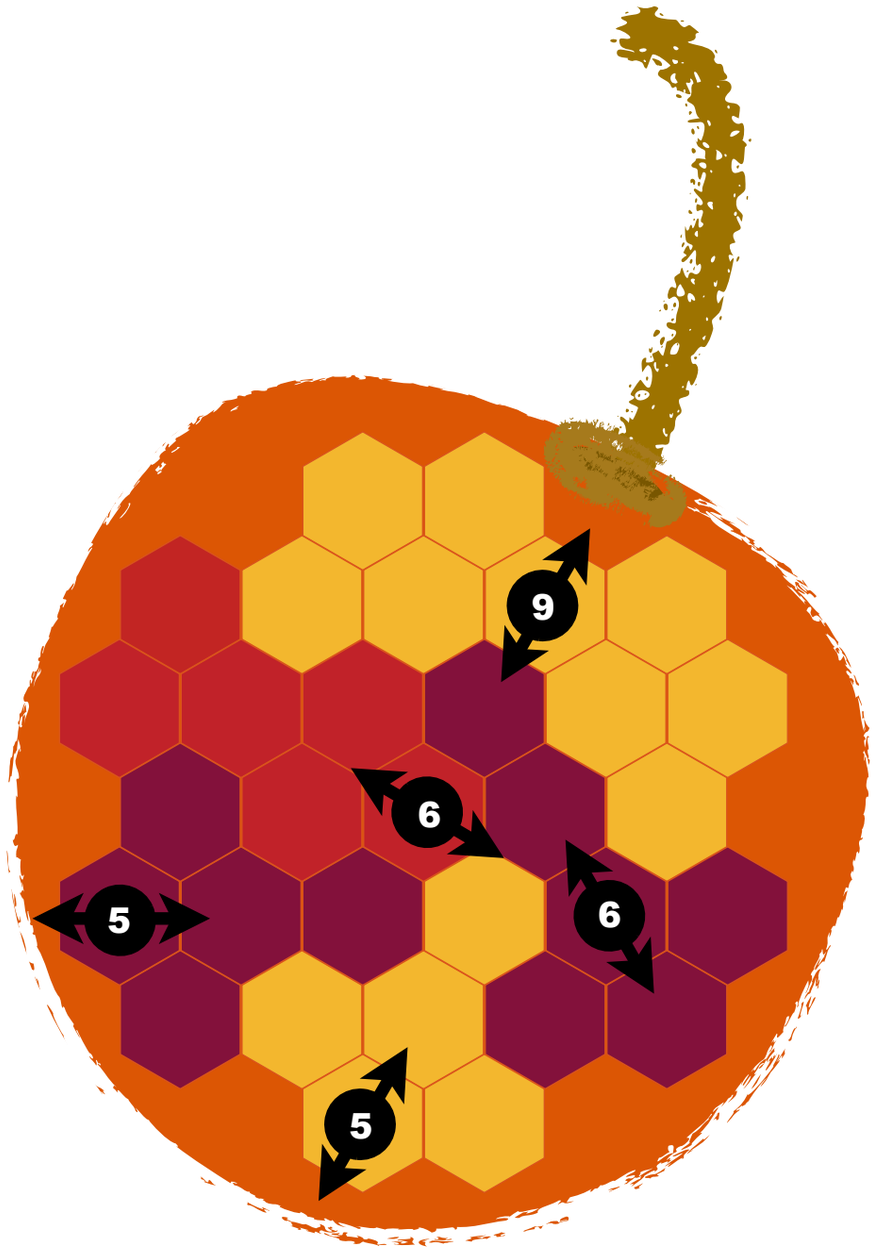


Kajitsu puzzles require fruit to be cut up. The size and symmetry of each piece are given by the pips. The five pieces that make up the cherry puzzle (left) have mirror symmetry.

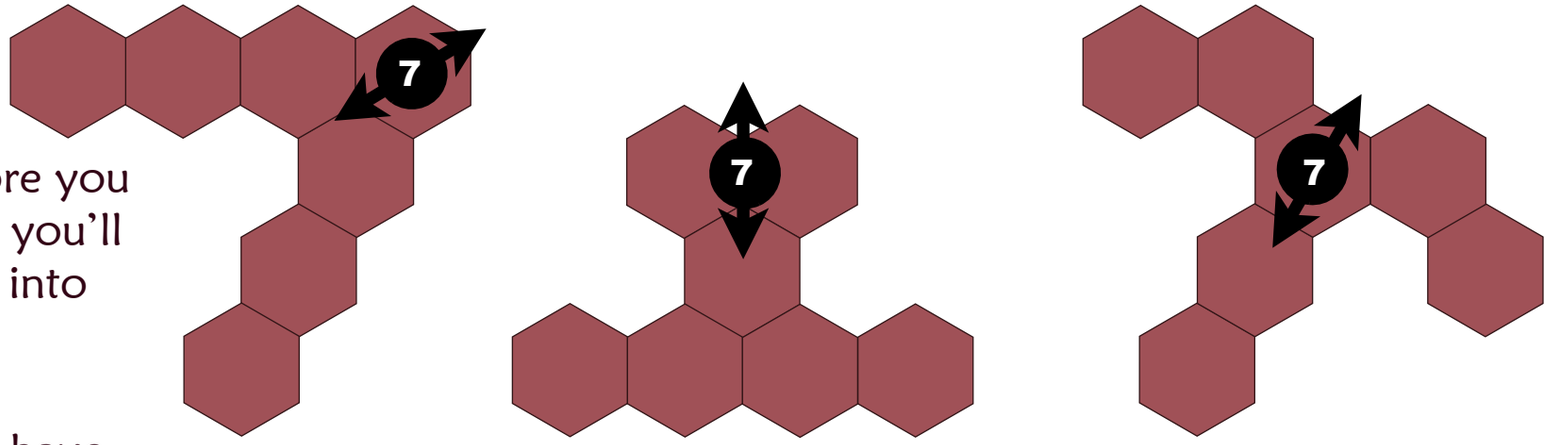
The solution to the puzzle (right) has five pieces with the correct size - each of them reflecting mirror symmetry.

Detailed rules follow...



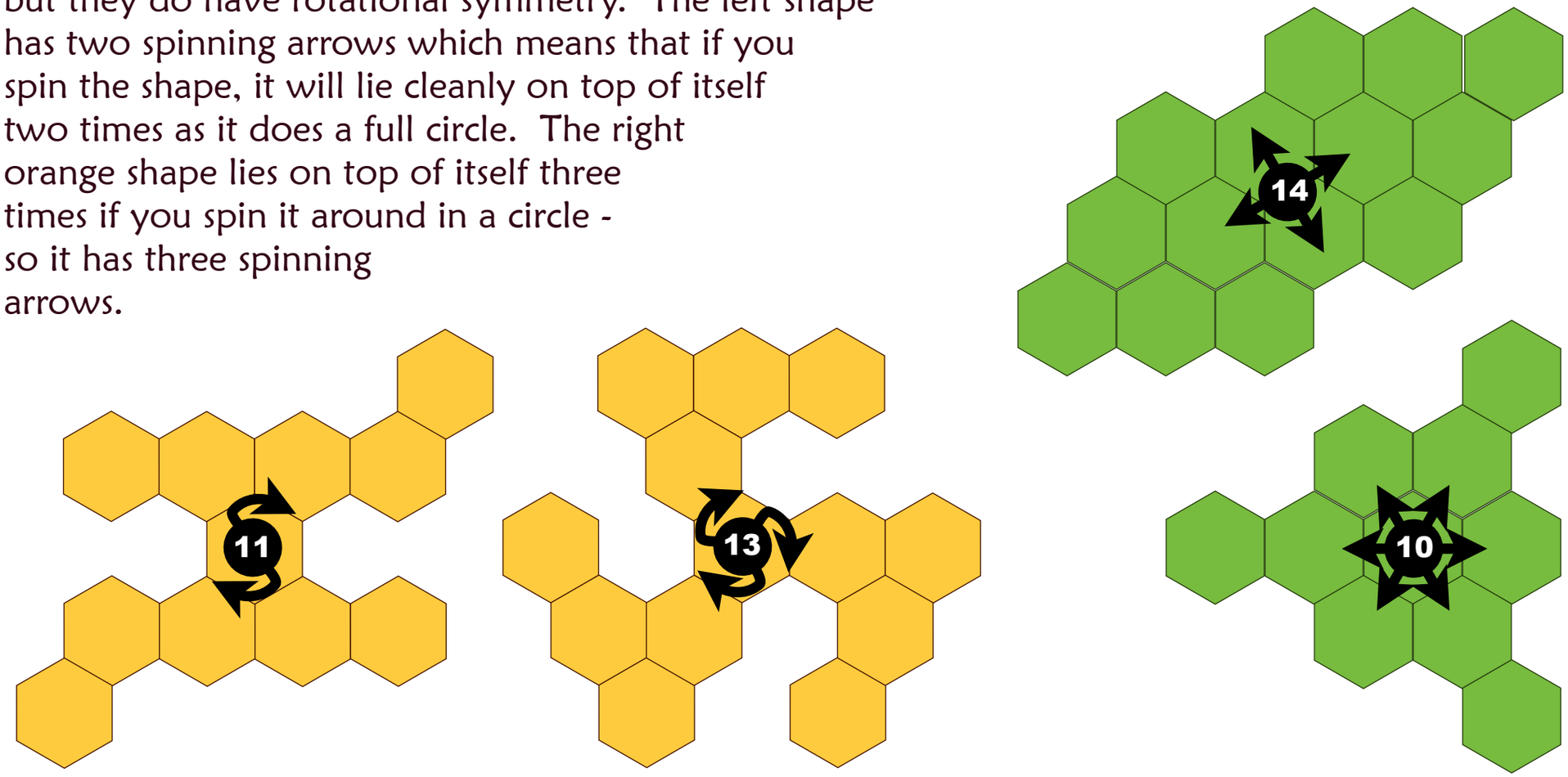
“Kajitsu”

is Japanese for fruit, but before you eat these puzzles, you’ll need to cut them into pieces.

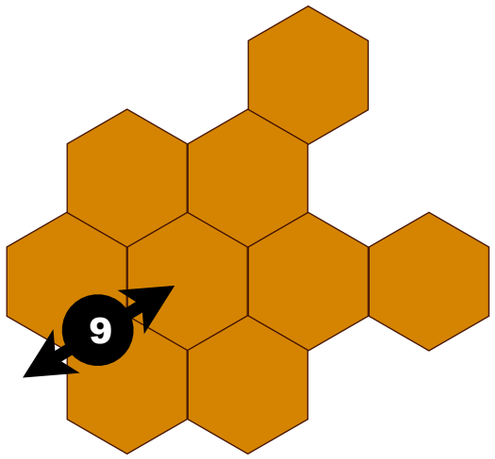


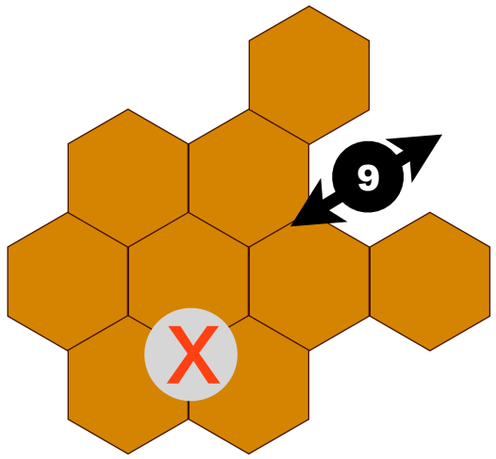
The pieces above have size 7 and mirror symmetry. They have size 7 because they are made up of 7 hexagons. Mirror symmetry means each piece can be folded so that the two halves lie one over the other. The arrows show the direction of the fold line. The 14 and 10 sized pieces have more than one line of mirror symmetry, so their pips ● have multiple arrows.

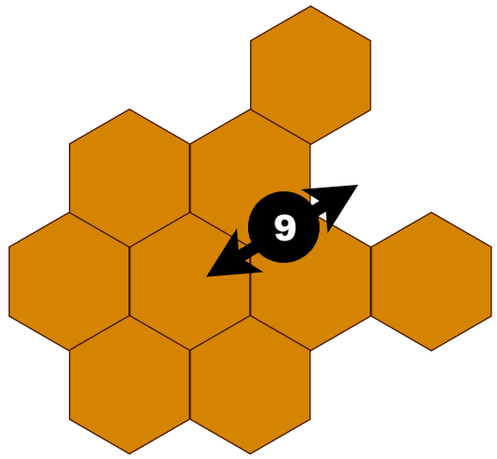
The 11 and 13 sized pieces do not have mirror symmetry, but they do have rotational symmetry. The left shape has two spinning arrows which means that if you spin the shape, it will lie cleanly on top of itself two times as it does a full circle. The right orange shape lies on top of itself three times if you spin it around in a circle - so it has three spinning arrows.



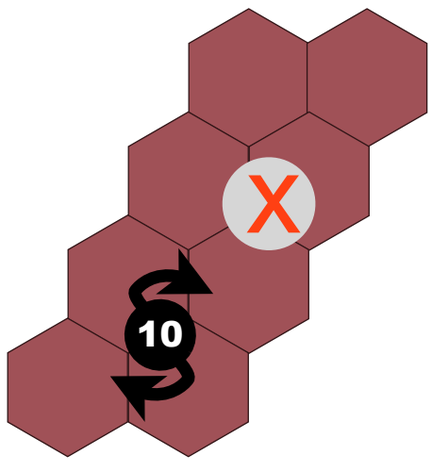
The pip identifies all the mirror symmetries of a piece. Arrows may extend outside a piece, but the pip must always be inside.

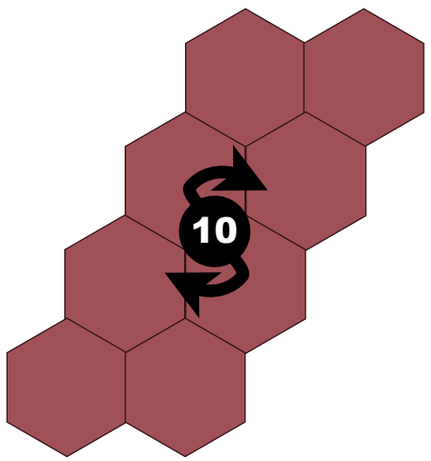


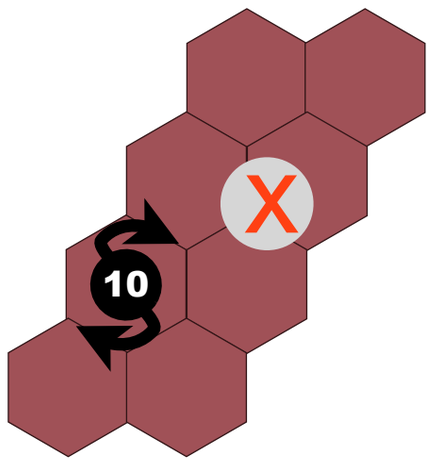




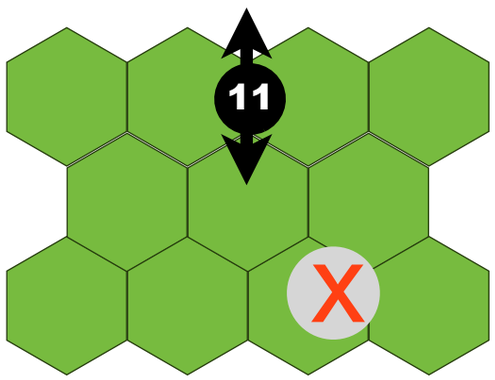
If a piece only has rotational symmetry, the pip will be central.

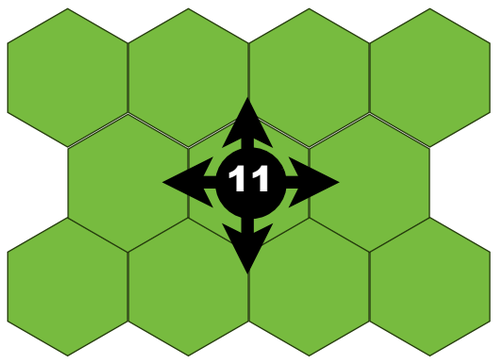


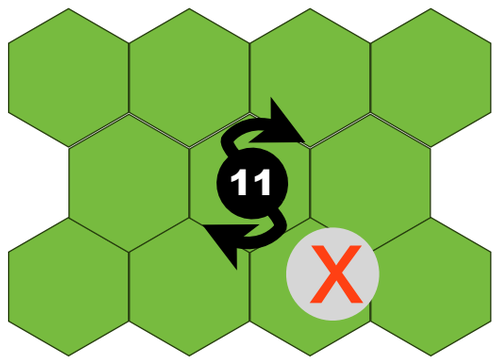




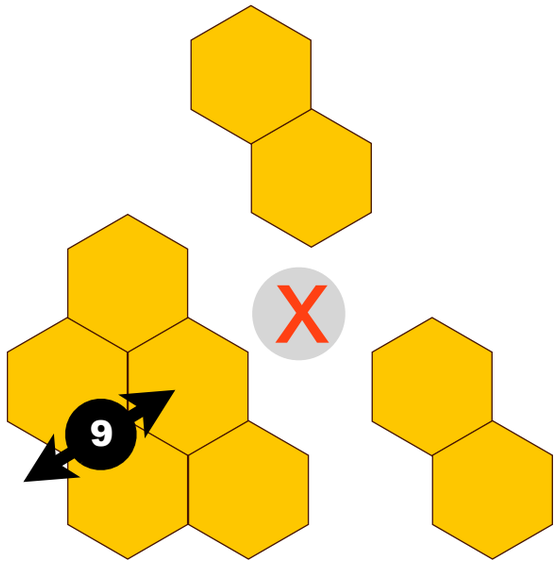
If a piece has both rotational and mirror symmetries - only the mirror symmetries are shown.

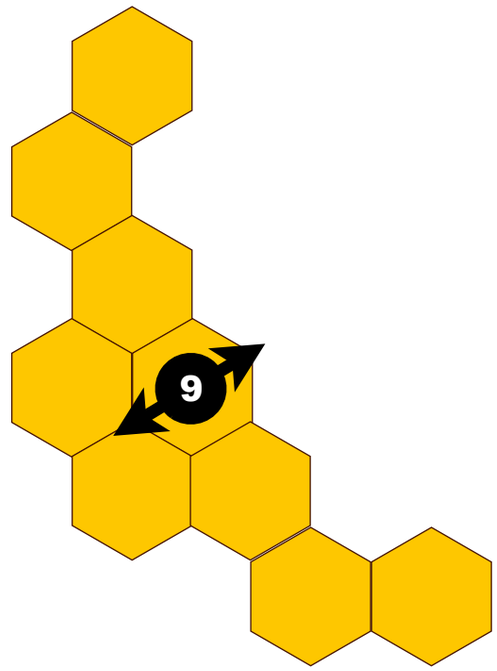


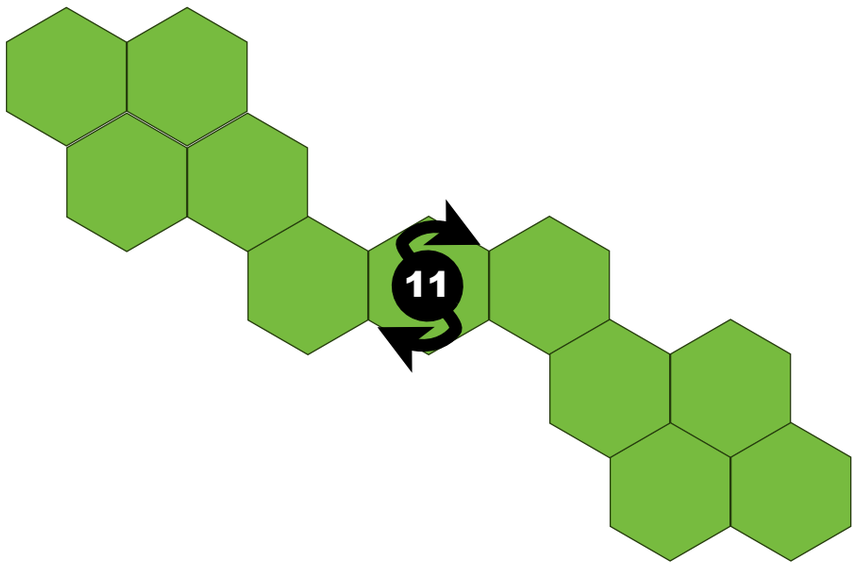


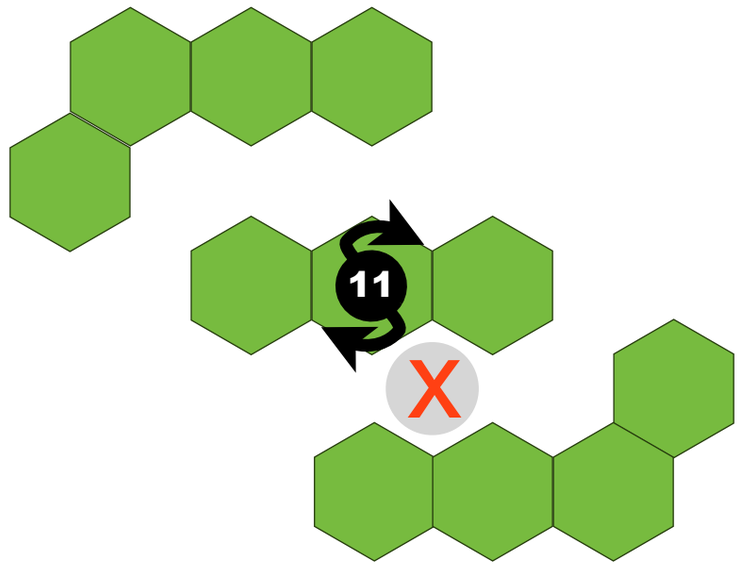


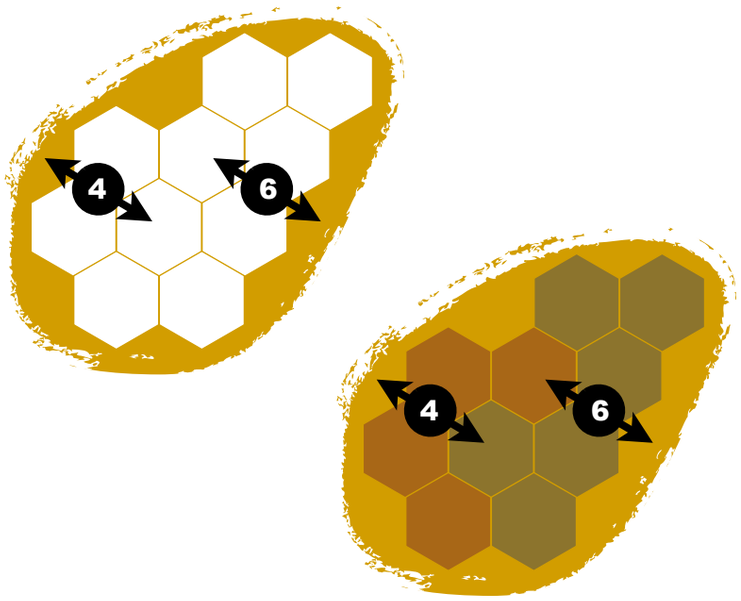
Each morsel must be connected.

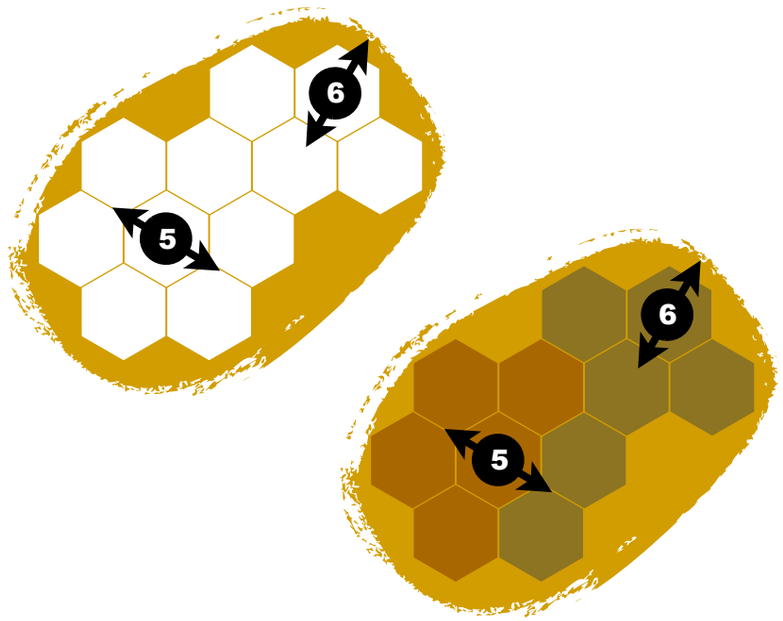






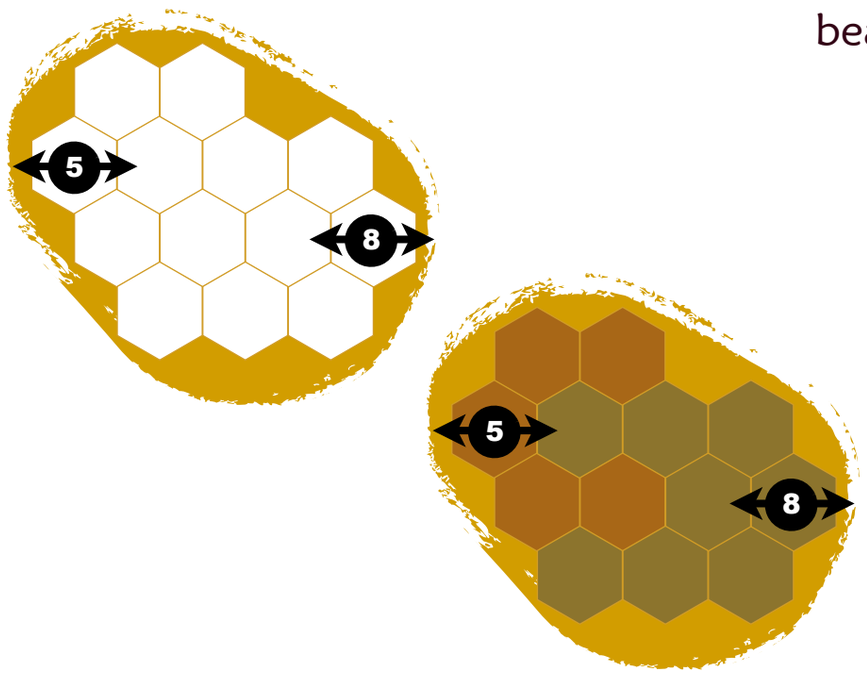


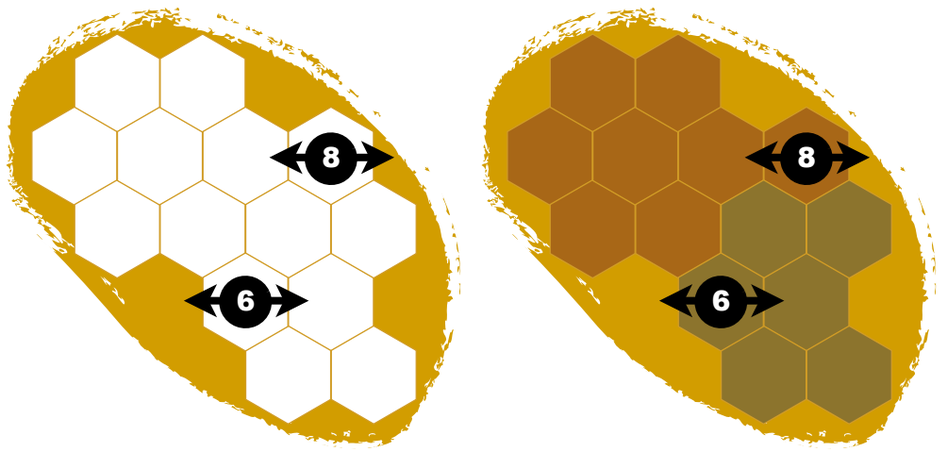




Some students will laugh at the ridiculousness of cutting up raisins and grapes - others will be inspired by beauty.

I've had students ask to have an extra worksheet so they can produce a beautiful copy.





Kajitsu in the Classroom

http://youtu.be/JHhN9LTPGzw

boring for 30% of students!

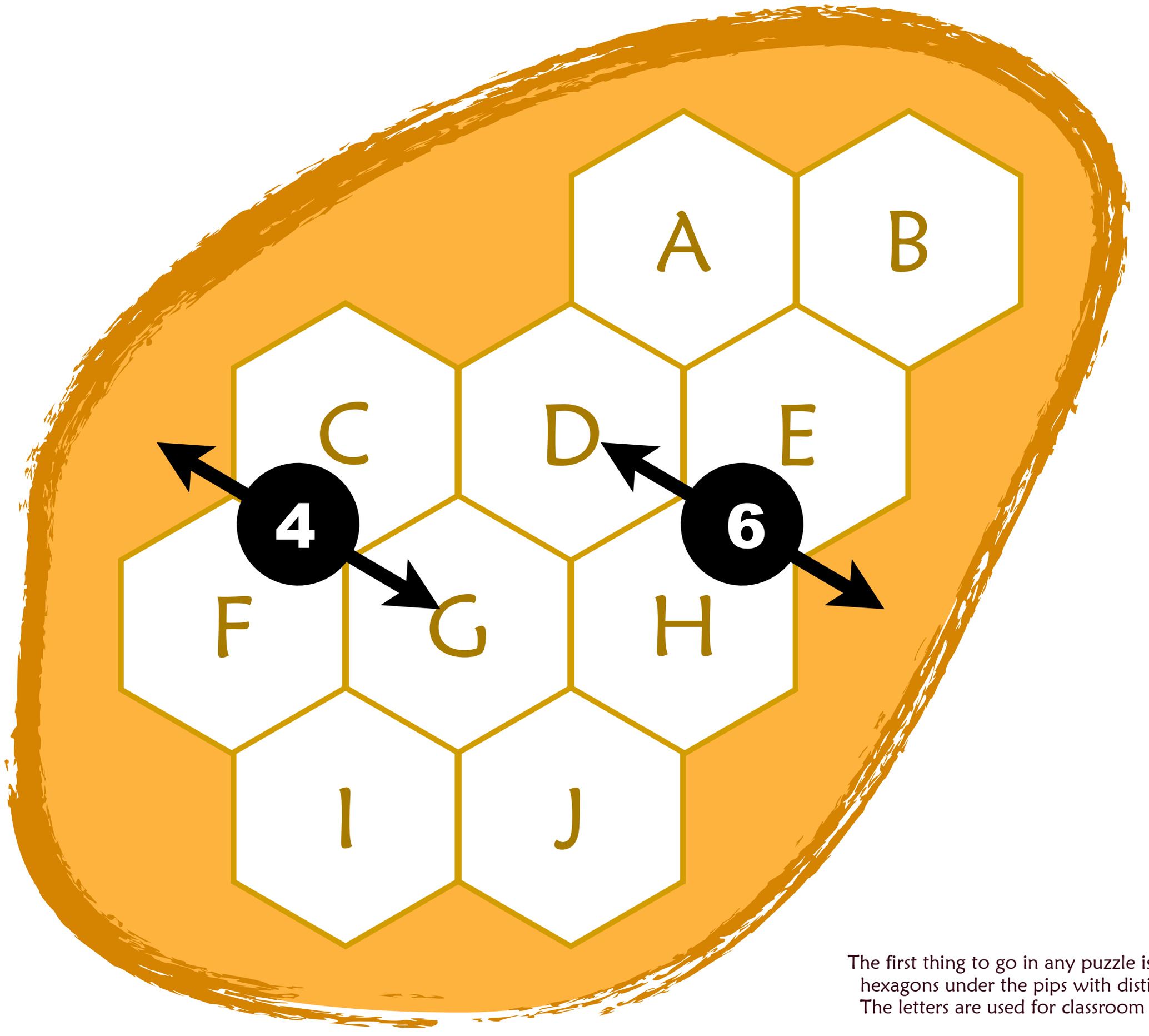
When introducing kajitsu to a classroom - instead of going through the rules on the previous pages - experiment jumping right in - asking the first student in the back left to choose what color to shade a specific hexagon. Then go on to the next student and the next... When a morsel has been created that lacks the required symmetry - then have fun with saying "failure!" and start again. At this point you can grudgingly reveal a rule and tell the failing group why they failed. This is a perfect excuse to give students a tongue-in-cheek failure that helps remove the stigma of failure from the classroom.

With this technique, you're baiting some of your difficult-to-engage students both emotionally and through humor. Top students can also have fun with this technique - trying to figure out the rules before they are revealed through classroom failures.

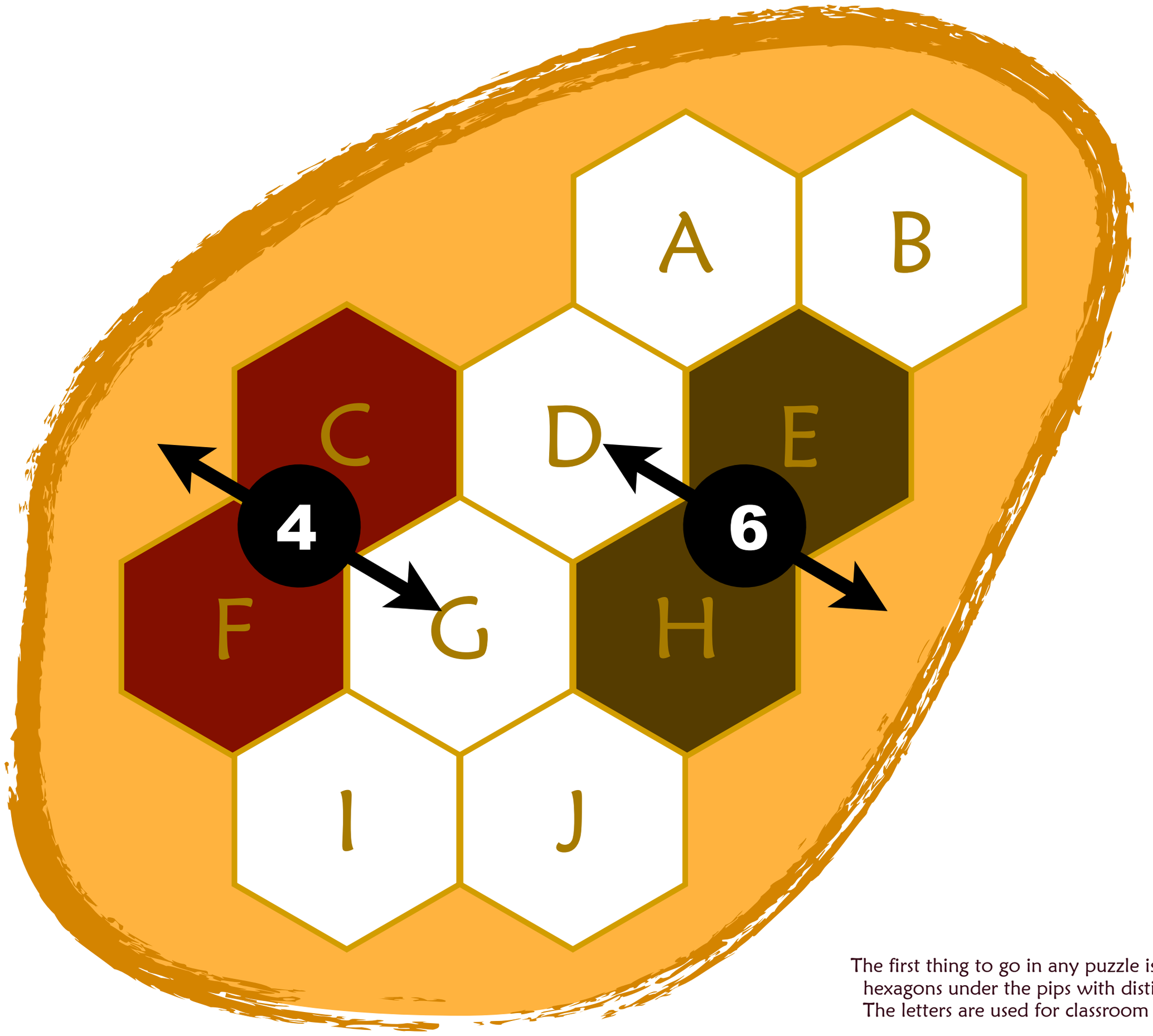
If you are going to teach through examples, most should be failures. This is much more compelling for the average student.

Another trick to engage students is ironically not to allow anyone to raise their hand if they know an answer. By allowing speedy students to shoot their hands up when they have an answer you rob slower students of a full-hearted success and reduce their interest in engaging. Instead of having hands raised you systematically get everyone to contribute. When worksheets are handed out, speedy students can race ahead. When they are finished a worksheet, quickly give them another so the slow students don't know that they are slow. Slow, ponderous thinkers are to be nurtured and their self-esteems subtly protected from your speedy students.

The following pages are an example of a classroom encounter. If you don't have the ability to shade in the hexagons, just reproduce the puzzles on a white board or discuss without shading in.



The first thing to go in any puzzle is shade the hexagons under the pips with distinct colors. The letters are used for classroom discussion.



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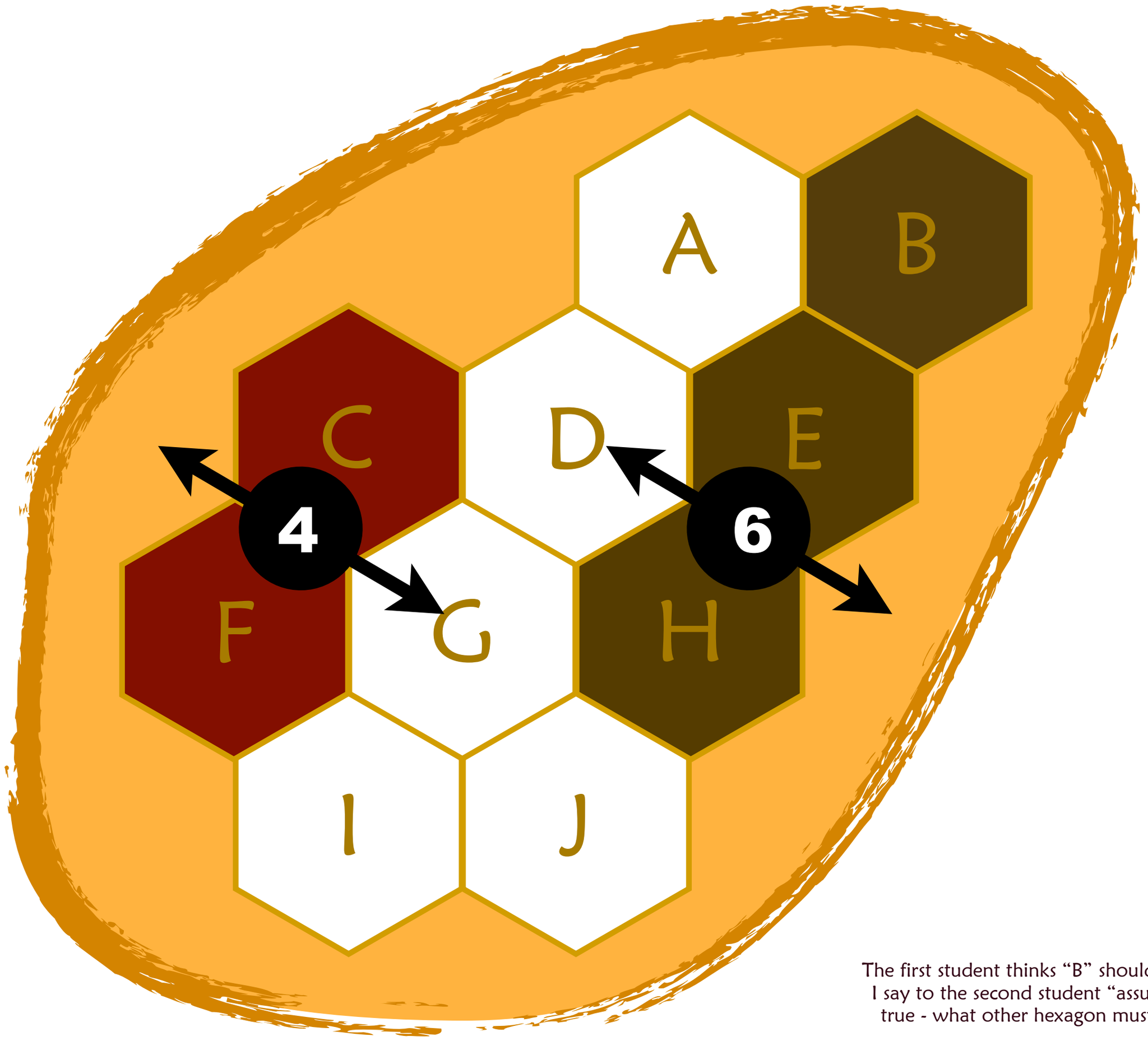
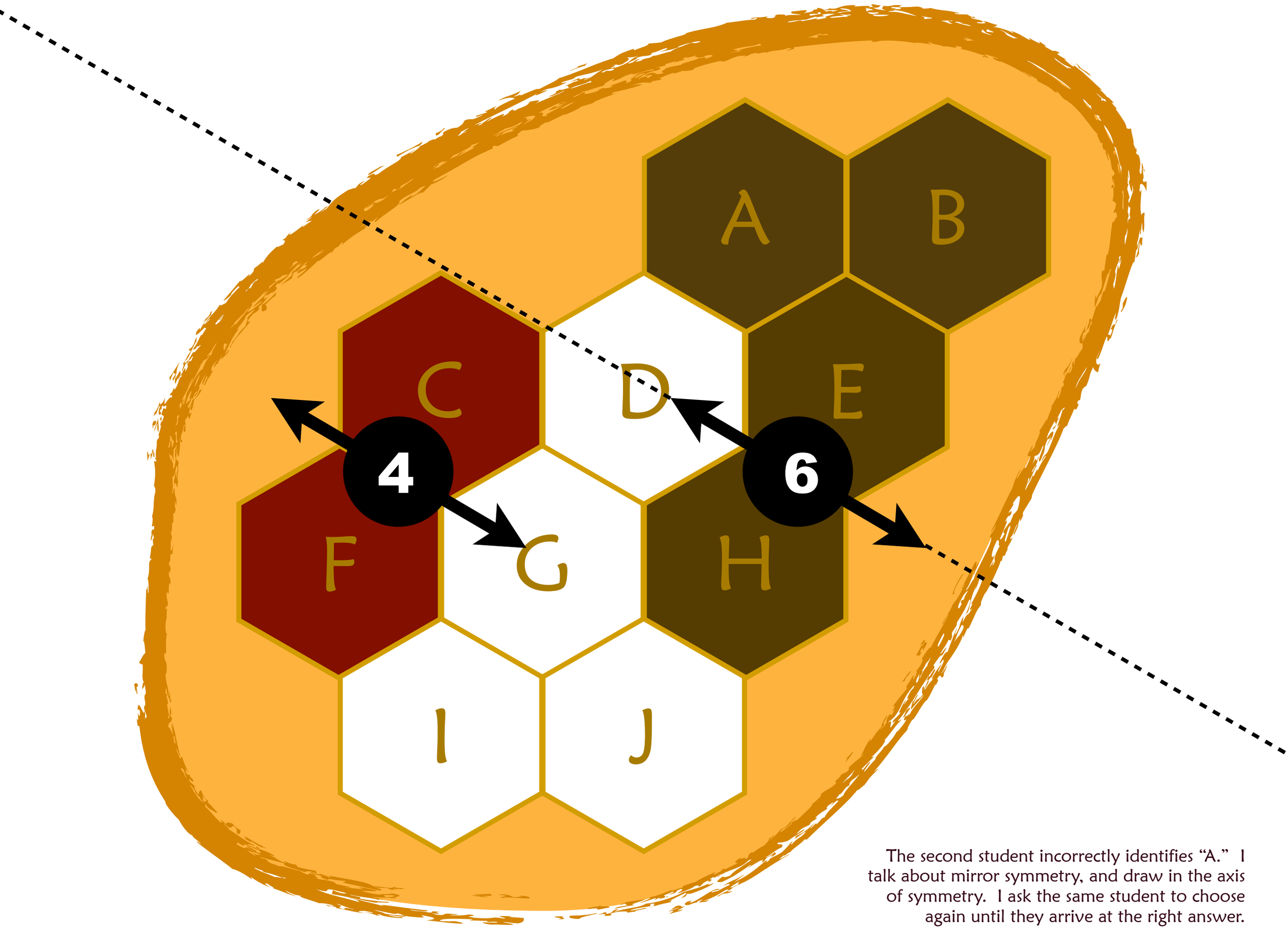
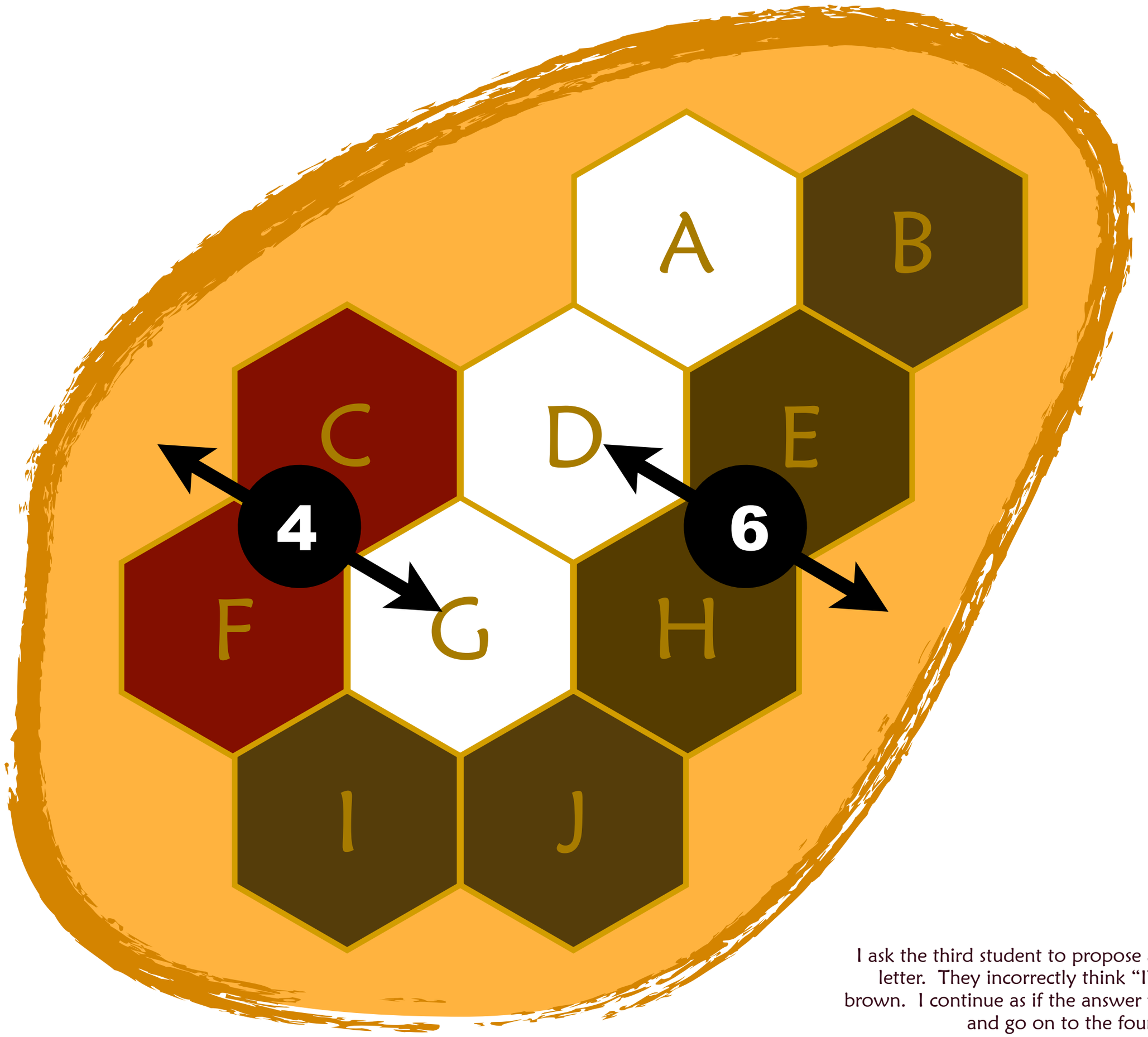


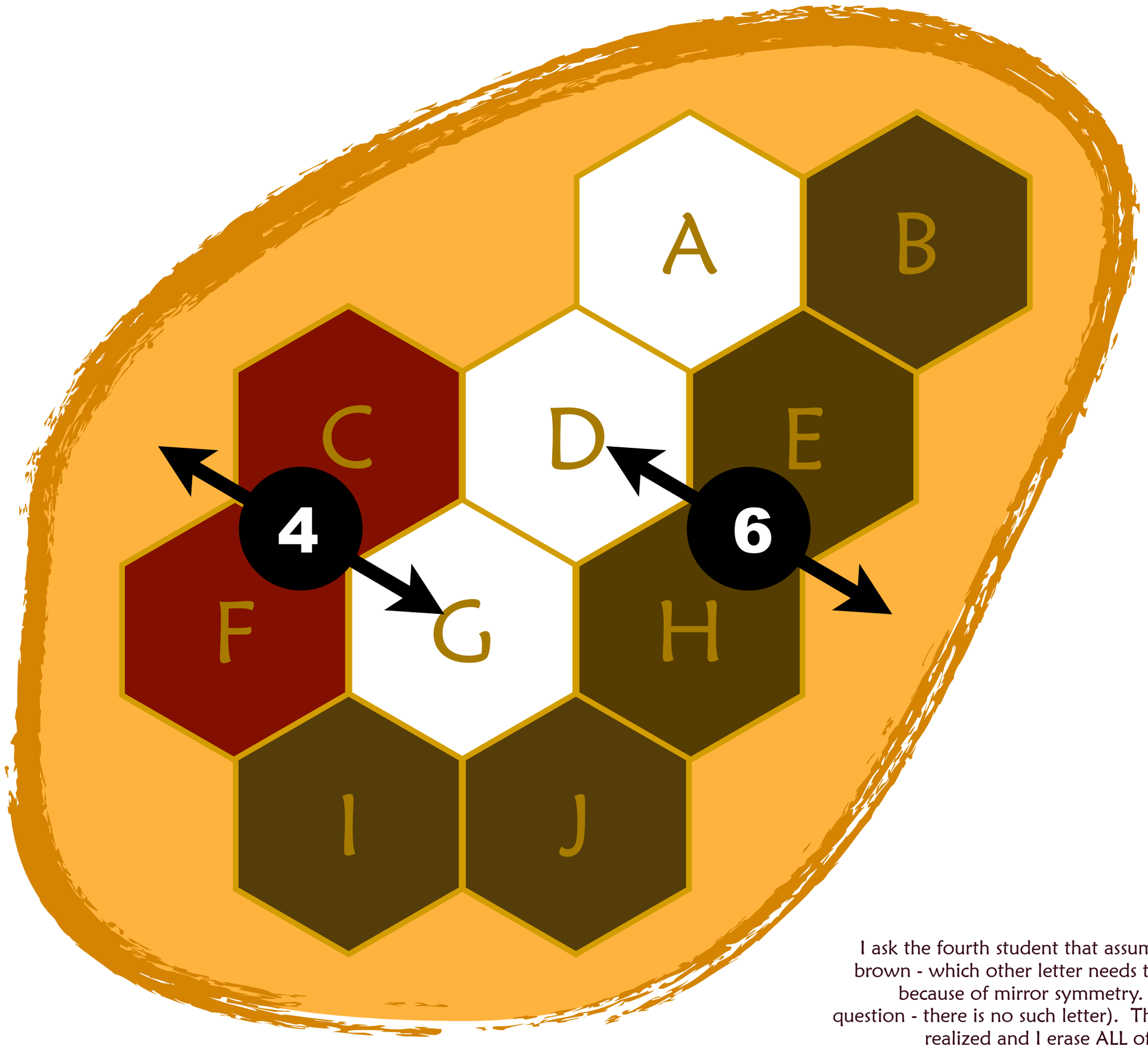
The first student thinks “B” should be brown.
I say to the second student “assuming that is true - what other hexagon must be brown?



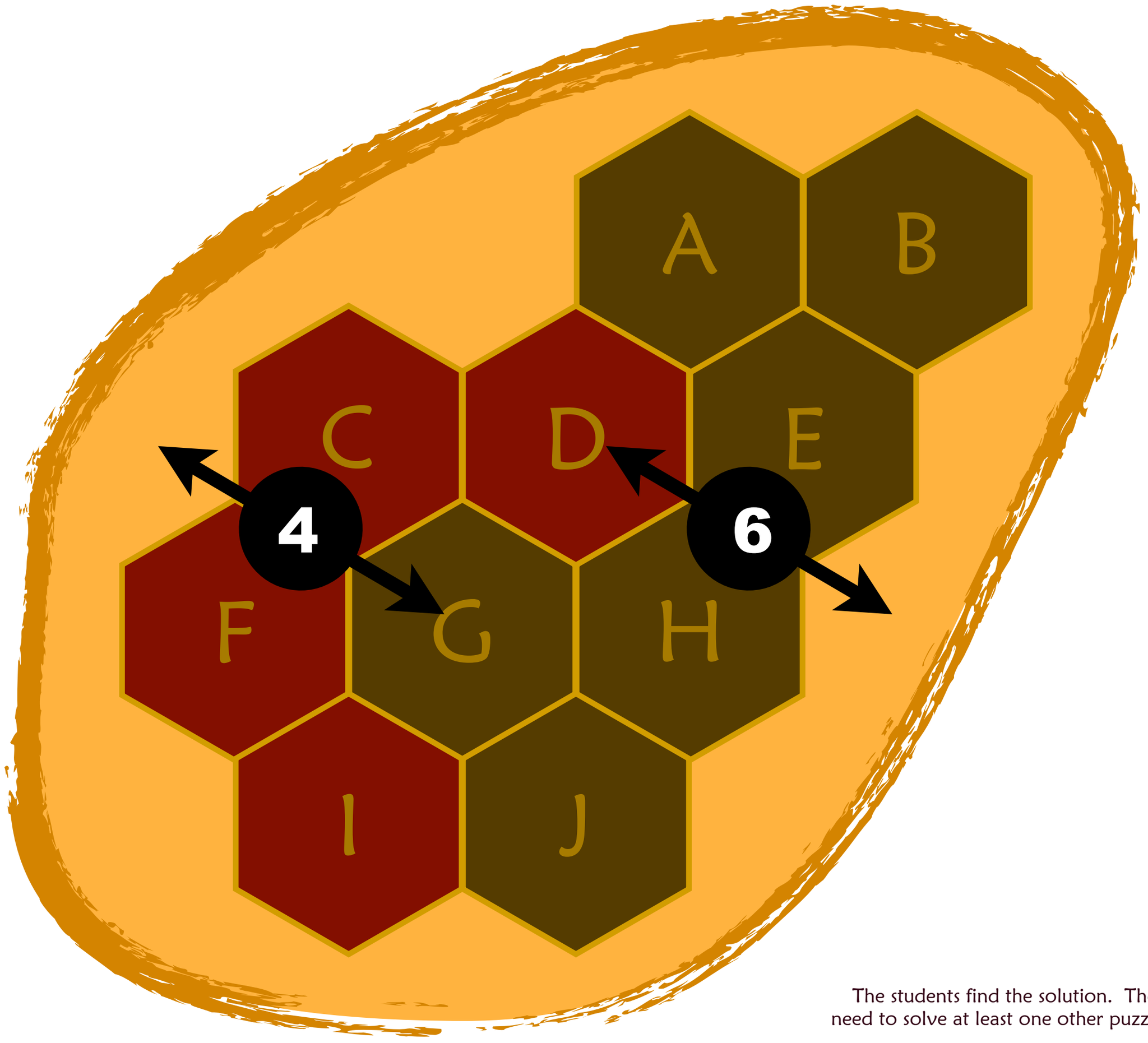
The second student incorrectly identifies “A.” I talk about mirror symmetry, and draw in the axis of symmetry. I ask the same student to choose again until they arrive at the right answer.



I ask the third student to propose a color and letter. They incorrectly think "I" should be brown. I continue as if the answer was correct and go on to the fourth student.



I ask the fourth student that assuming “I” was brown - which other letter needs to be brown because of mirror symmetry. (It is a trick question - there is no such letter). The mistake is realized and I erase ALL of the puzzle.



The students find the solution. They probably need to solve at least one other puzzle as a class.

Standards for Mathematical Practice

All MathPickle puzzle designs, including 果実 kajitsu, are guaranteed to engage a wide spectrum of student abilities while targeting the following Standards for Mathematical Practice:

MP1 Toughen up!
This is problem solving where our students develop grit and resiliency in the face of nasty, thorny problems. It is the most sought after skill for our students.

MP3 Work together!
This is collaborative problem solving in which students discuss their strategies to solve a problem and identify missteps in a failed solution. MathPickle recommends pairing up students for all its puzzles.

MP6 Be precise!
This is where our students learn to communicate using precise terminology. MathPickle encourages students not only to use the precise terms of others, but to invent and rigorously define their own terms.

MP7 Be observant!
One of the things that the human brain does very well is identify pattern. We sometimes do this too well and identify patterns that don't really exist.

(http://www.corestandards.org/Math/Practice/)

Common Core State Standards

果実 kajitsu targets the following Common Core State Standards:

Grade 4

CCSS.MATH.CONTENT.4.G.A.3
Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Grade 8

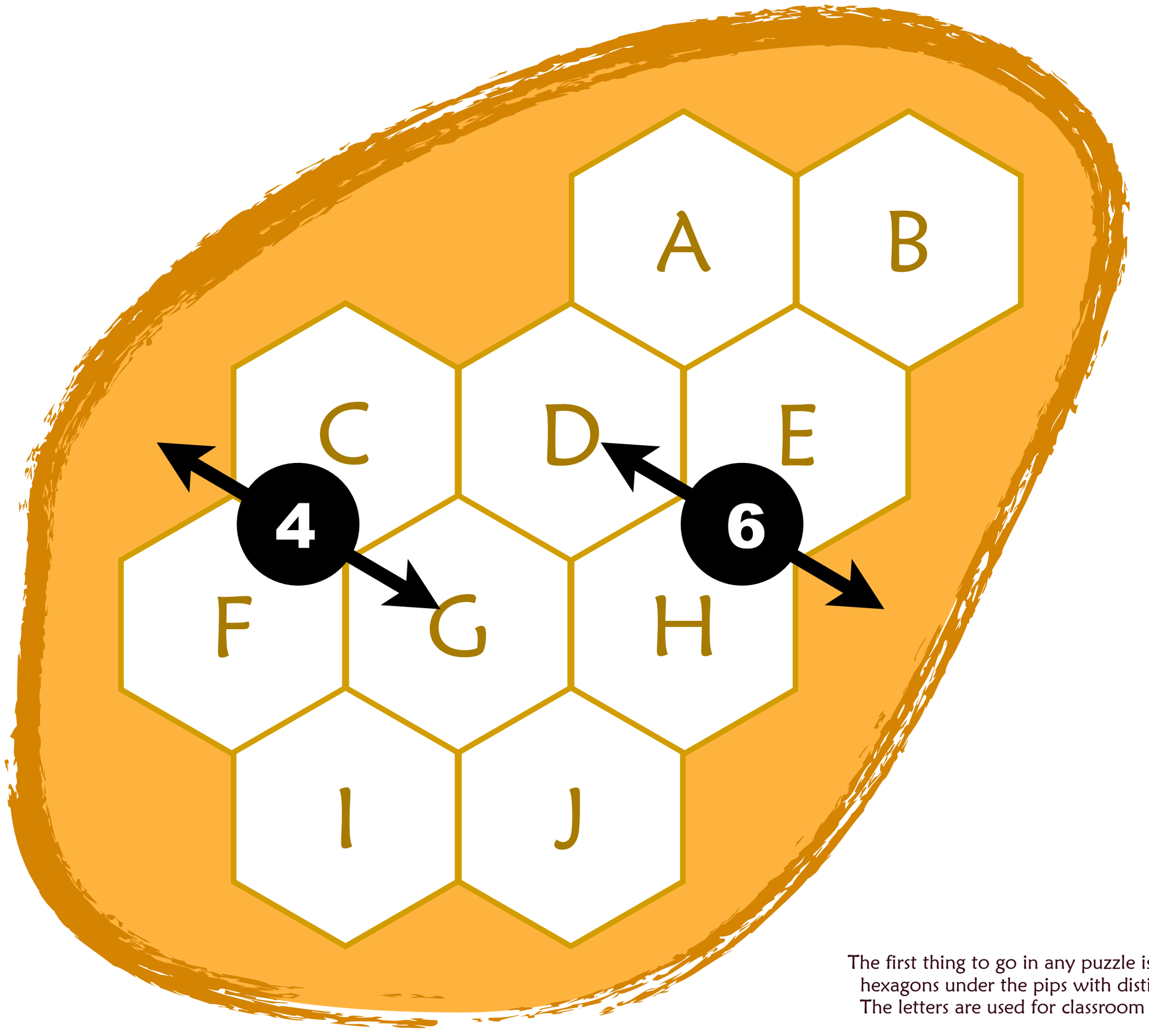
CCSS.MATH.CONTENT.8.G.A.1
Verify experimentally the properties of rotations, reflections, and translations:

CCSS.MATH.CONTENT.8.G.A.1.A
Lines are taken to lines, and line segments to line segments of the same length.

CCSS.MATH.CONTENT.8.G.A.1.B
Angles are taken to angles of the same measure.

CCSS.MATH.CONTENT.8.G.A.1.C
Parallel lines are taken to parallel lines.

果実 kajitsu belongs in both the grade 4 and grade 8 classroom. This ability to appeal to a wide range of ages and abilities is typical of beautiful puzzles.



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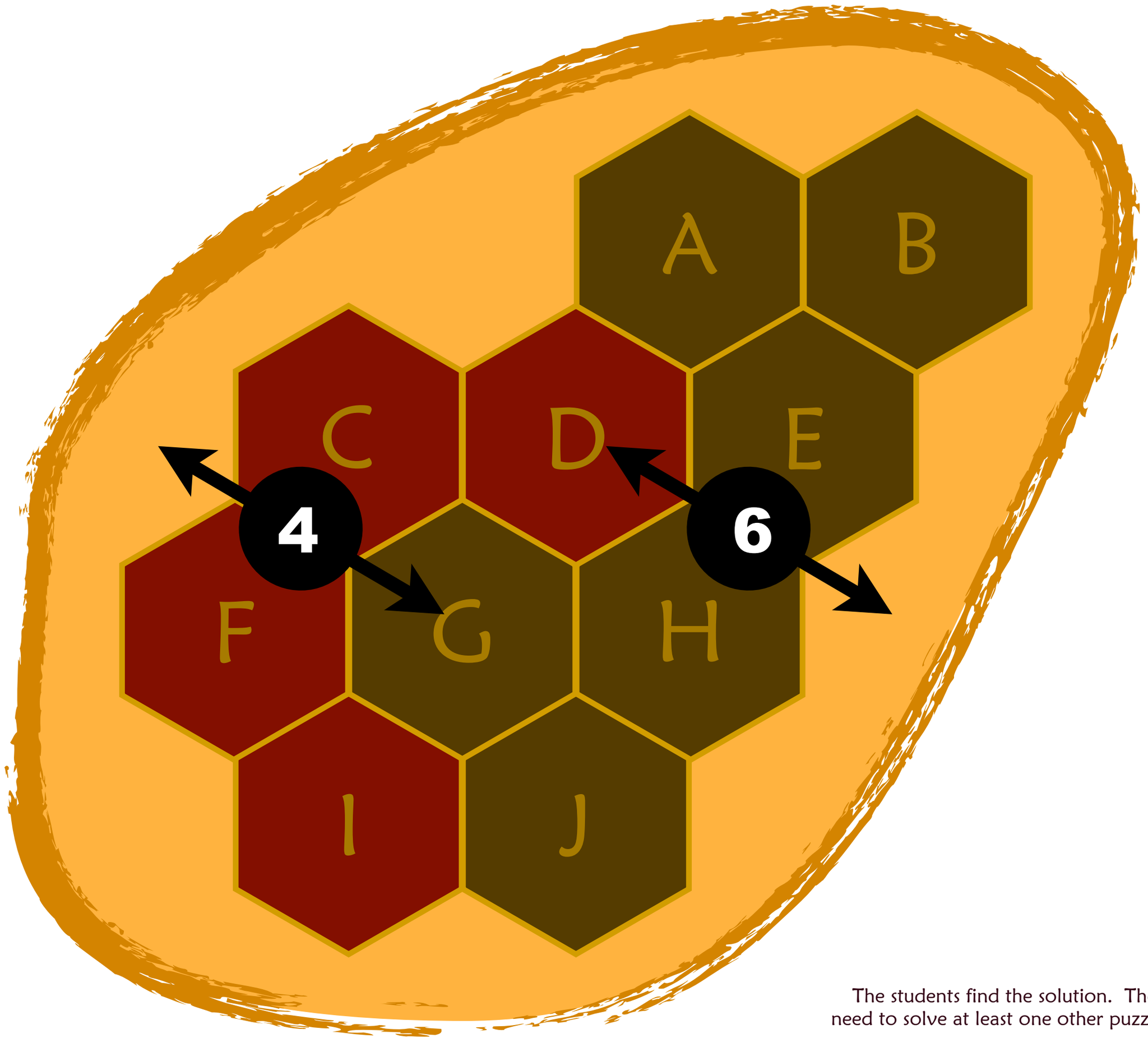

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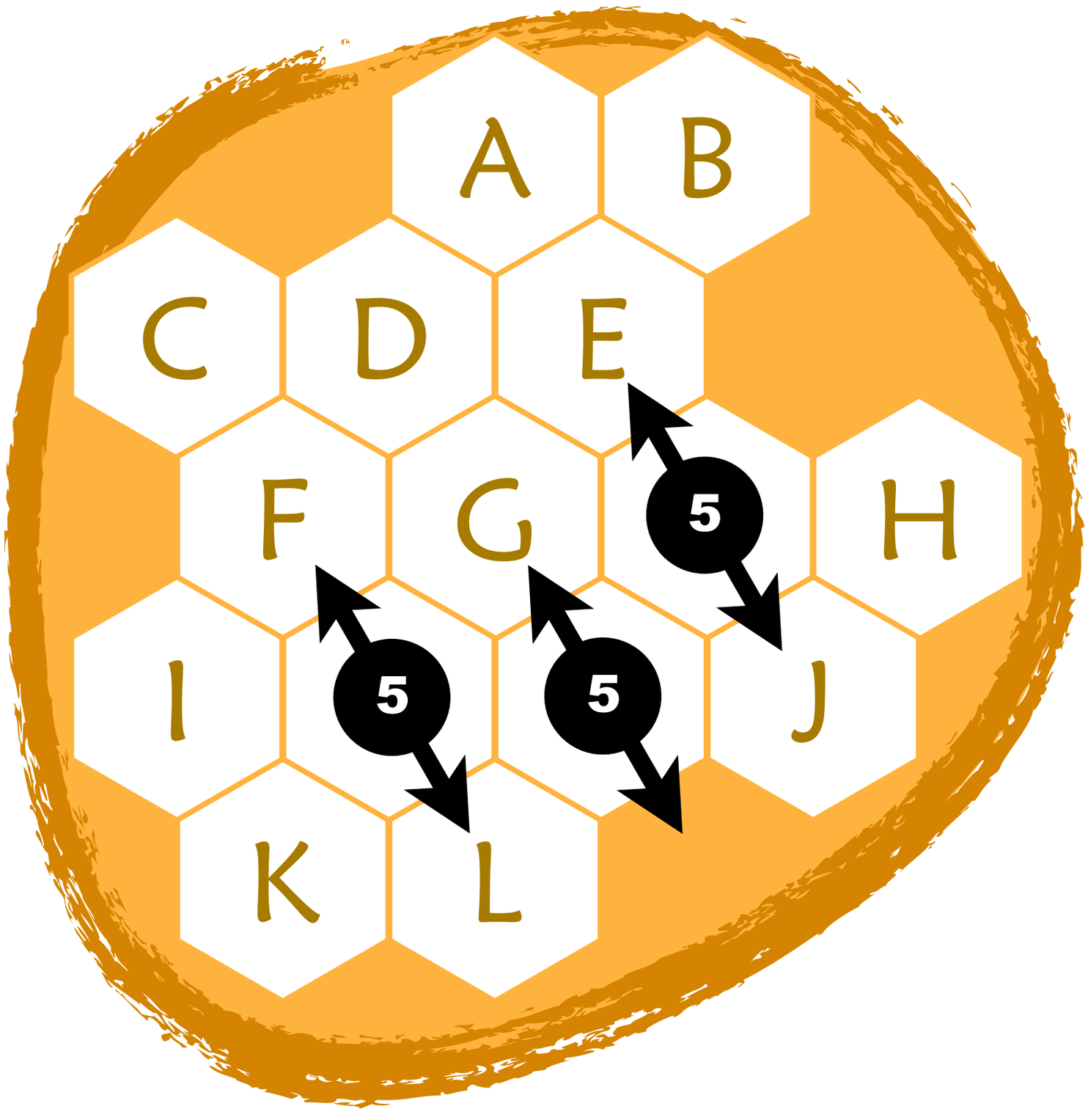
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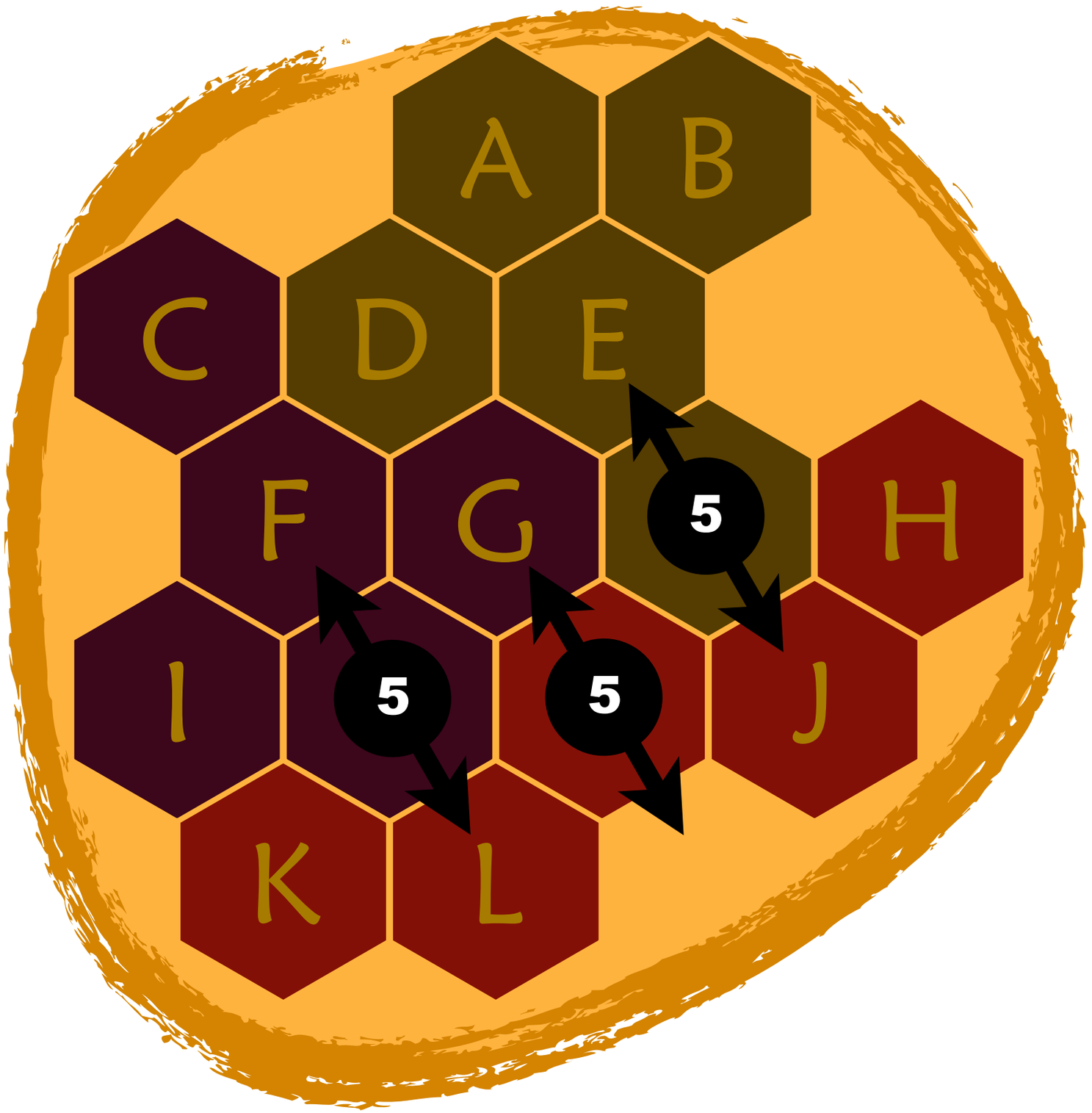
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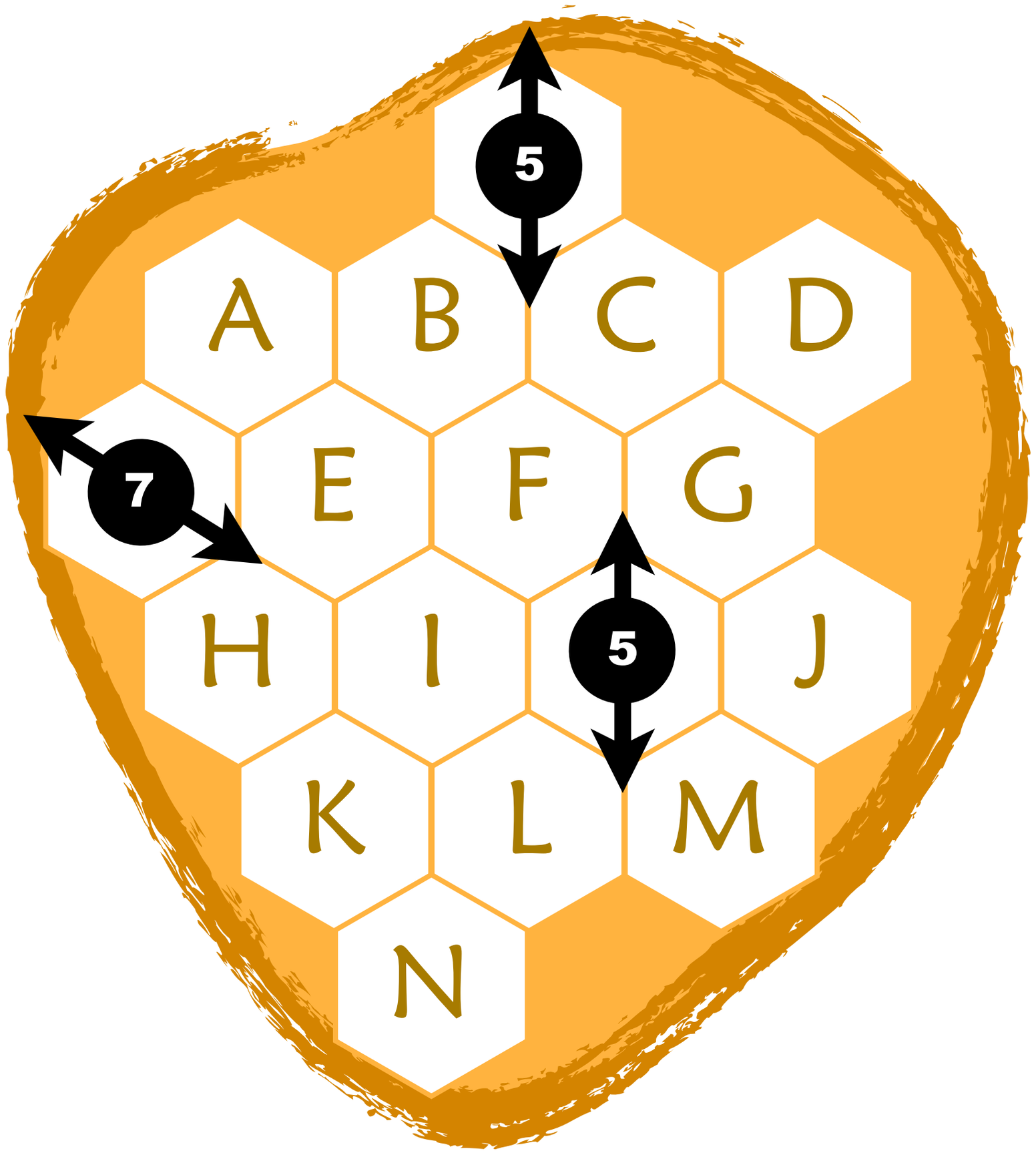
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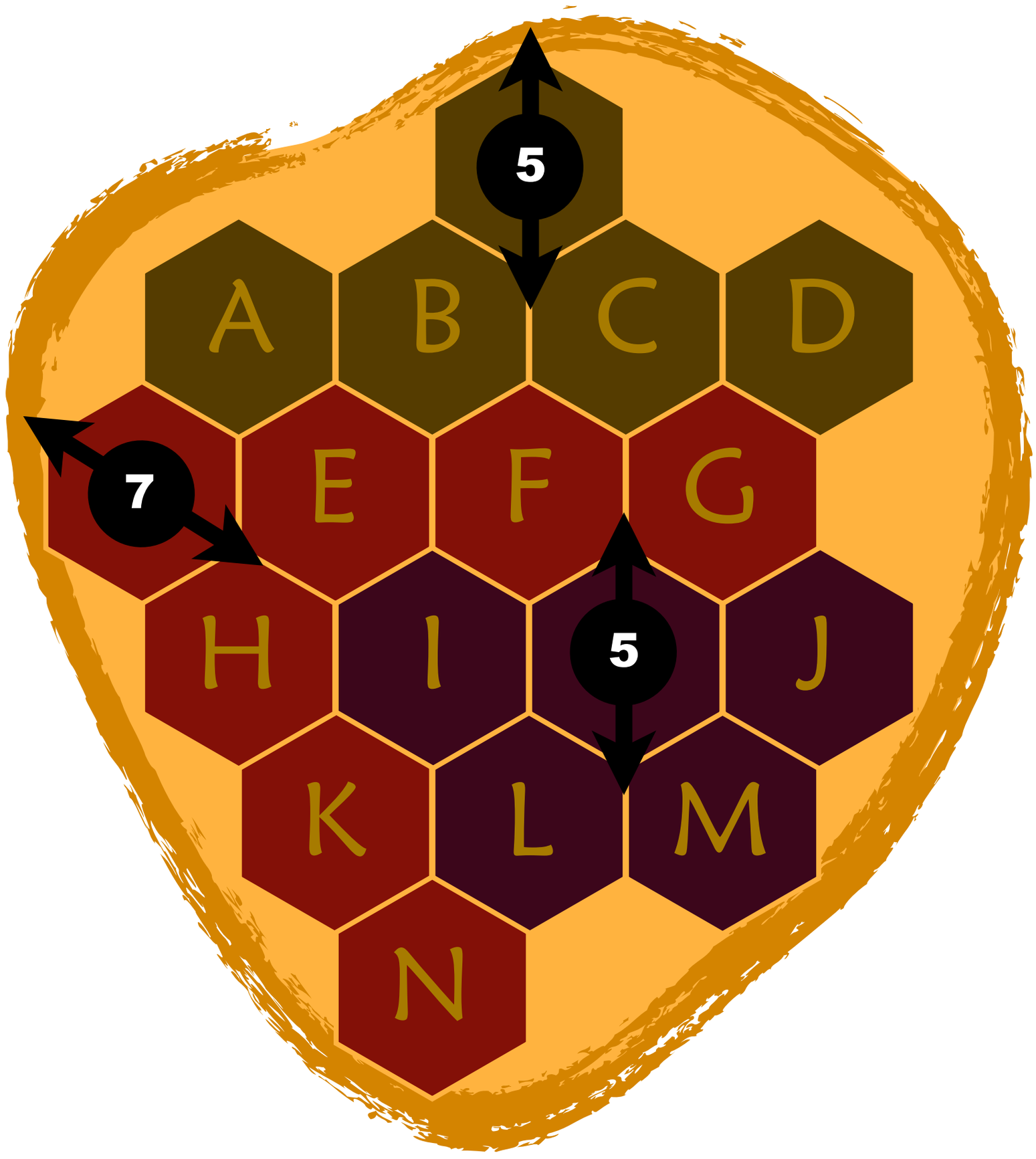
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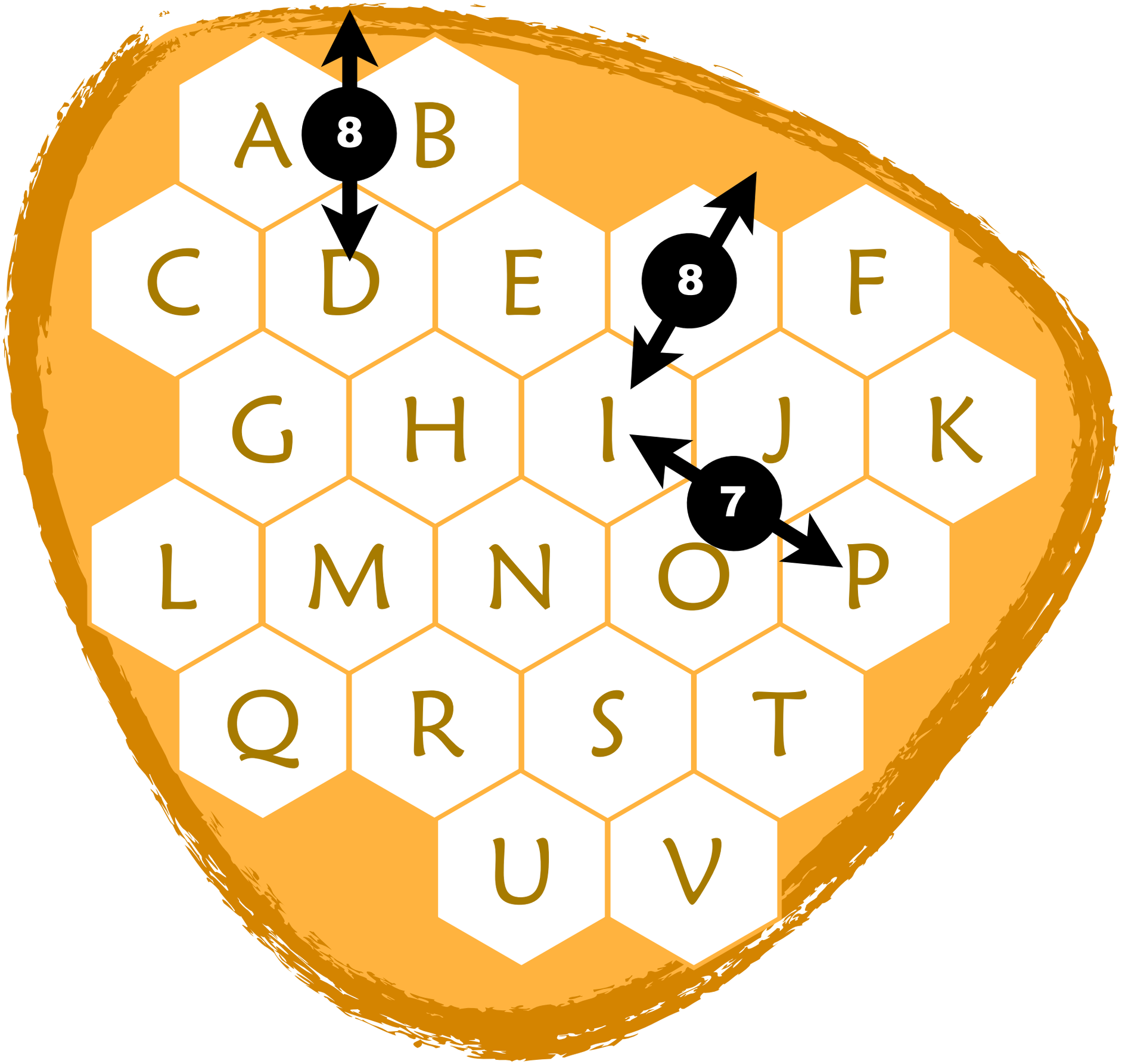
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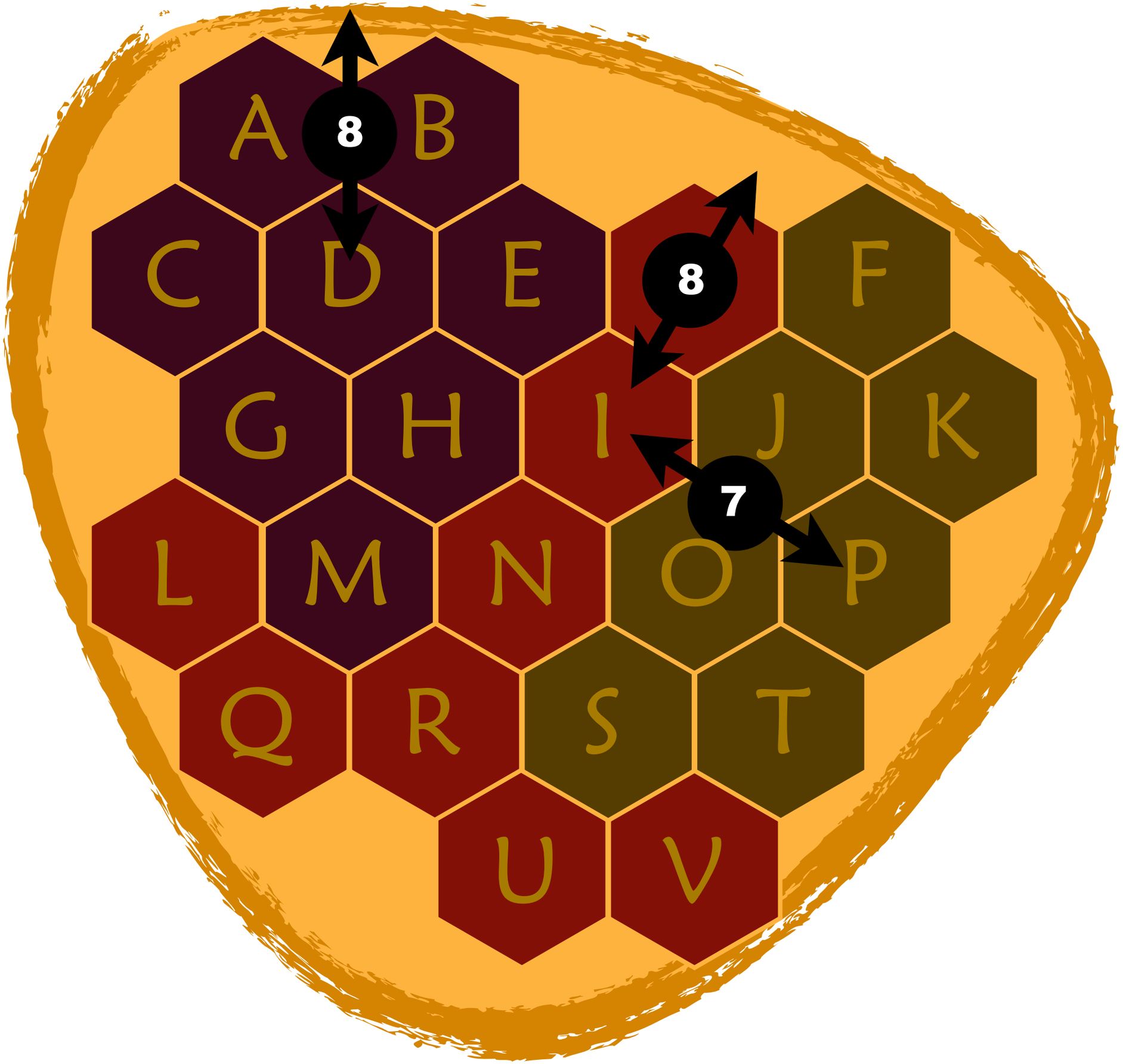
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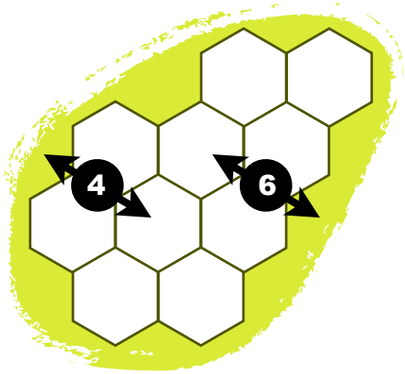
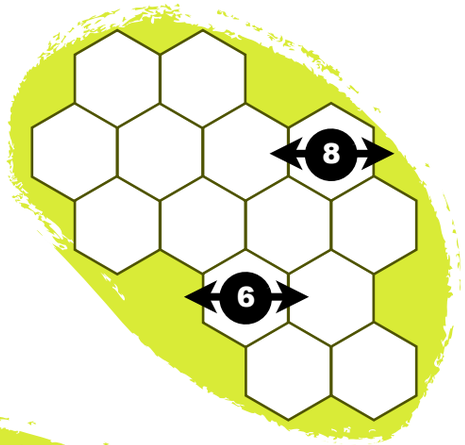
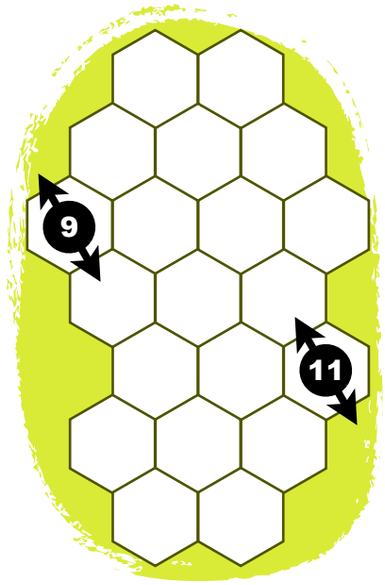
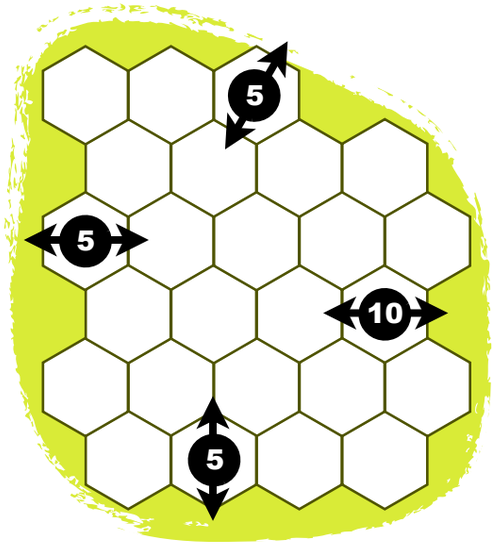
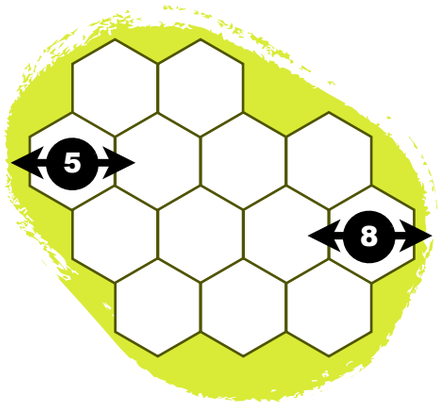
Grapes

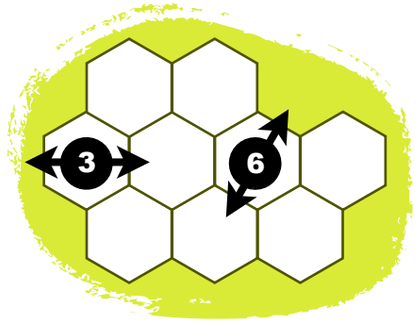


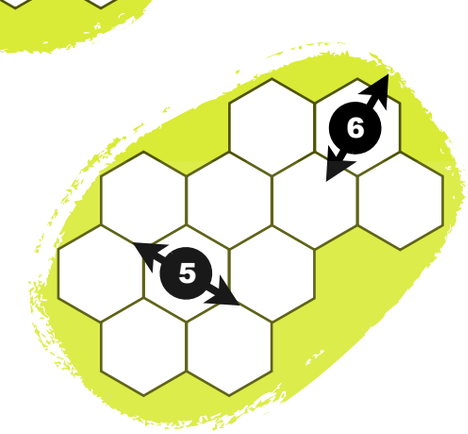


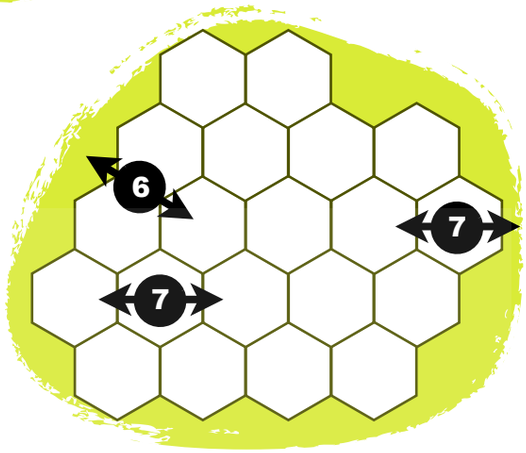


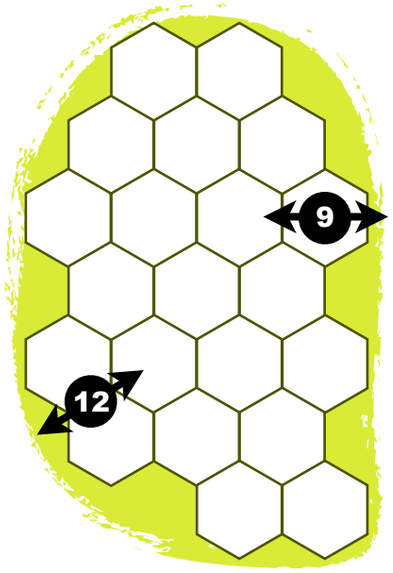


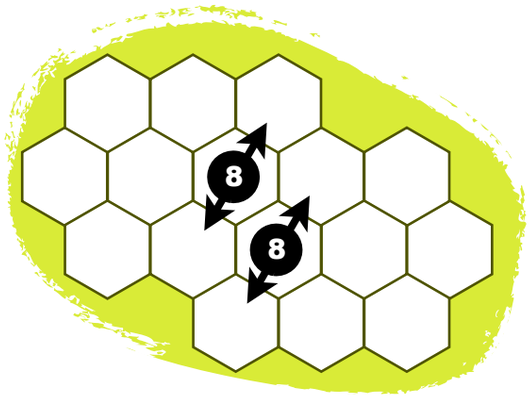


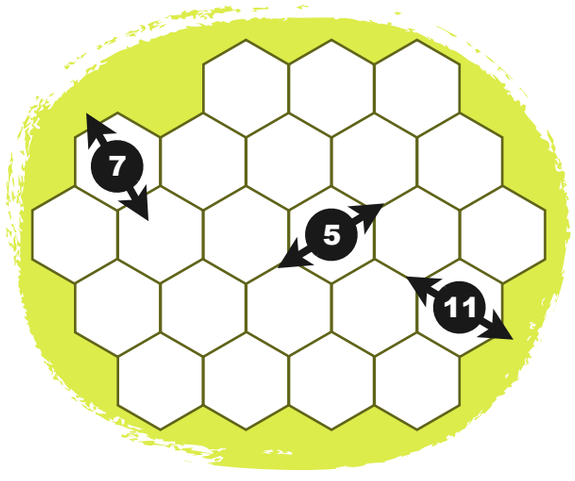


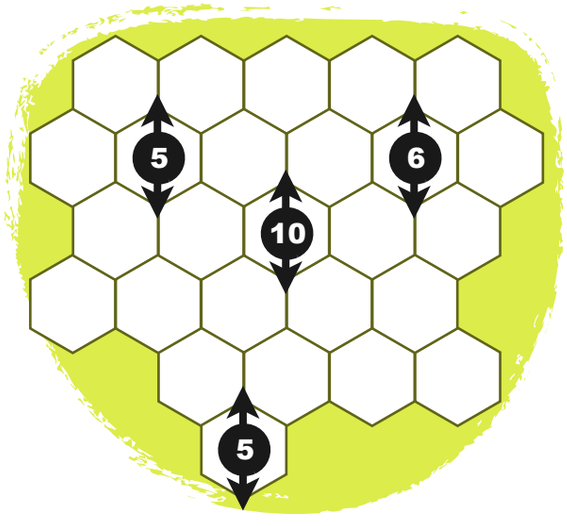


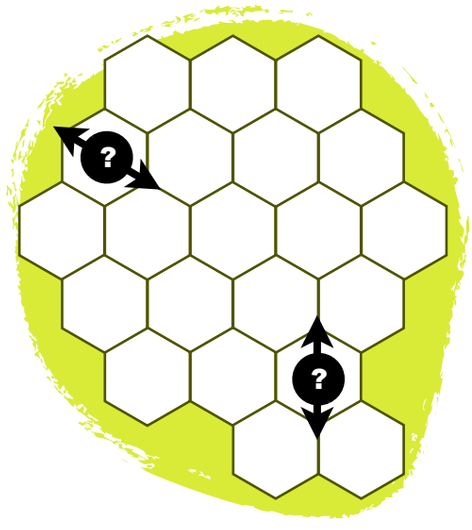


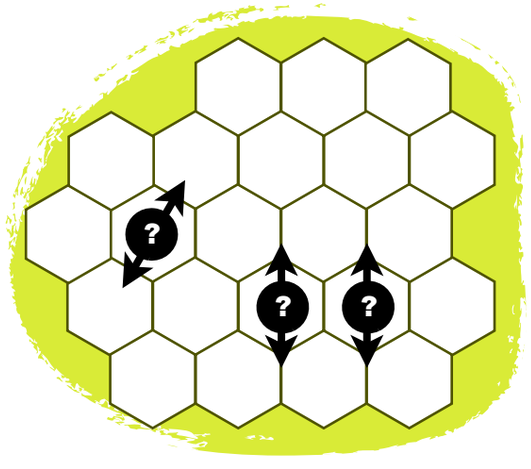






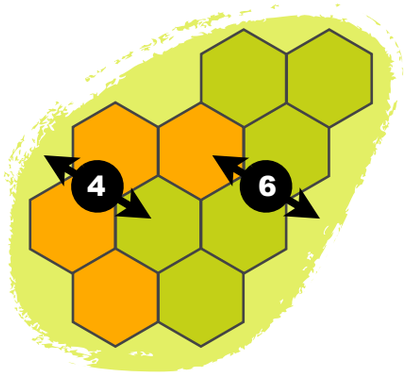


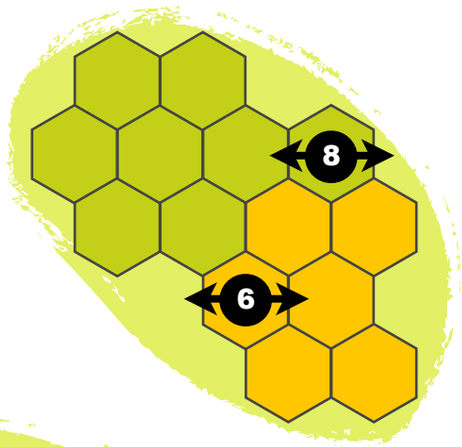


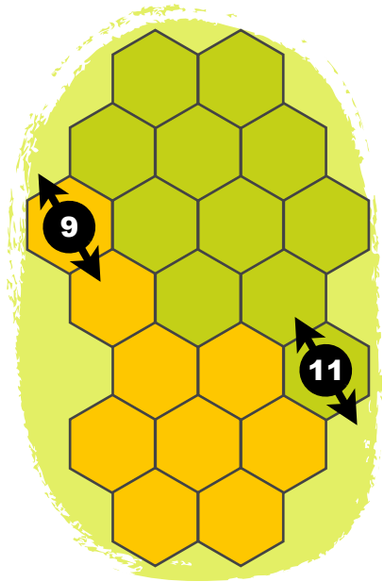


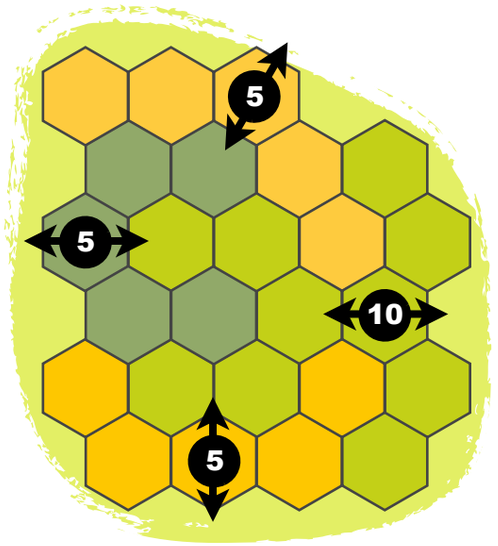
Grapes

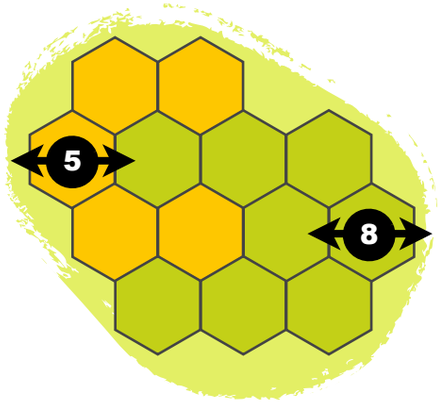
solutions

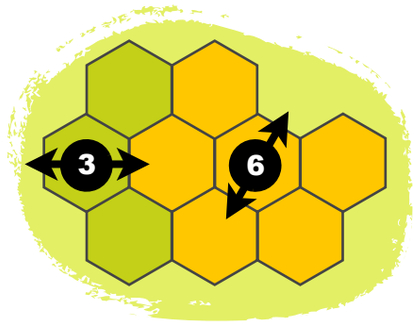


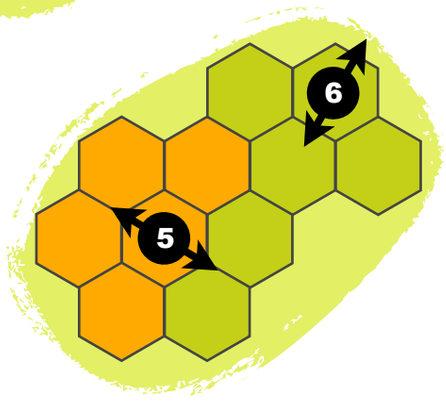


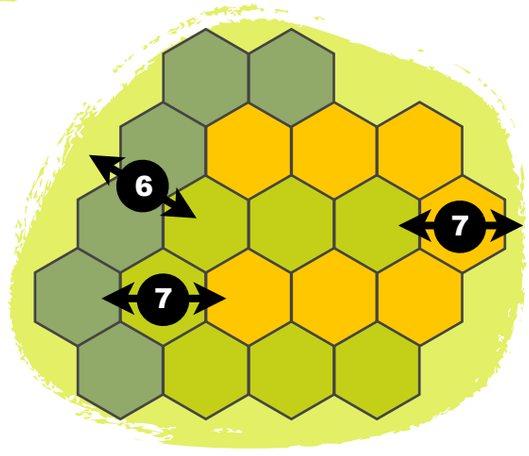


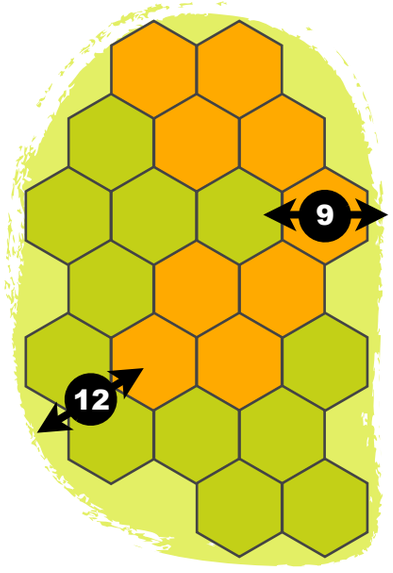


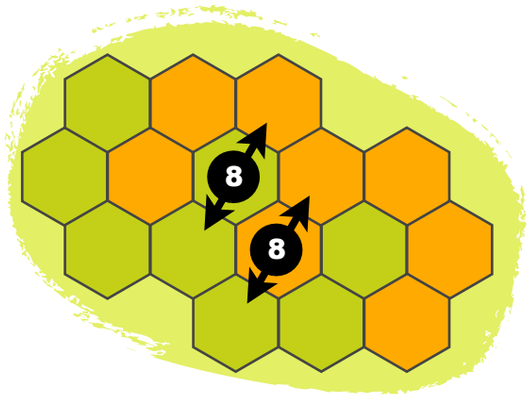


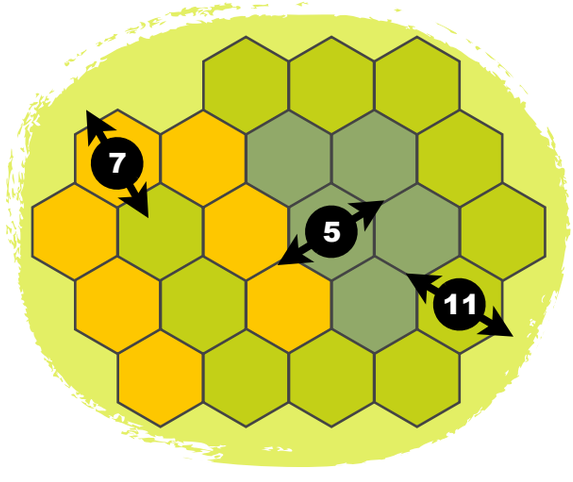


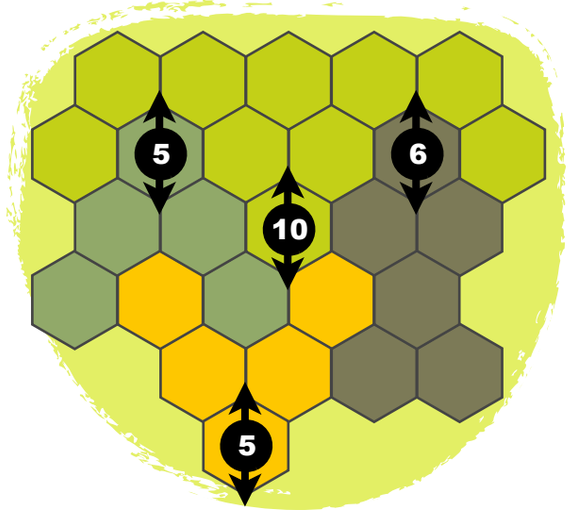


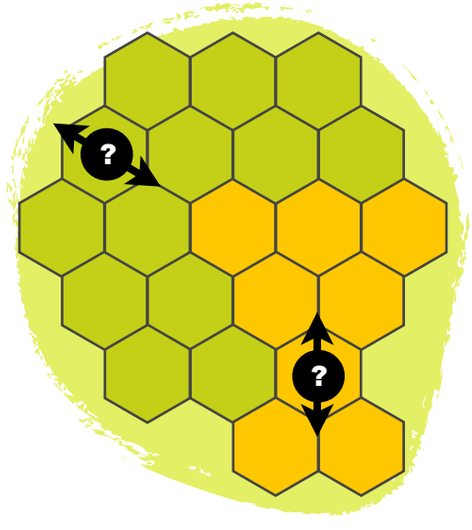


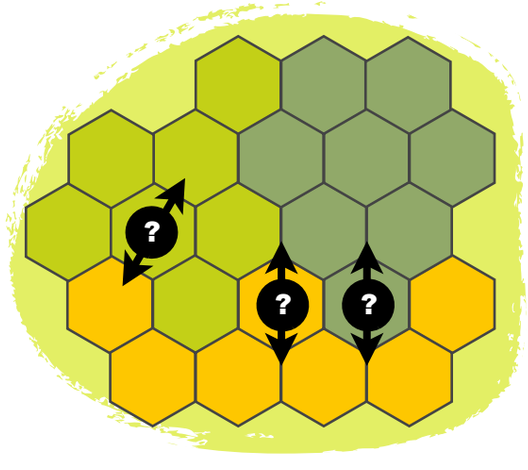




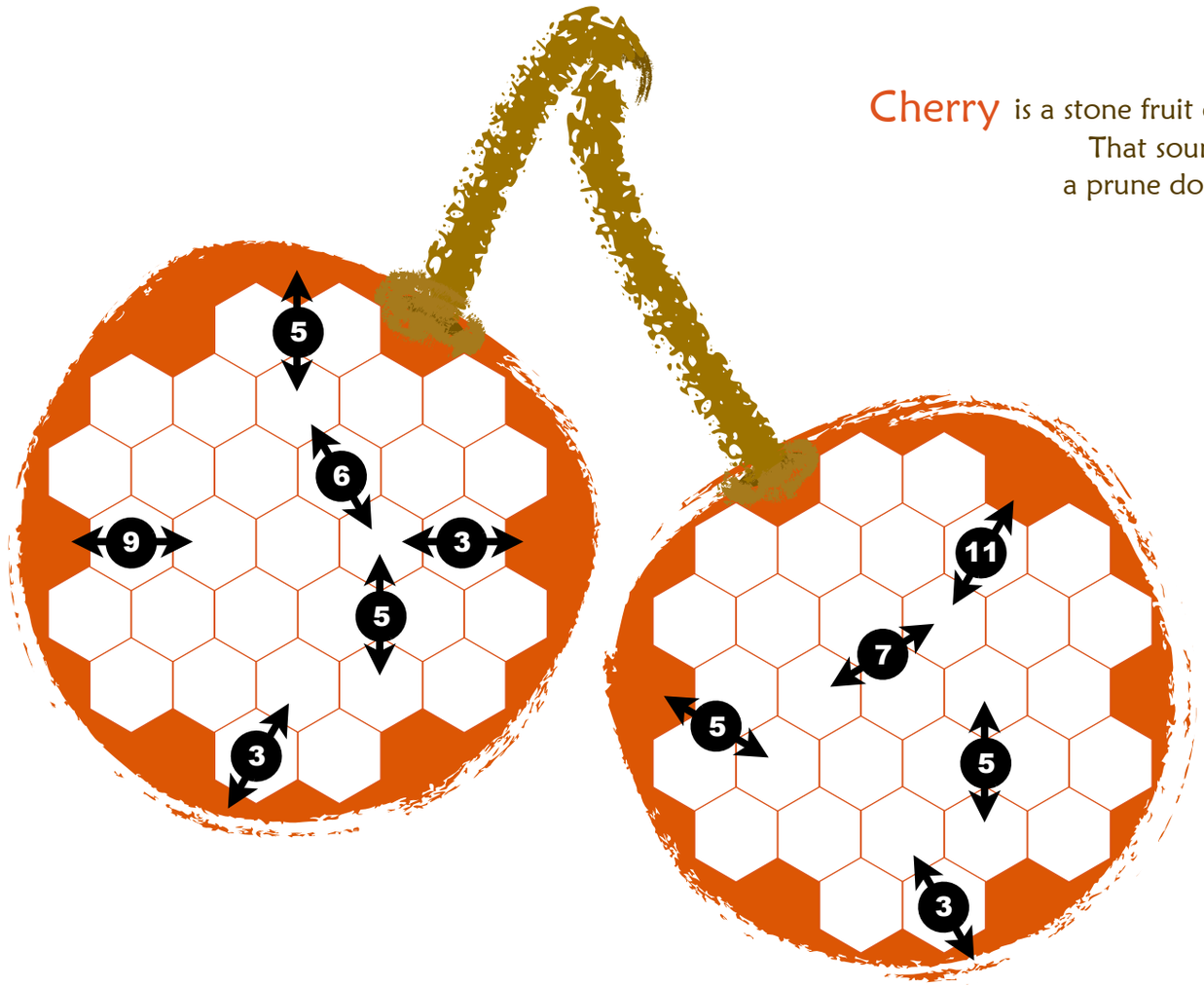


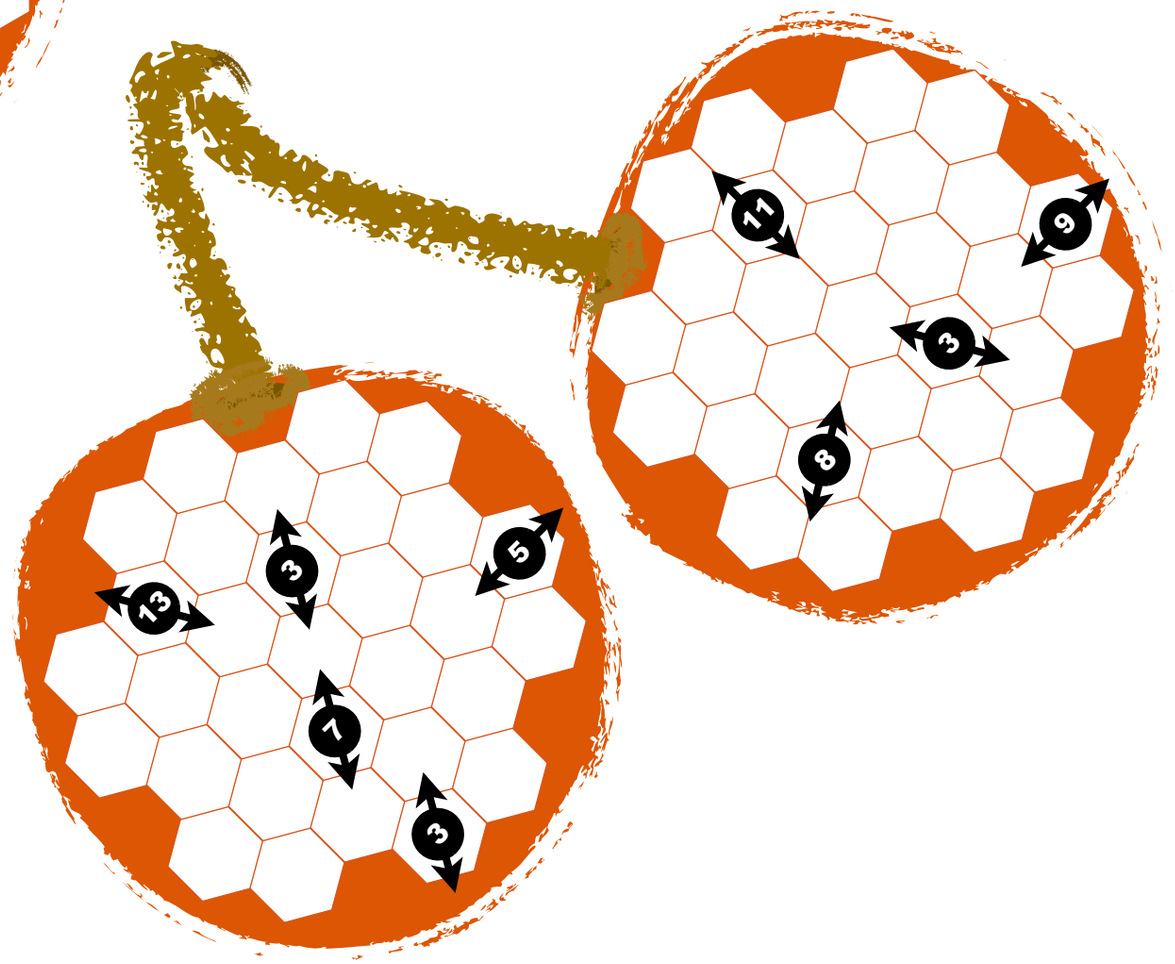


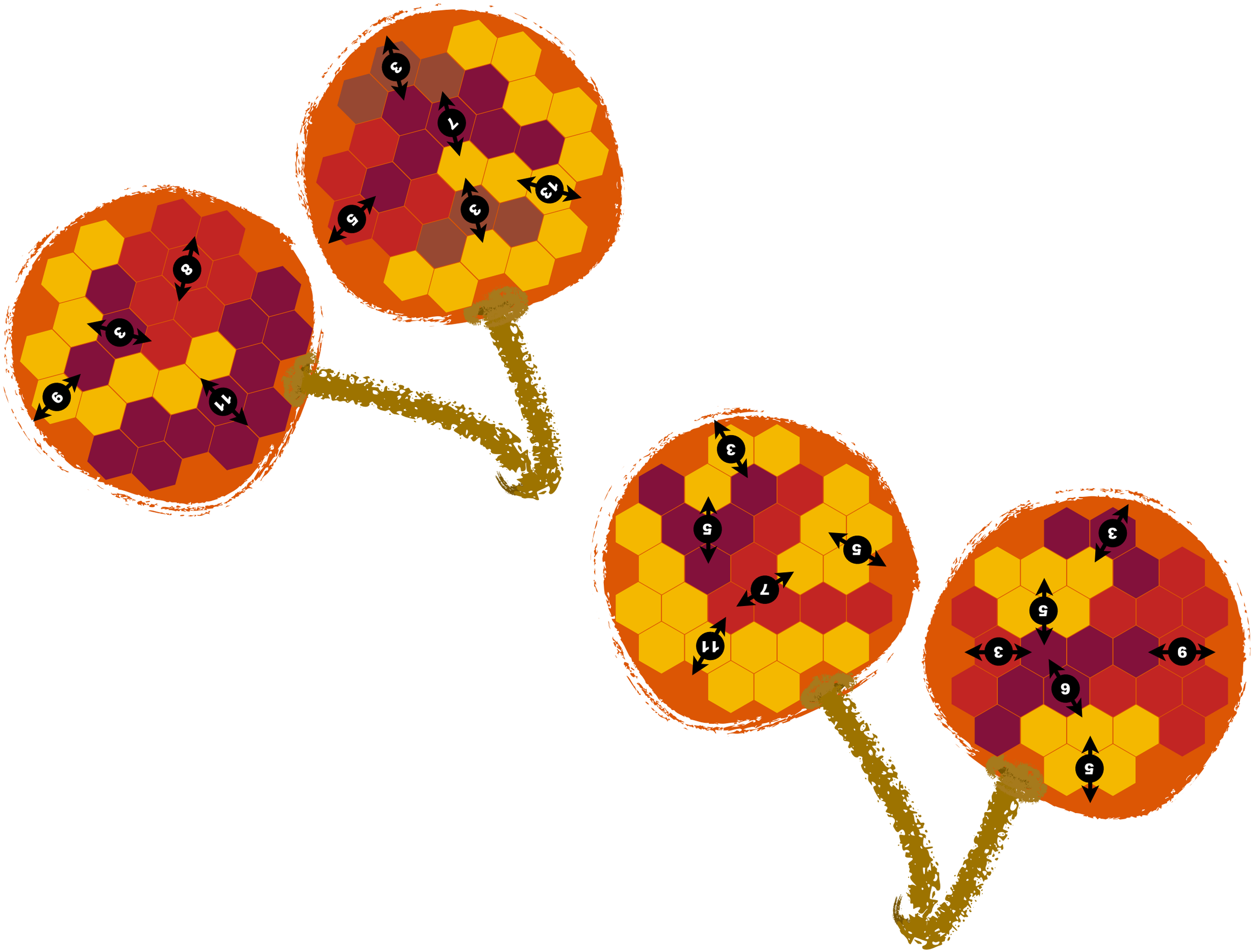




Cherry is a stone fruit of the genus Prunus.
That sounds more like
a prune doesn't it.



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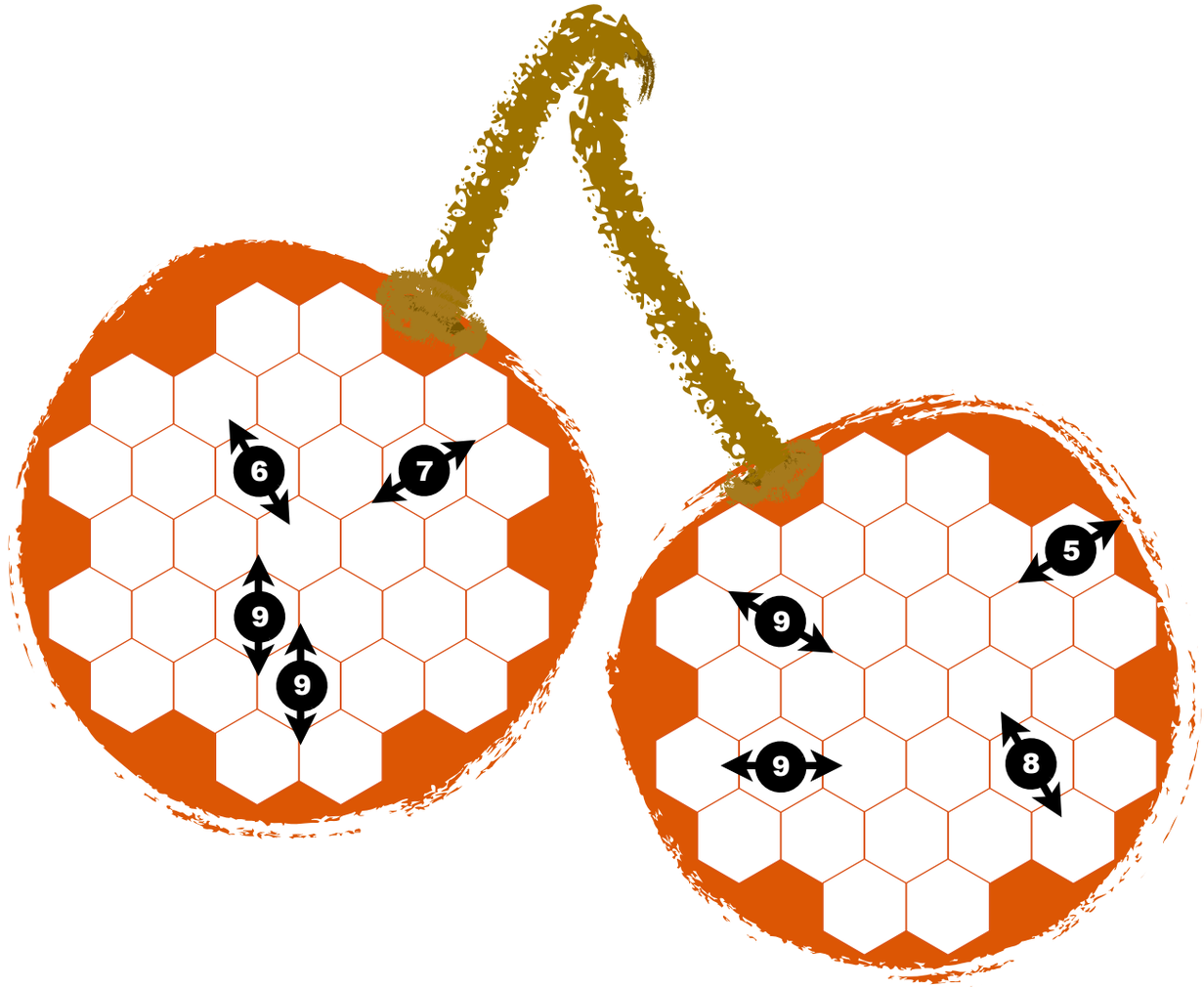
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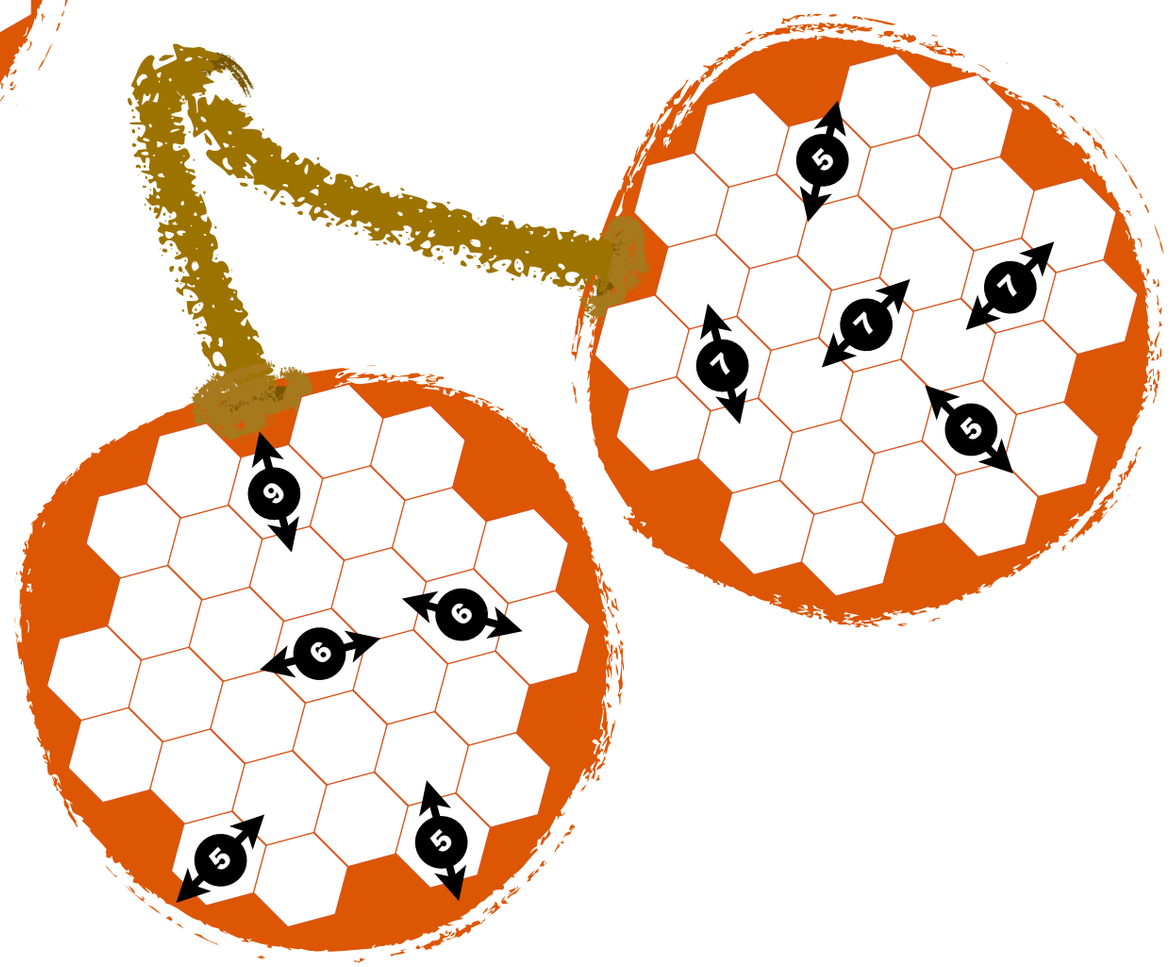
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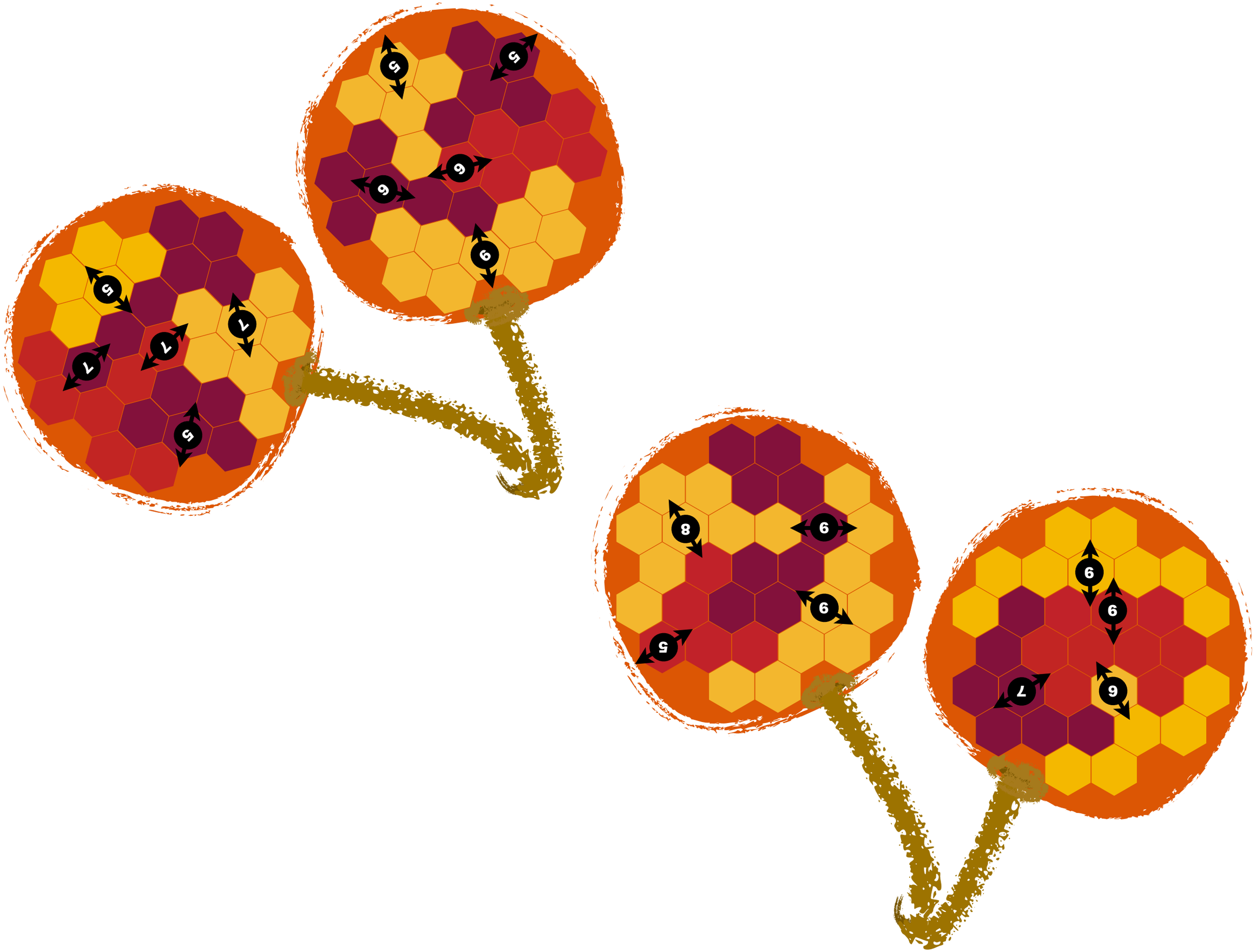
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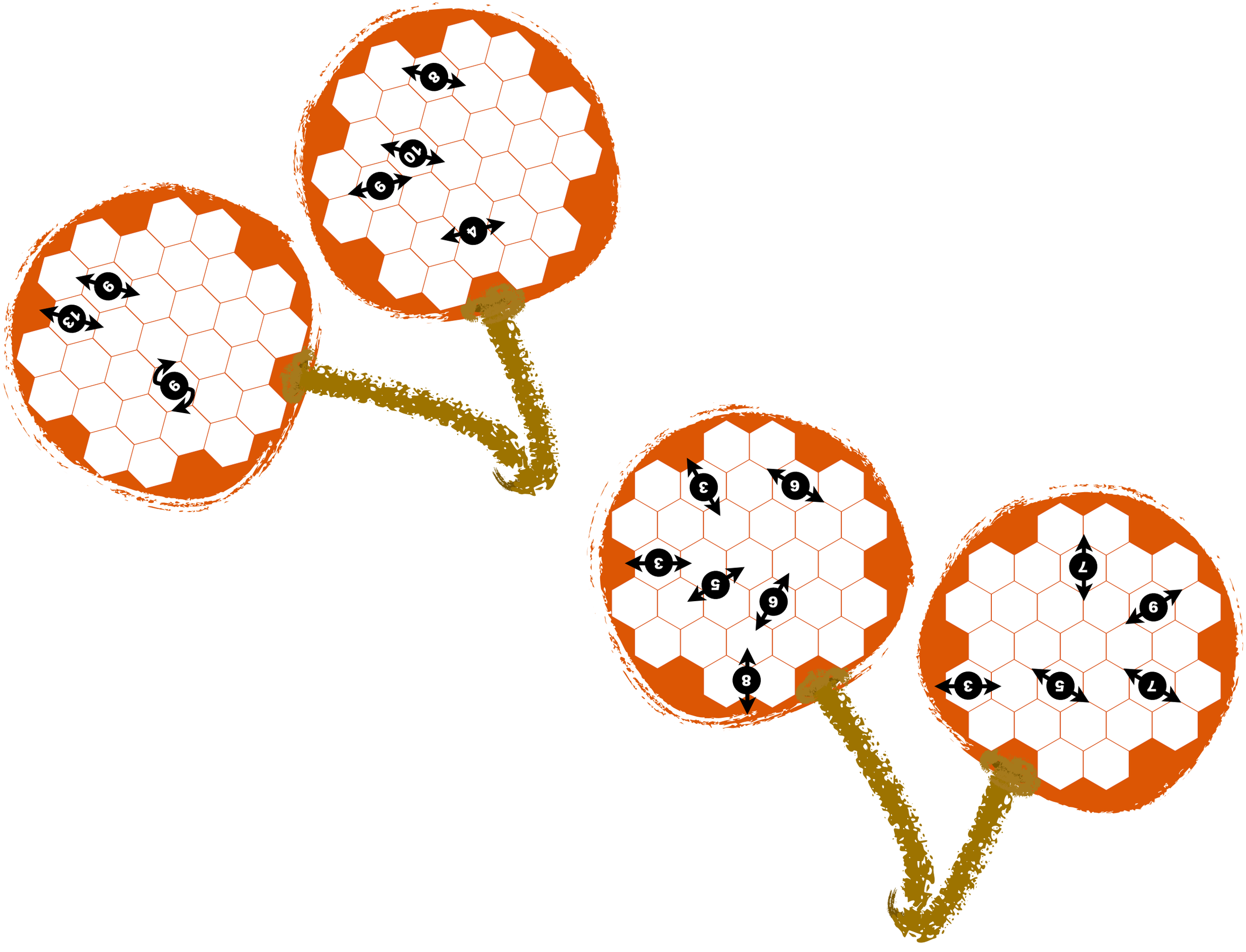
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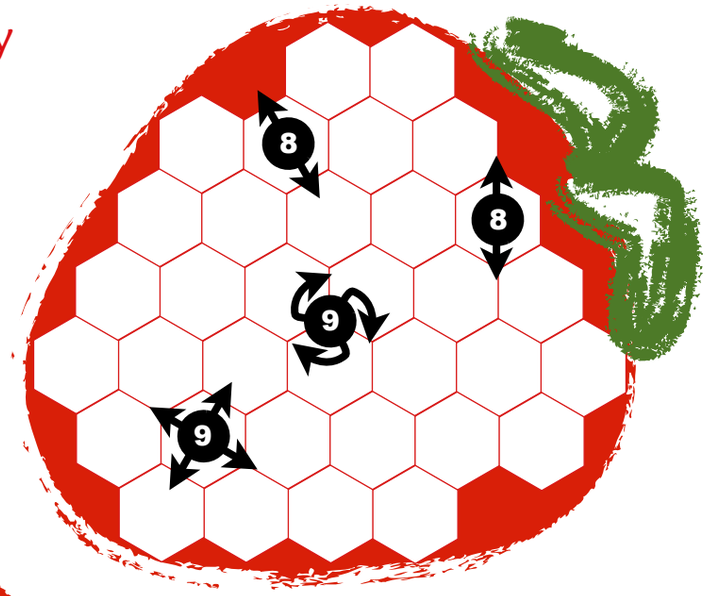
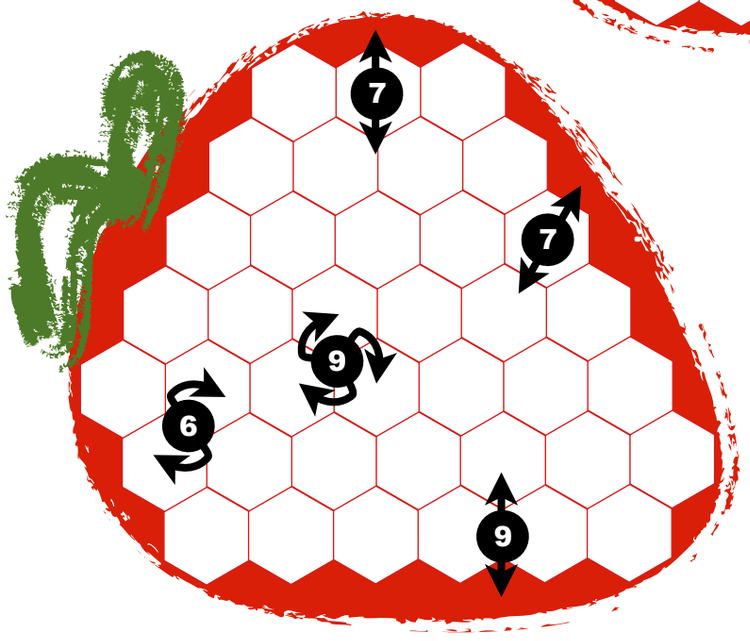
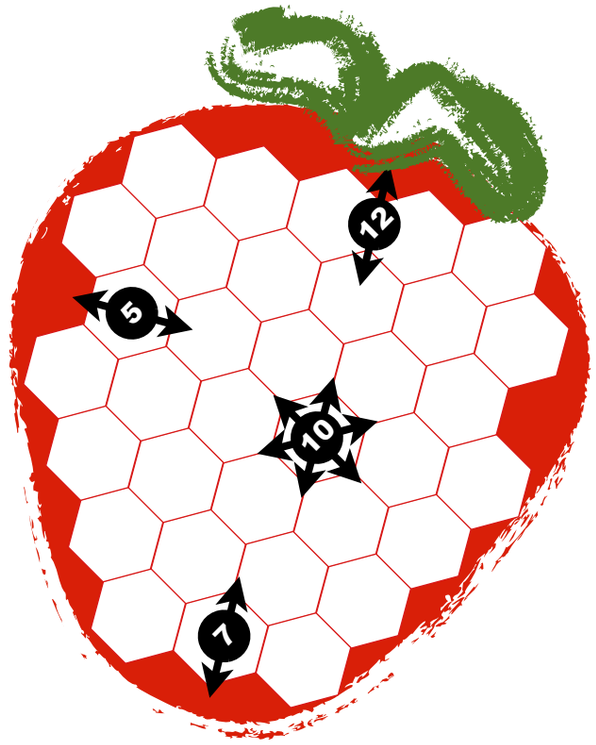
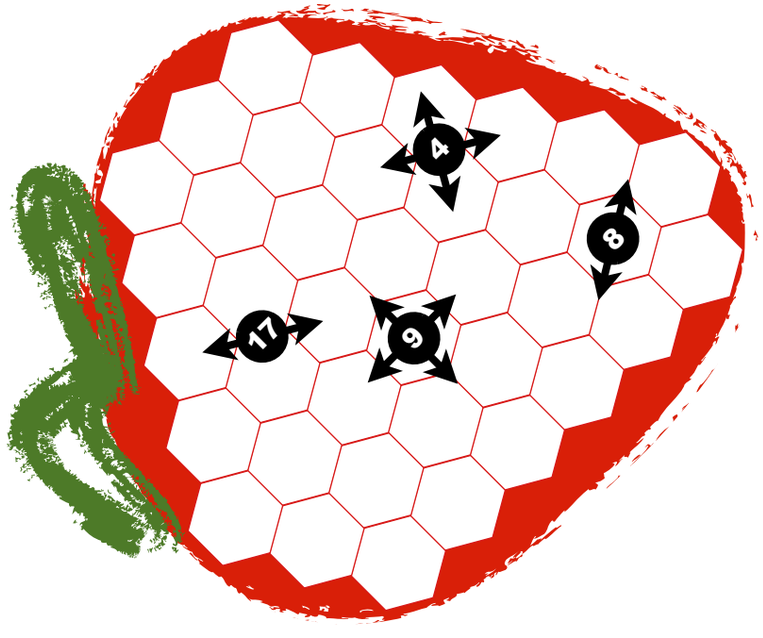
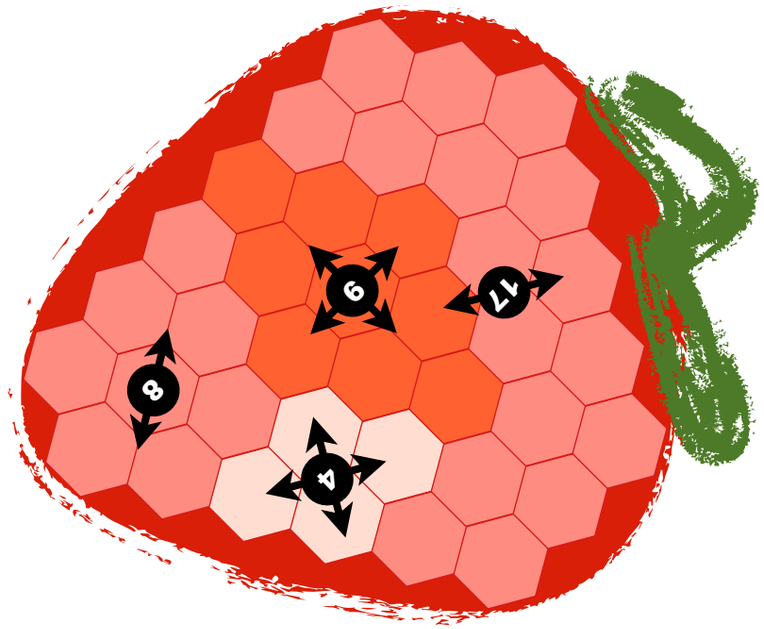
Four million tons of strawberry
are produced each year.







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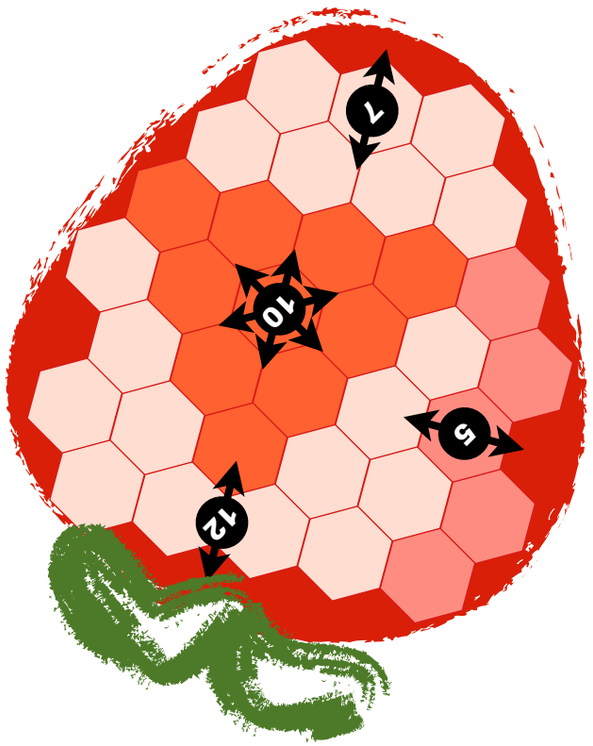
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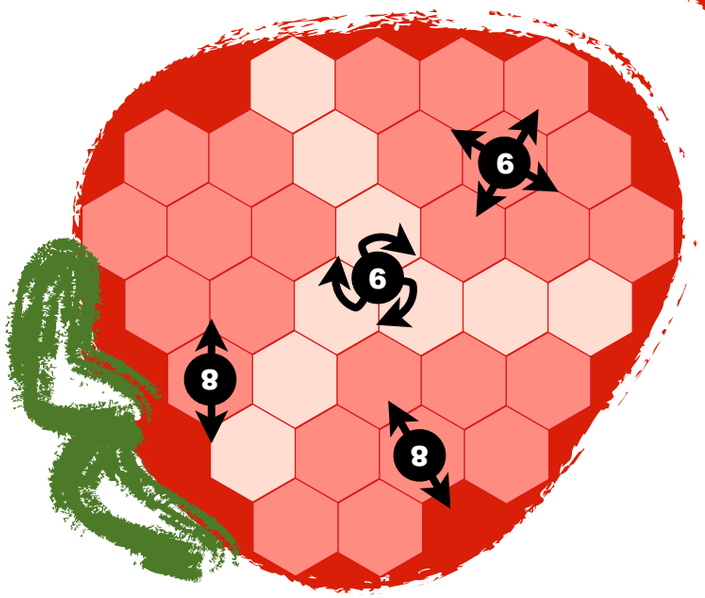
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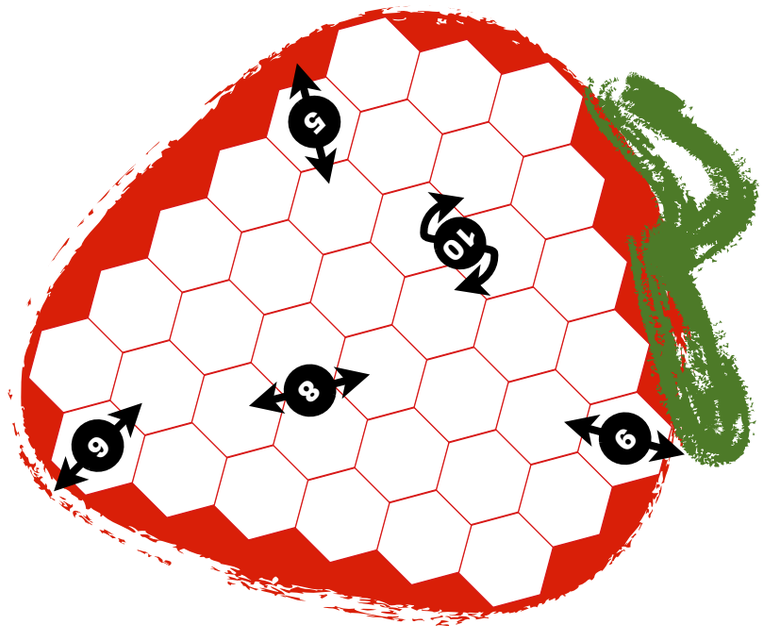
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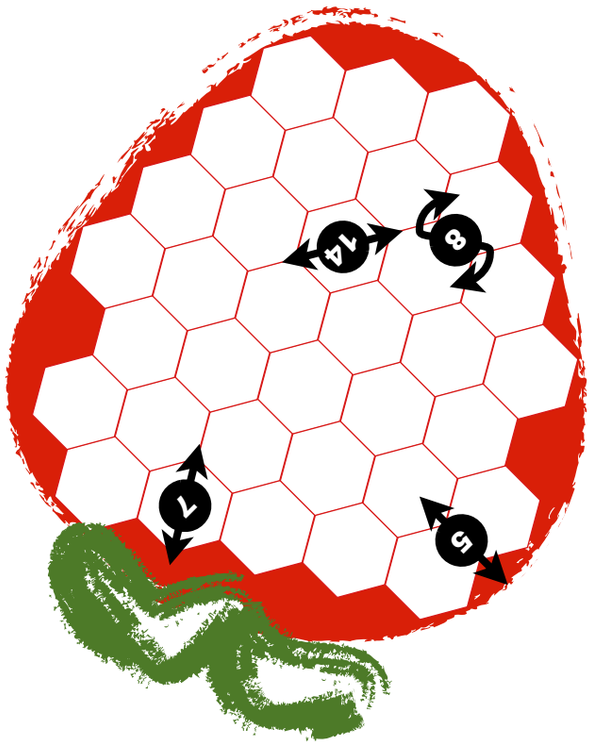
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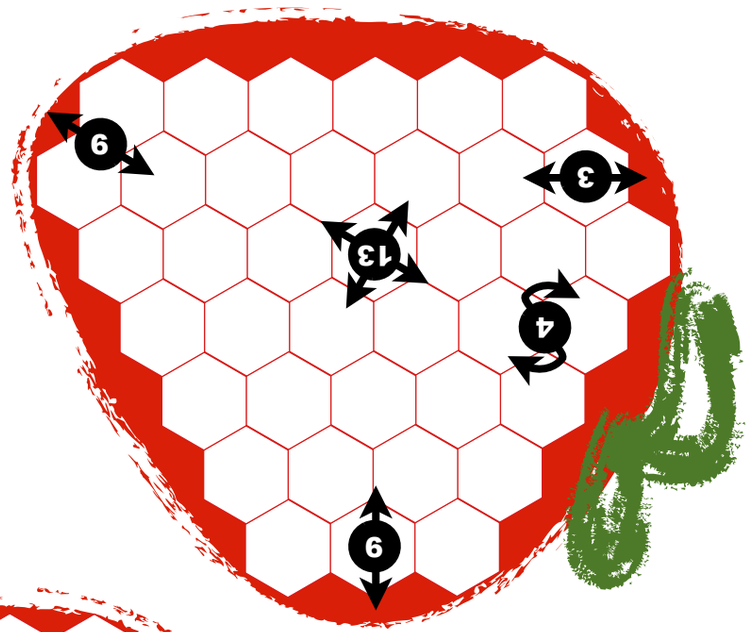
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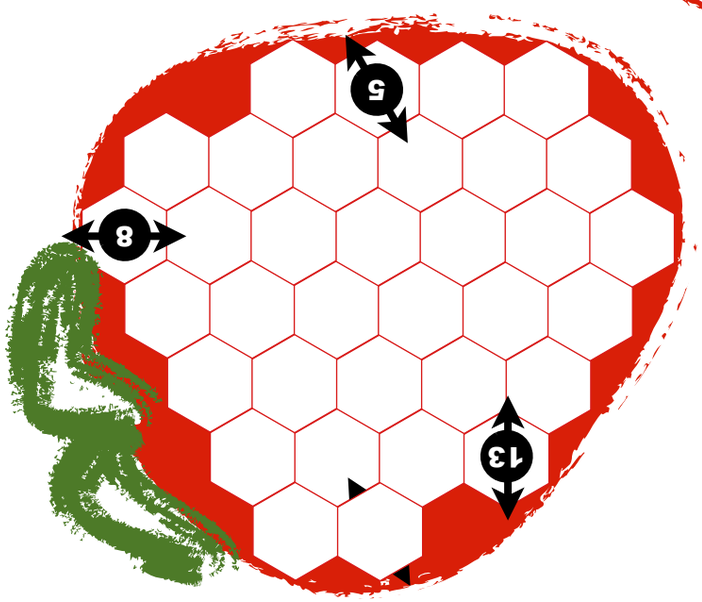
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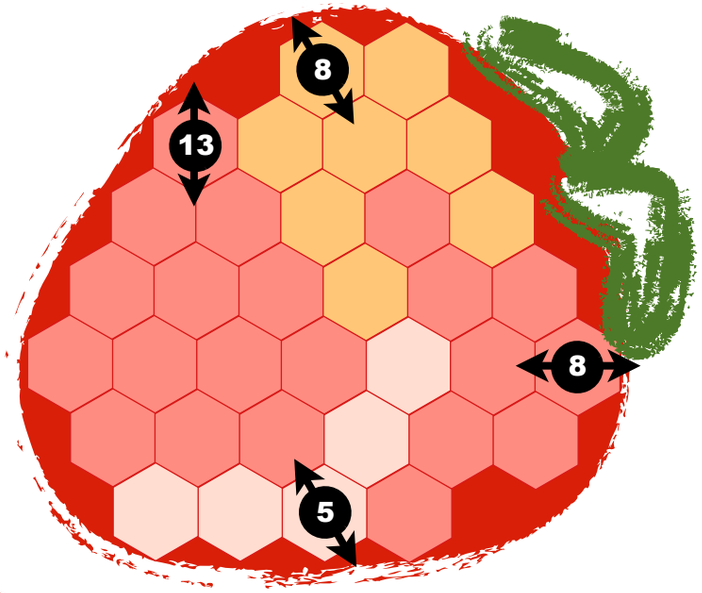
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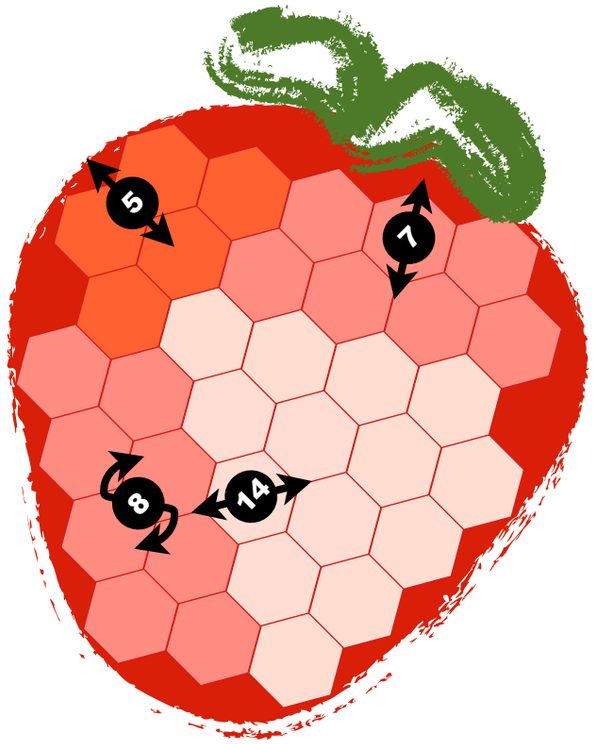
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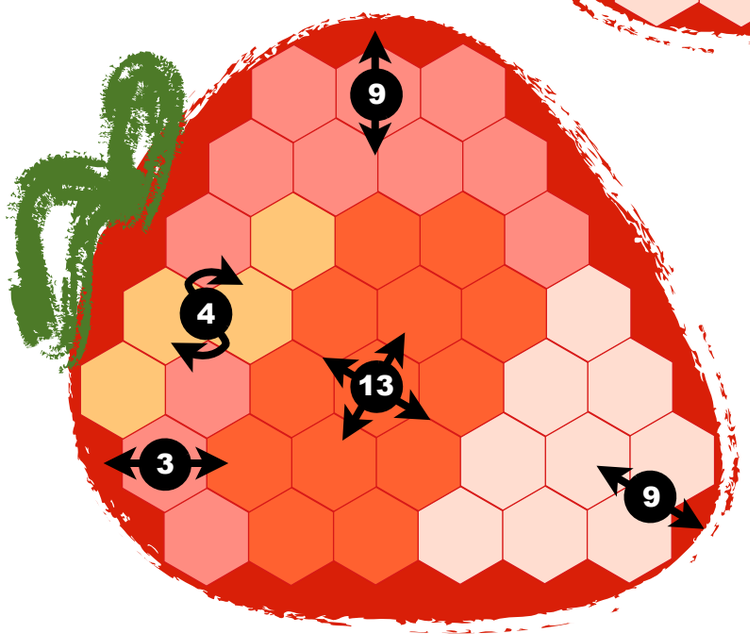
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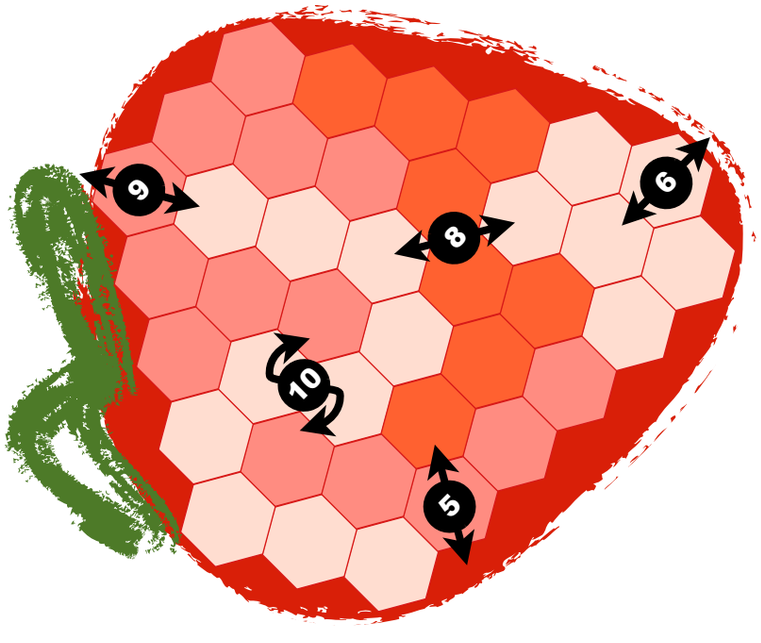
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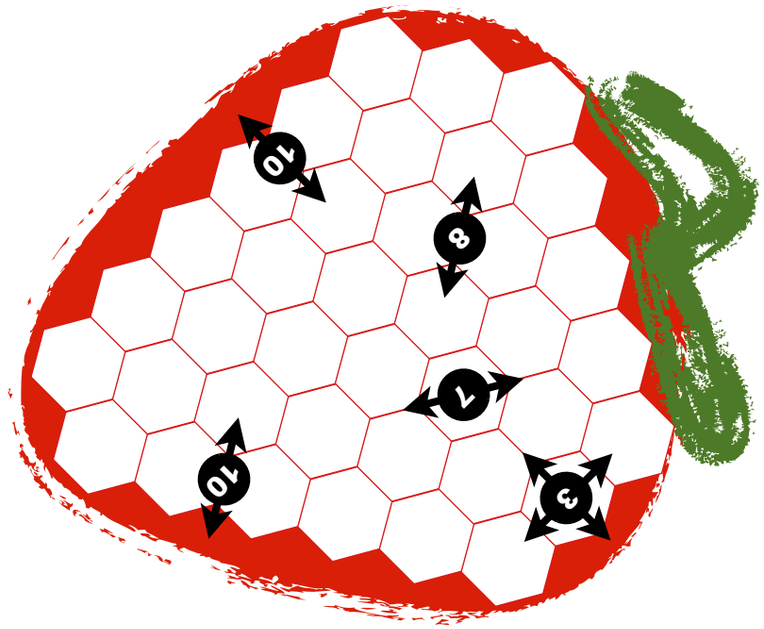
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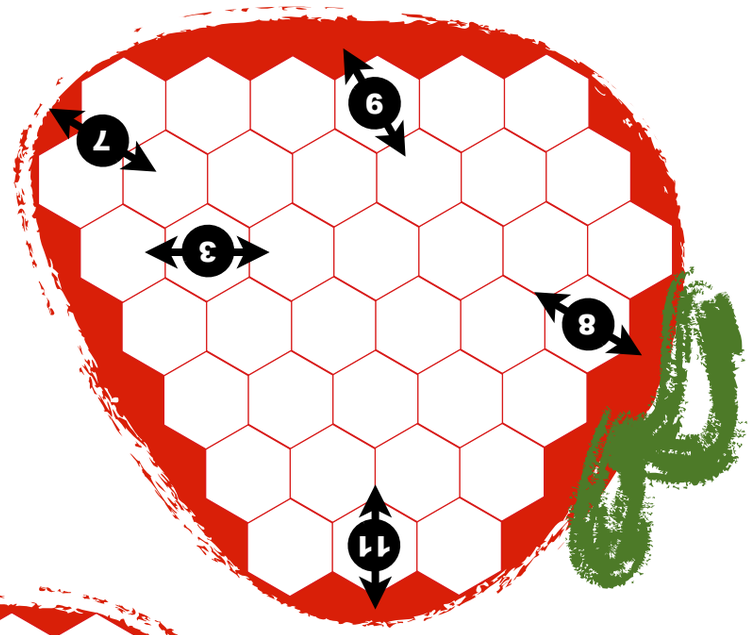
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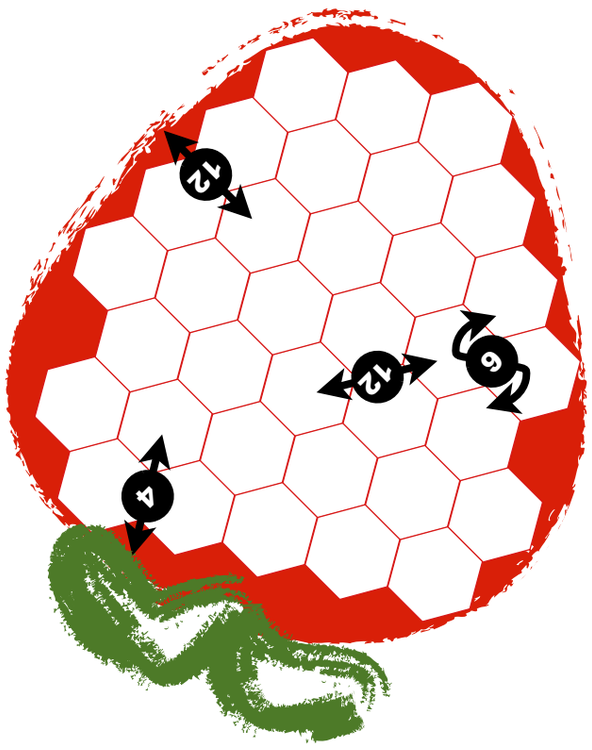
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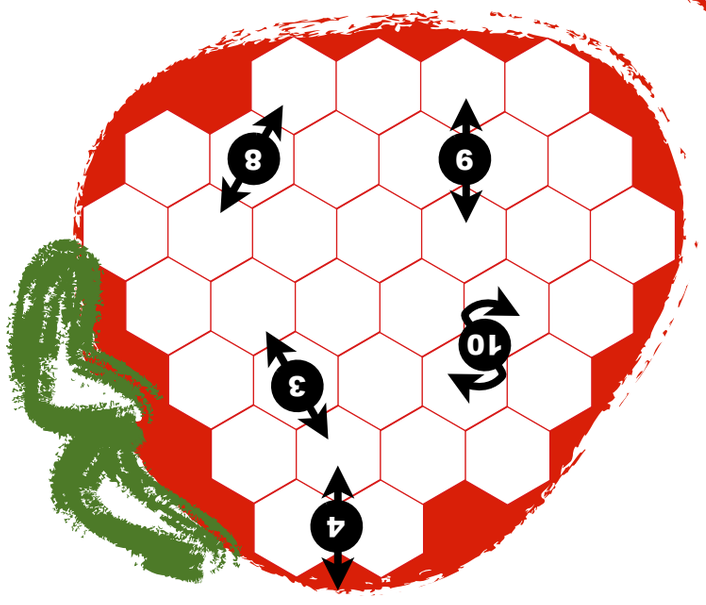
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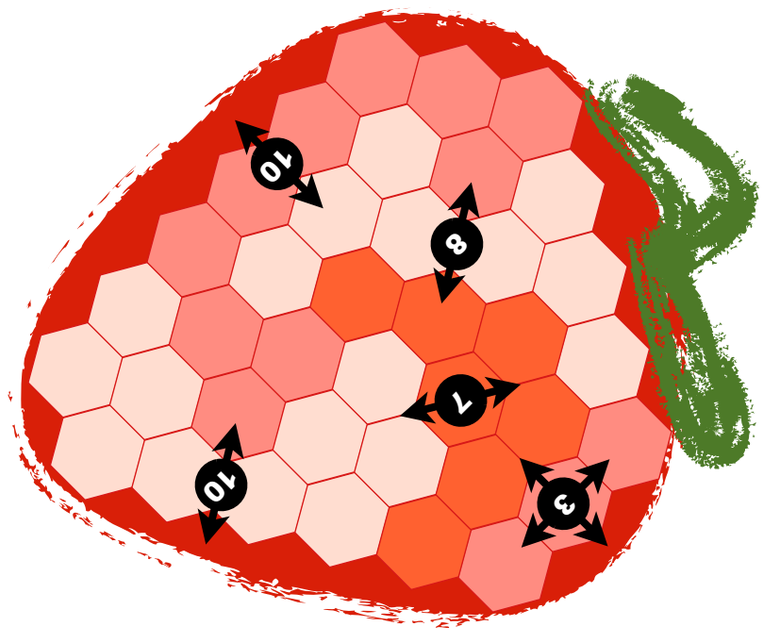
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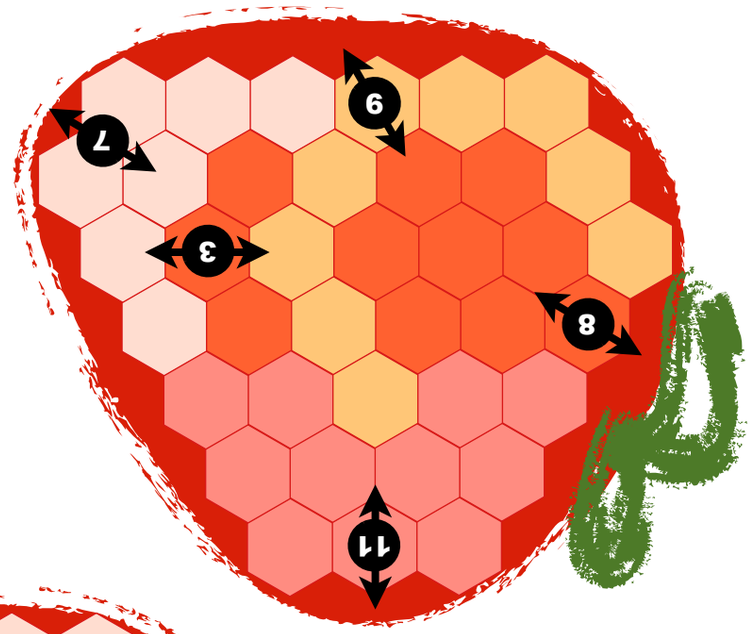
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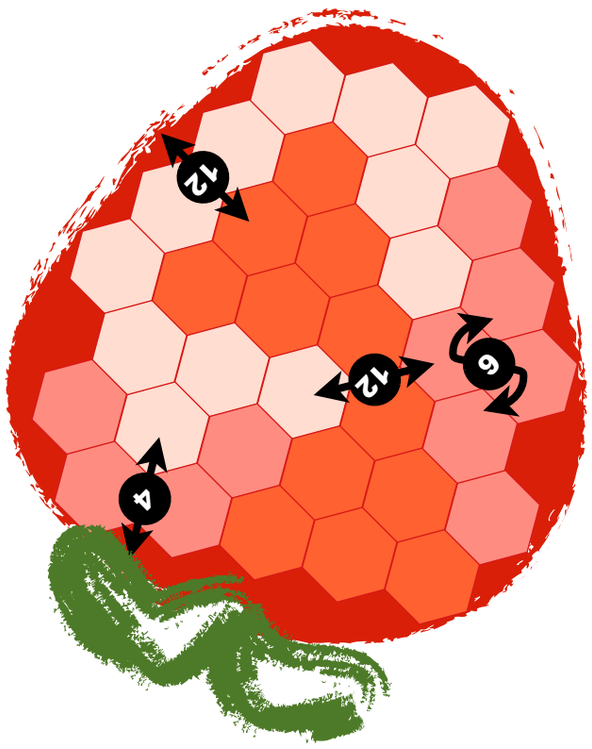
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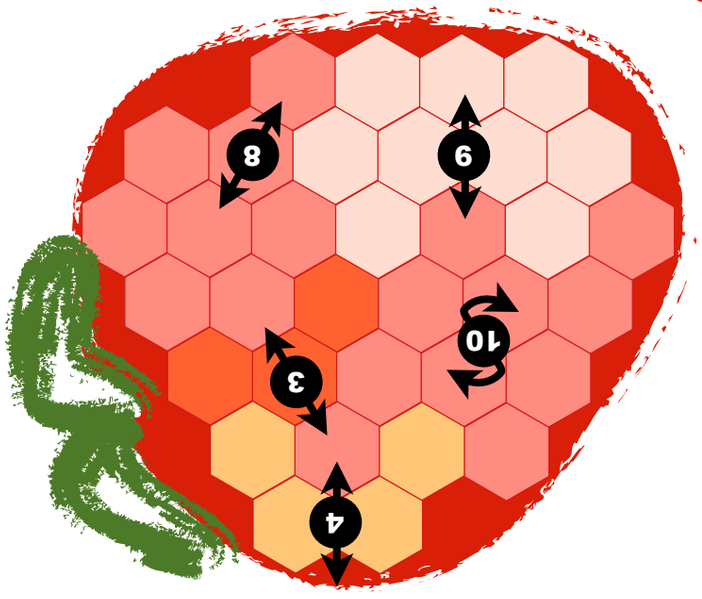
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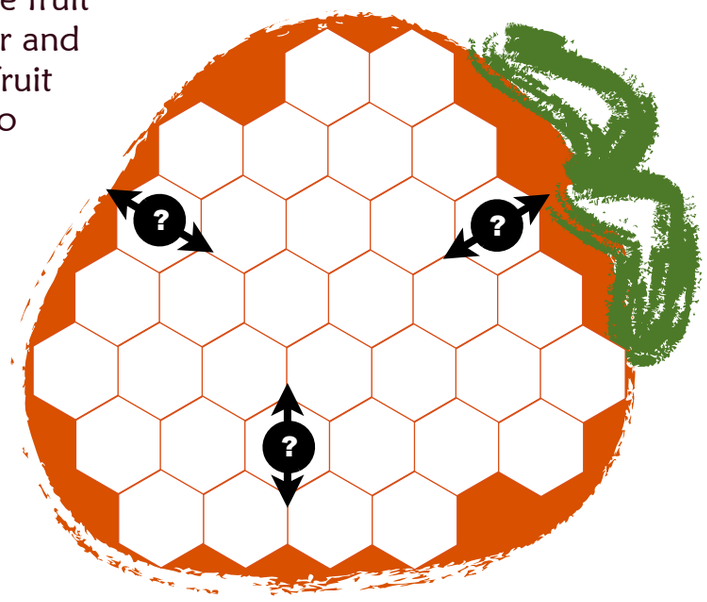
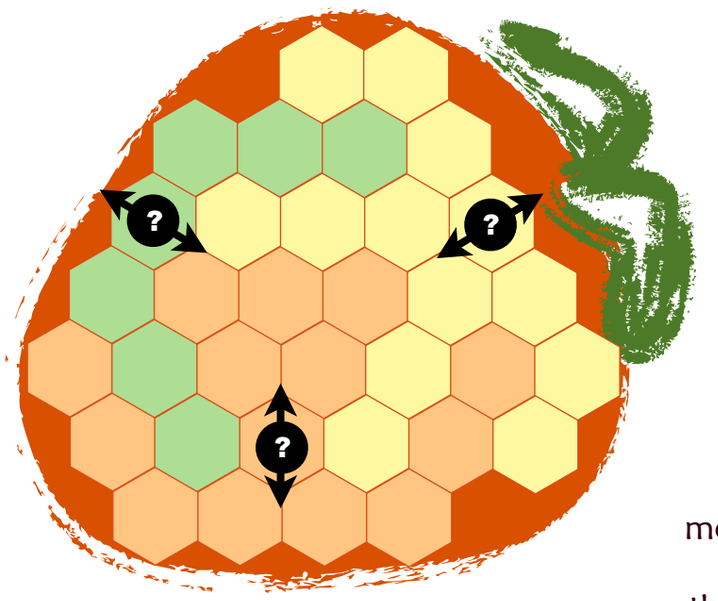
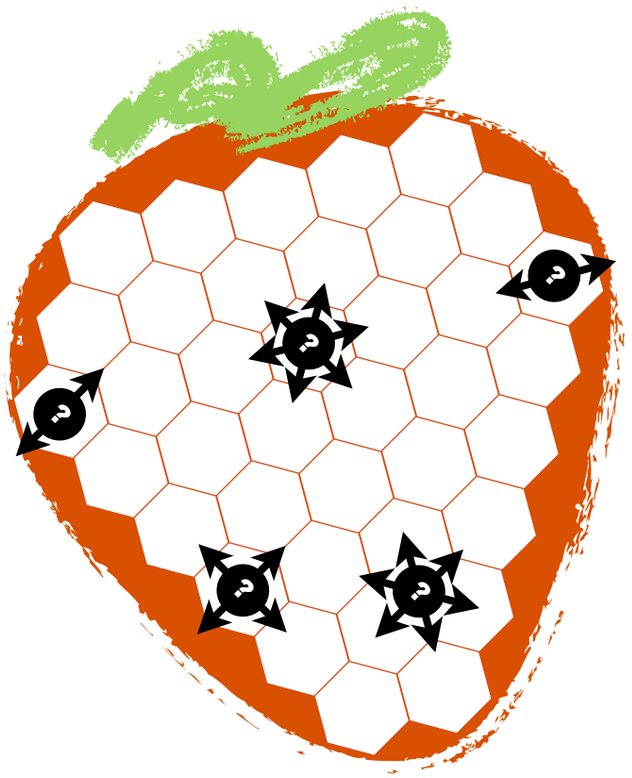
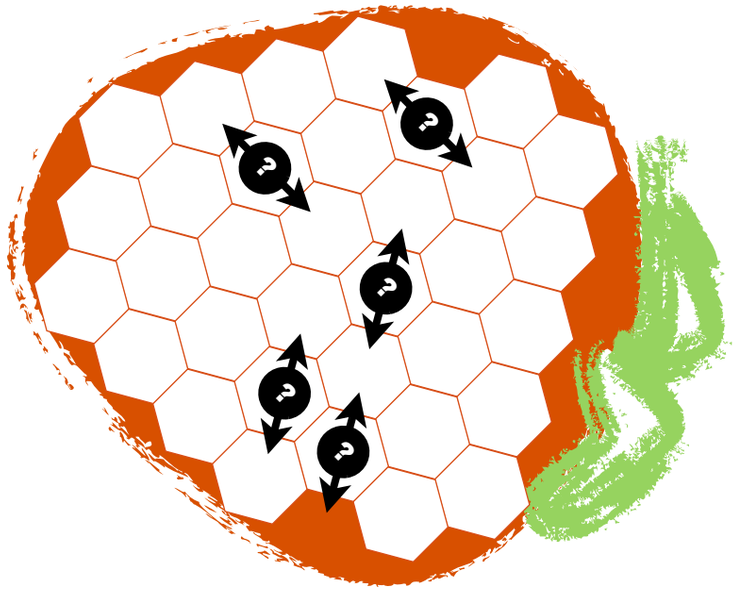
Disappointingly, some fruit is unripe - hard, bitter and unpleasant. Unripe fruit can still be cut up into symmetric pieces, but there are no clues about the size of the unripe pieces.

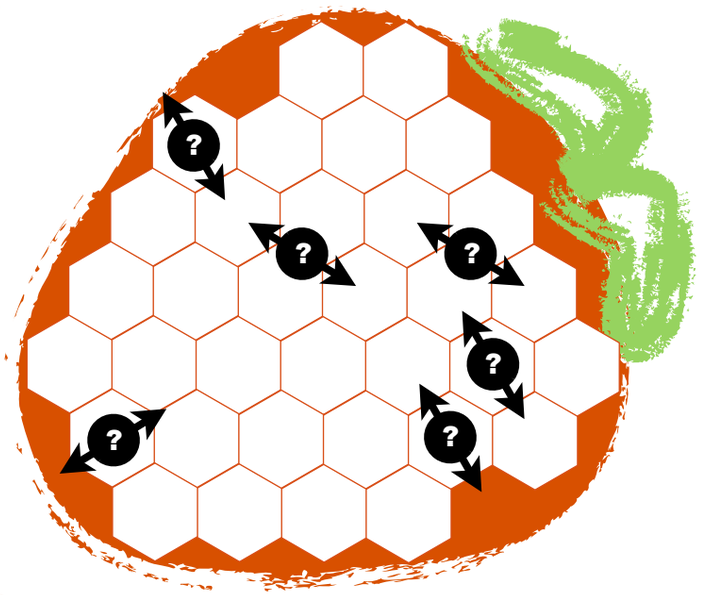


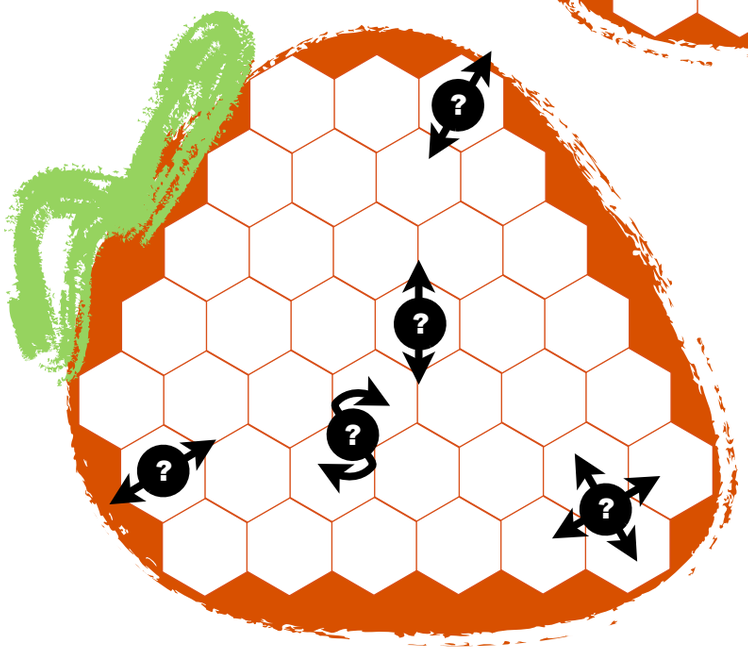


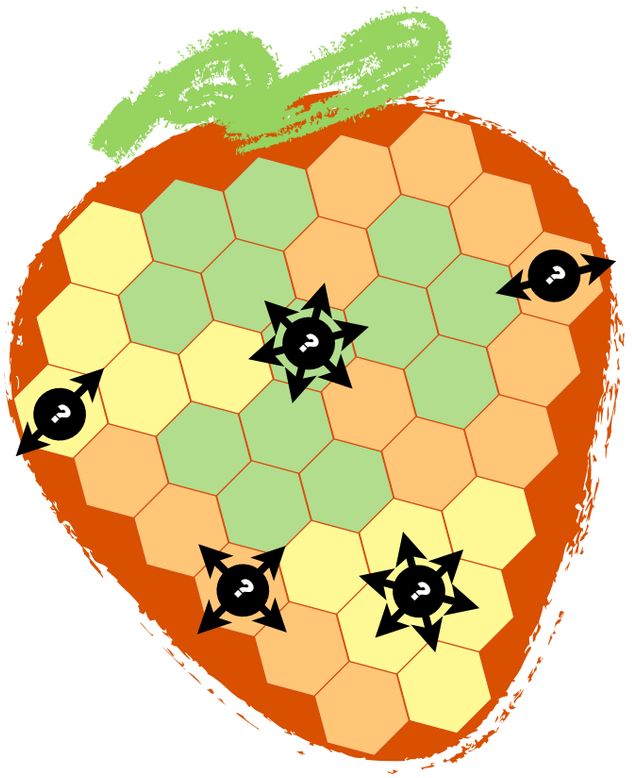
The unripe strawberry has this solution if you ever wanted to slice it up, but most students will be too scared to try these harder puzzles!

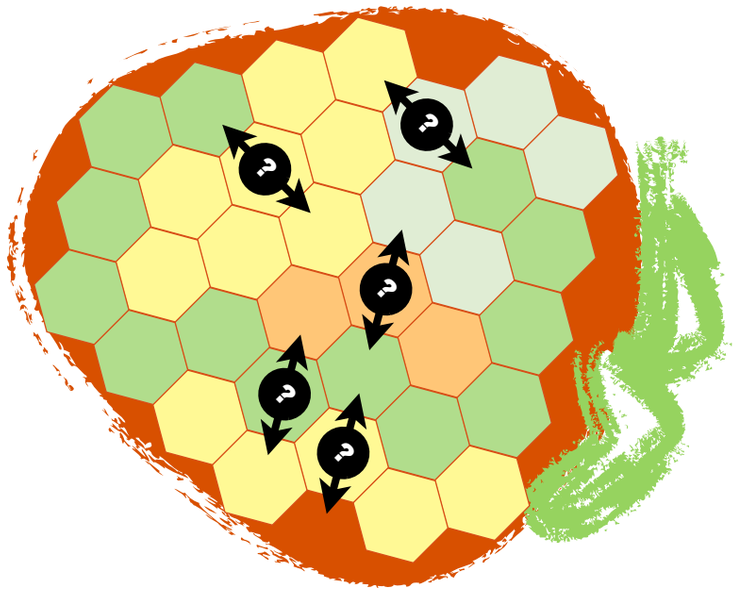


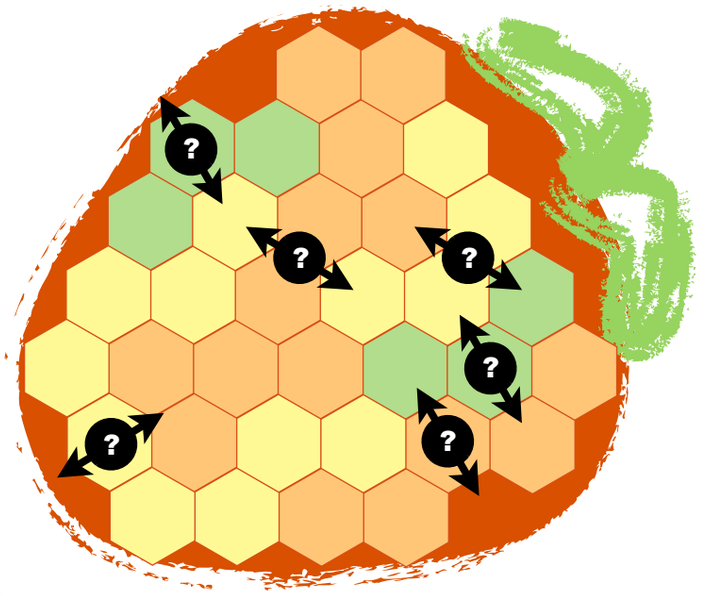


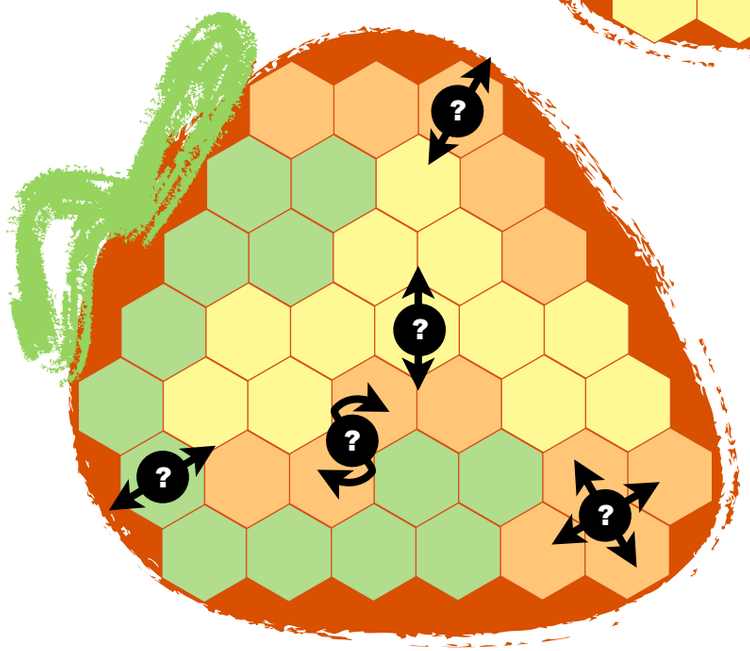


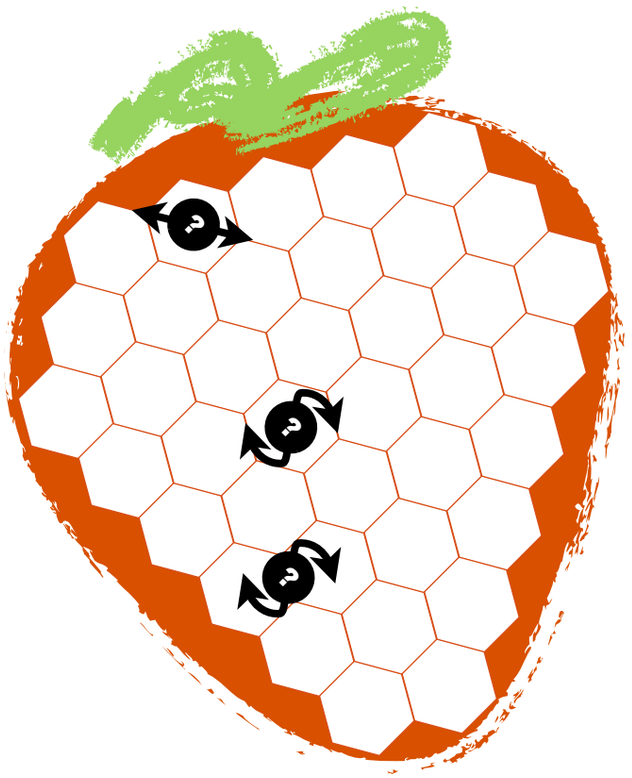


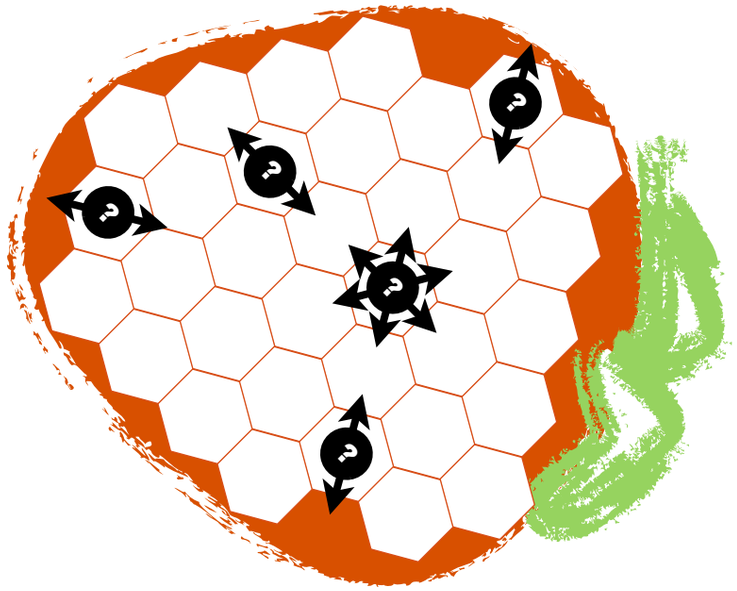


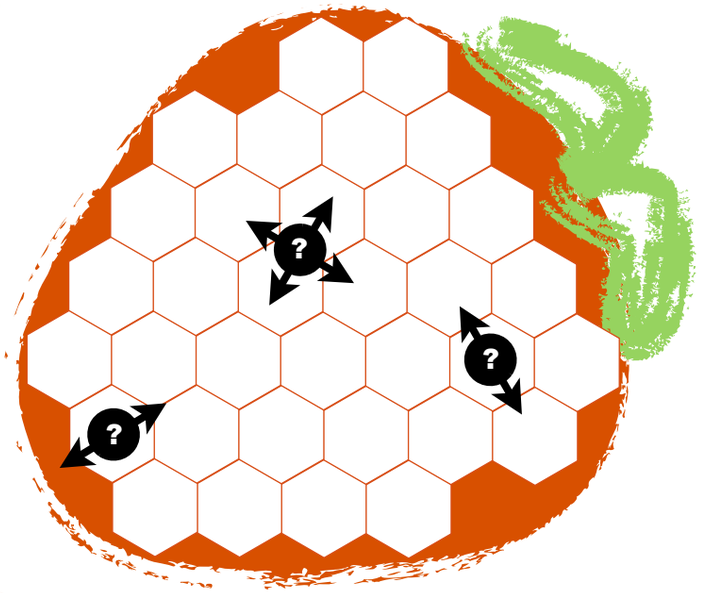


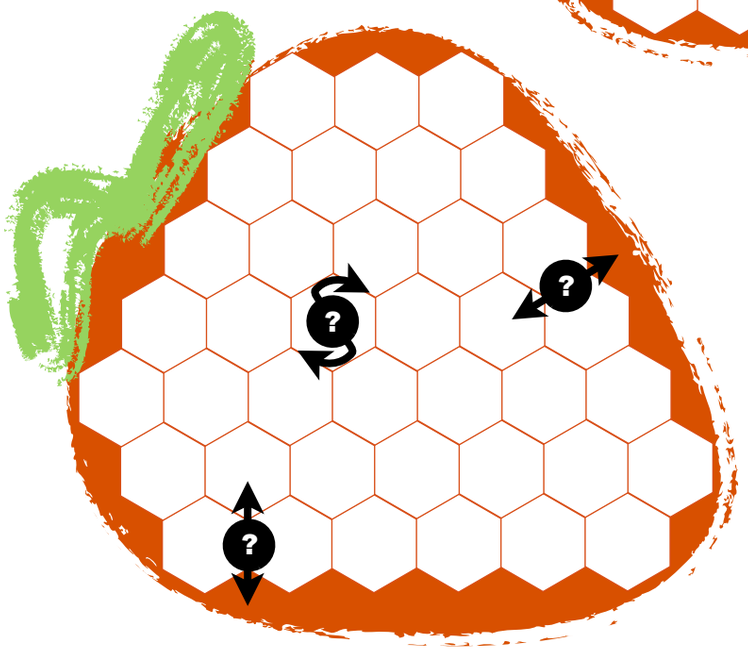


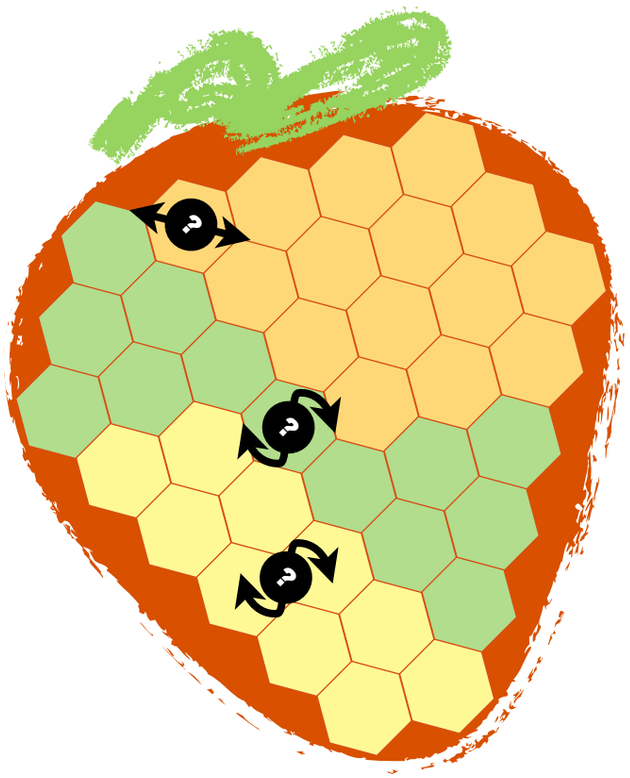


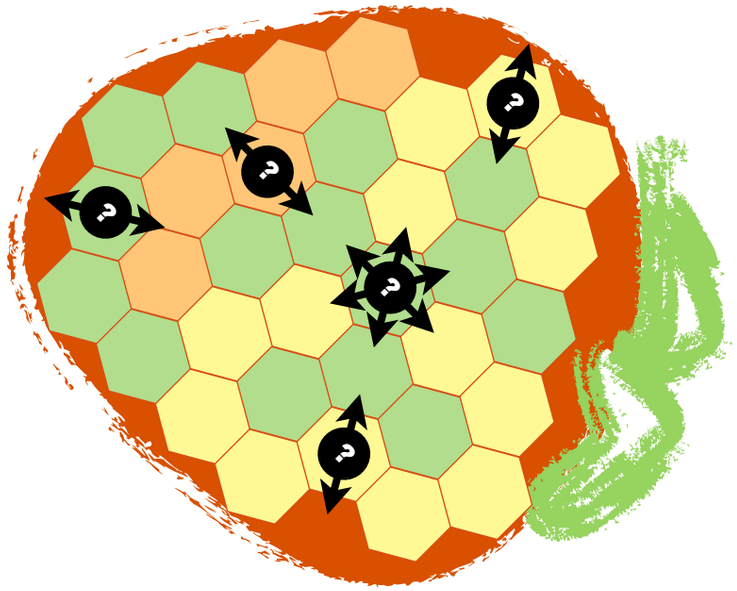


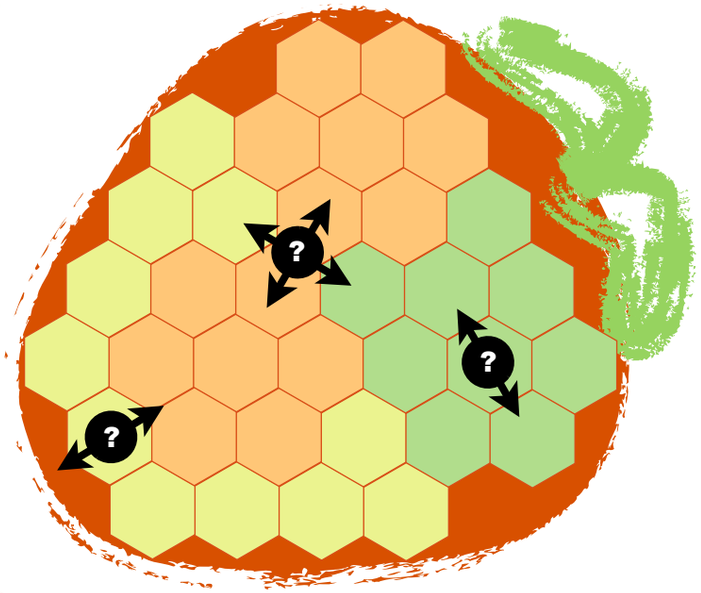


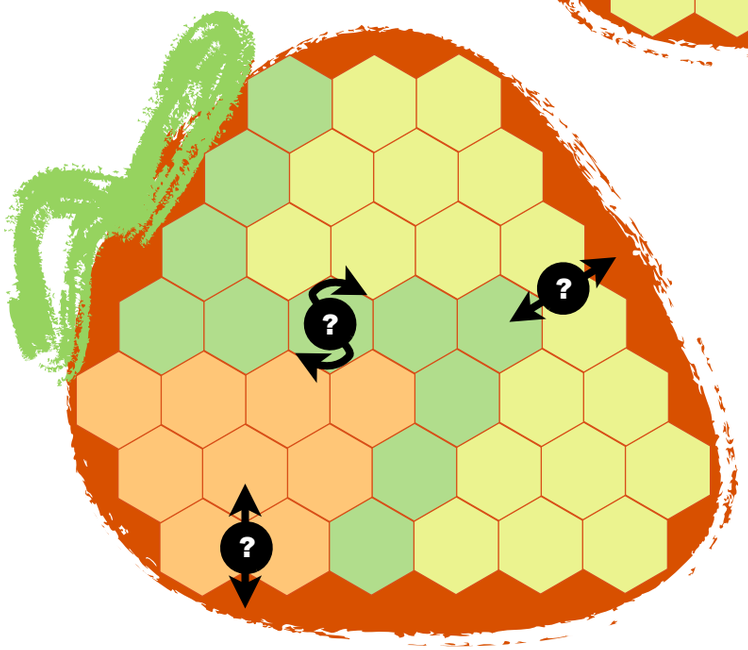




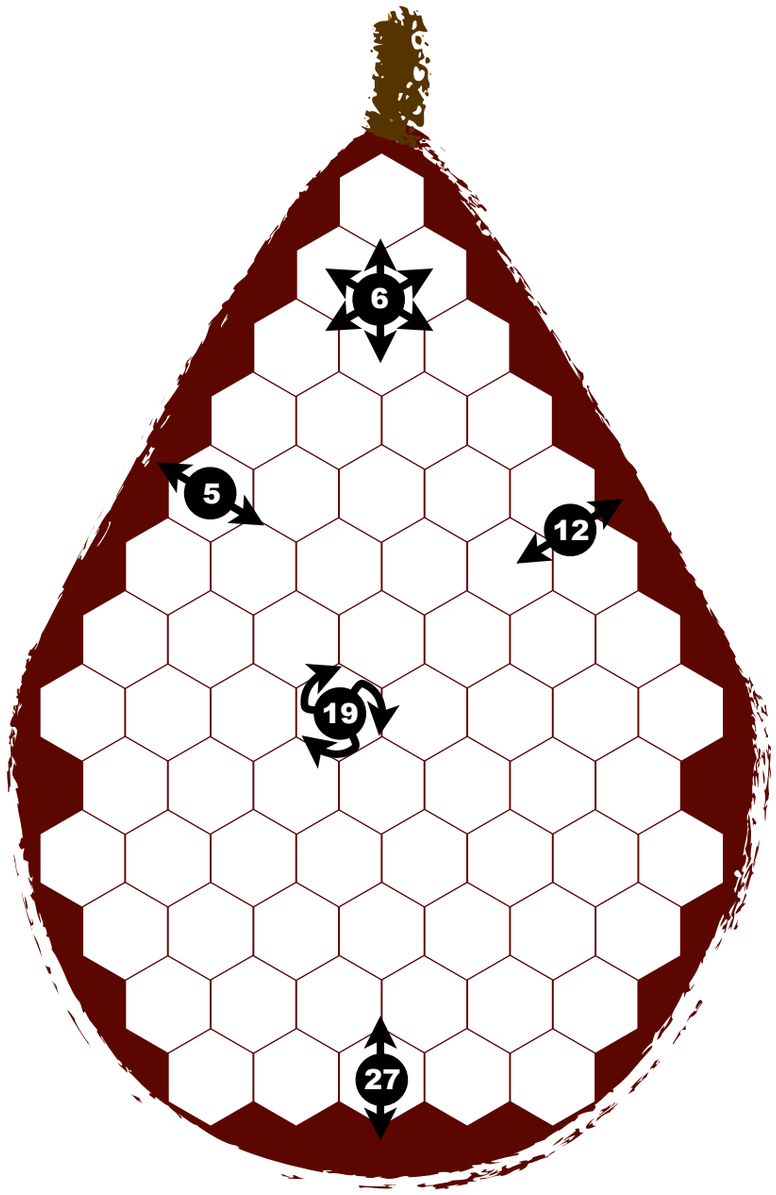


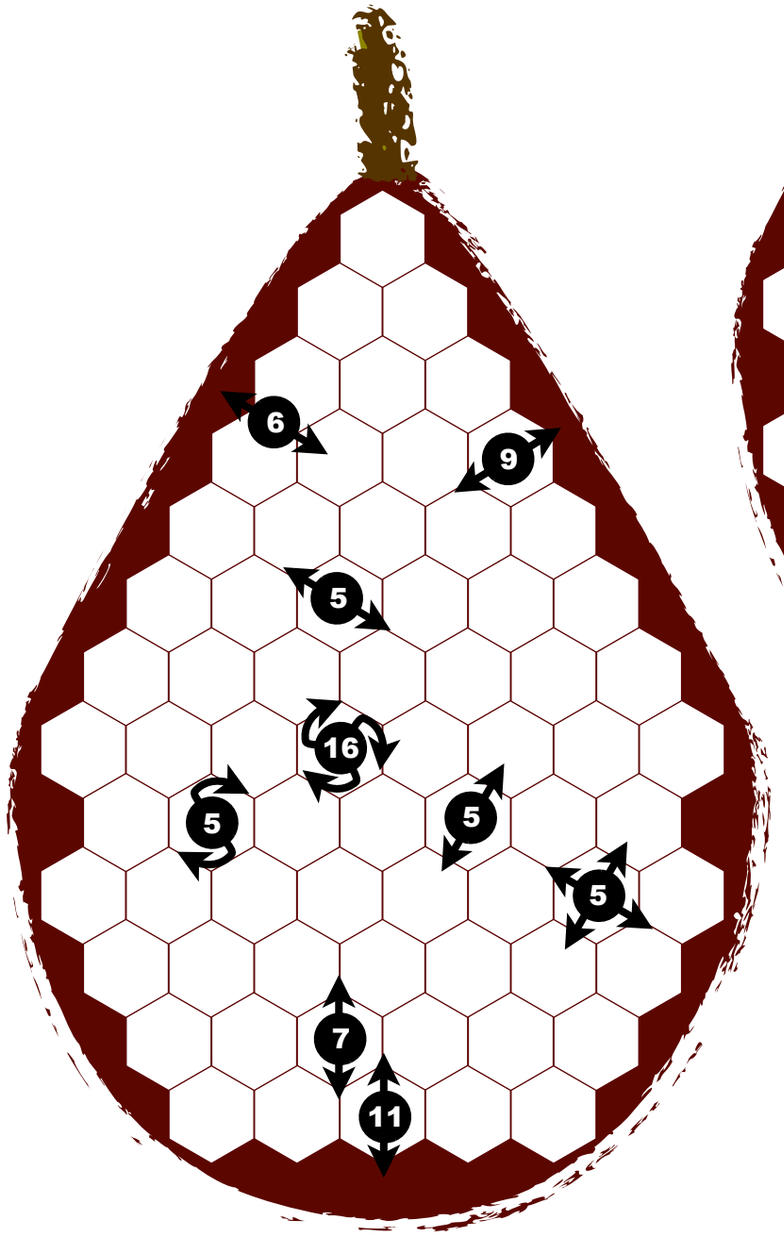


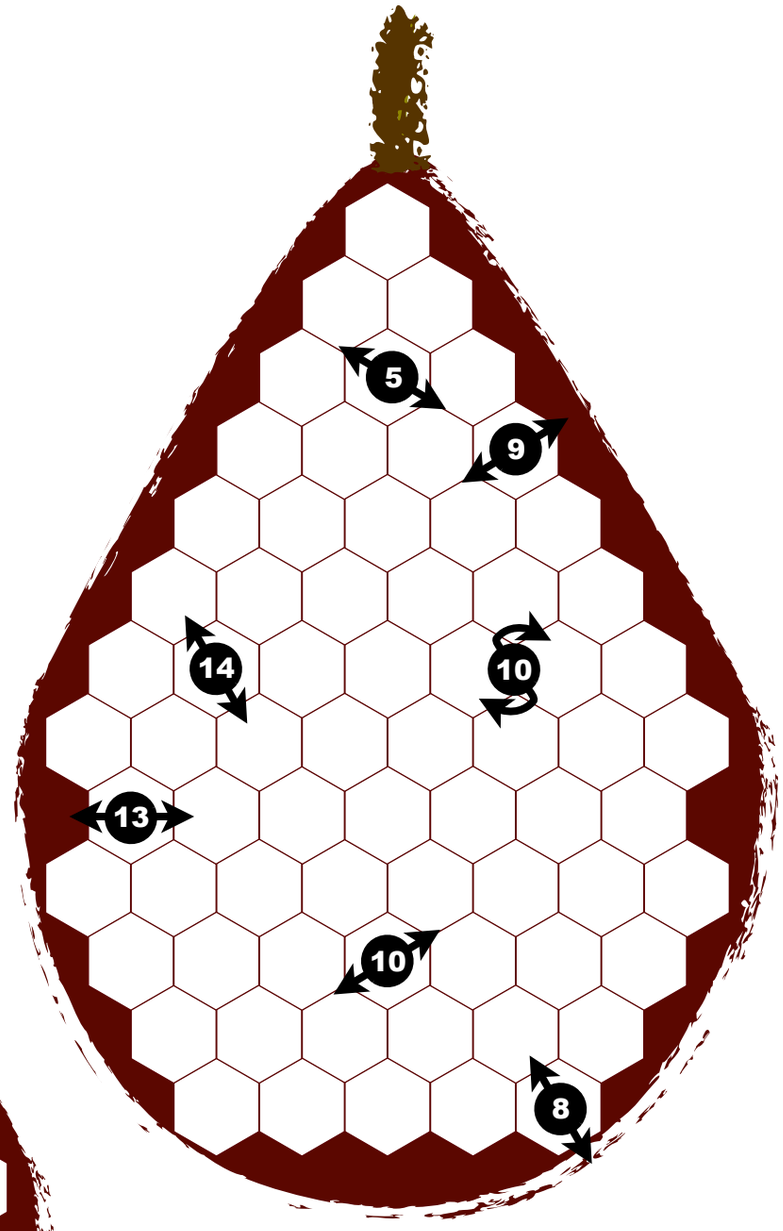


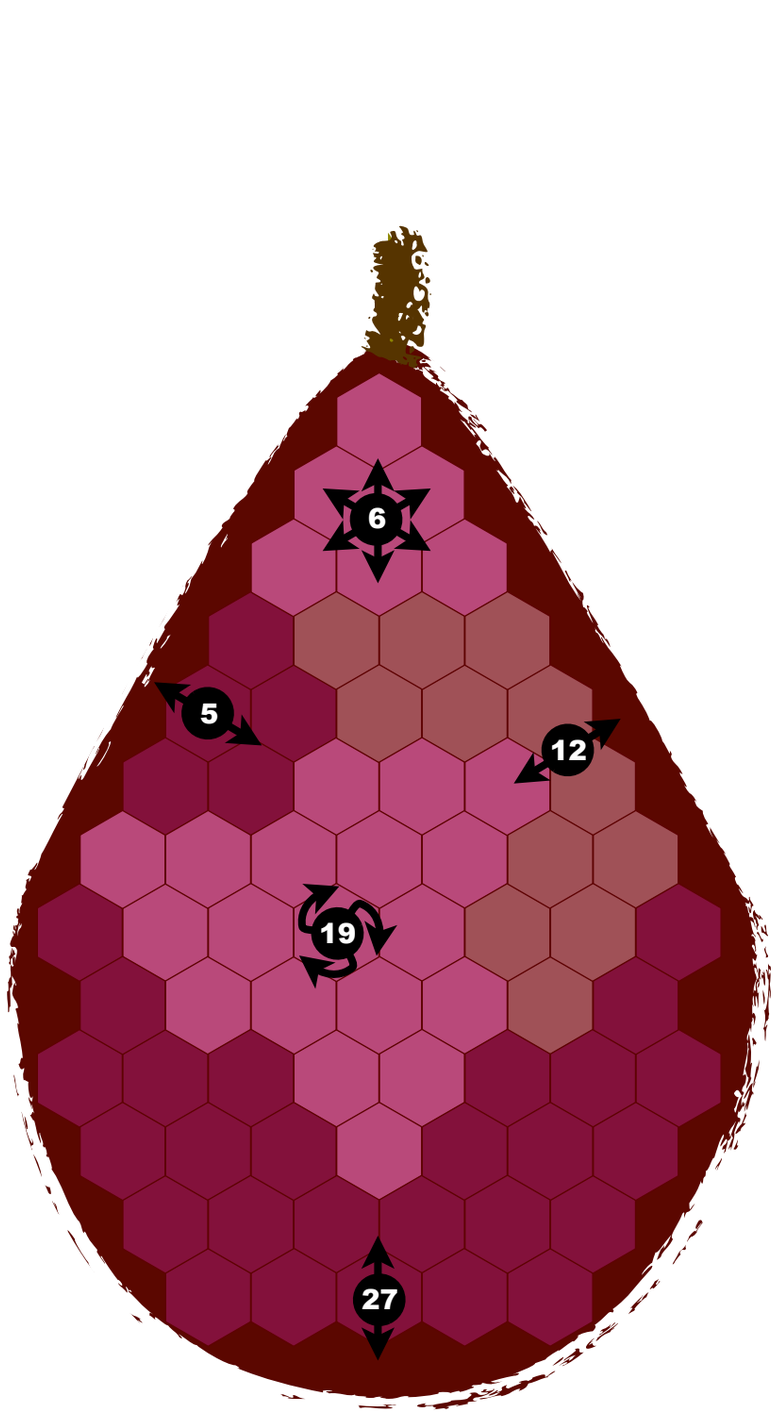


Turkey produces the most figs
followed by Egypt.





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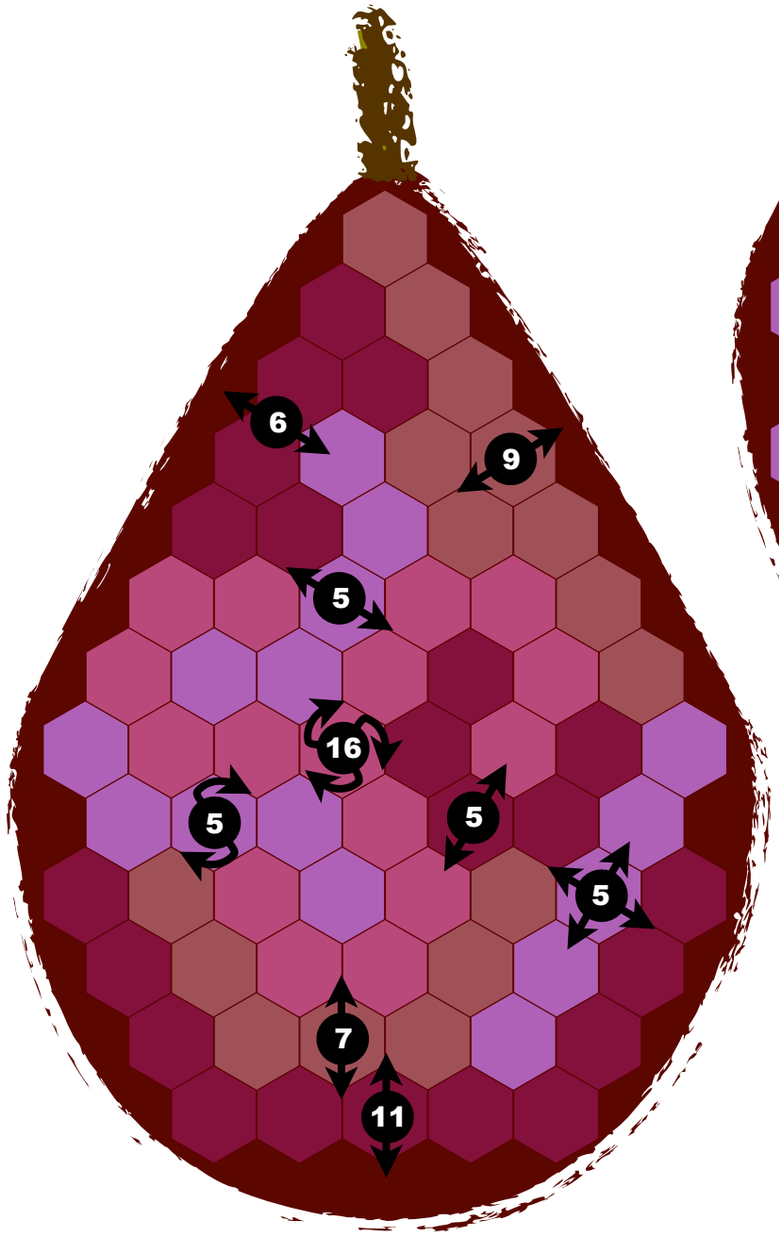
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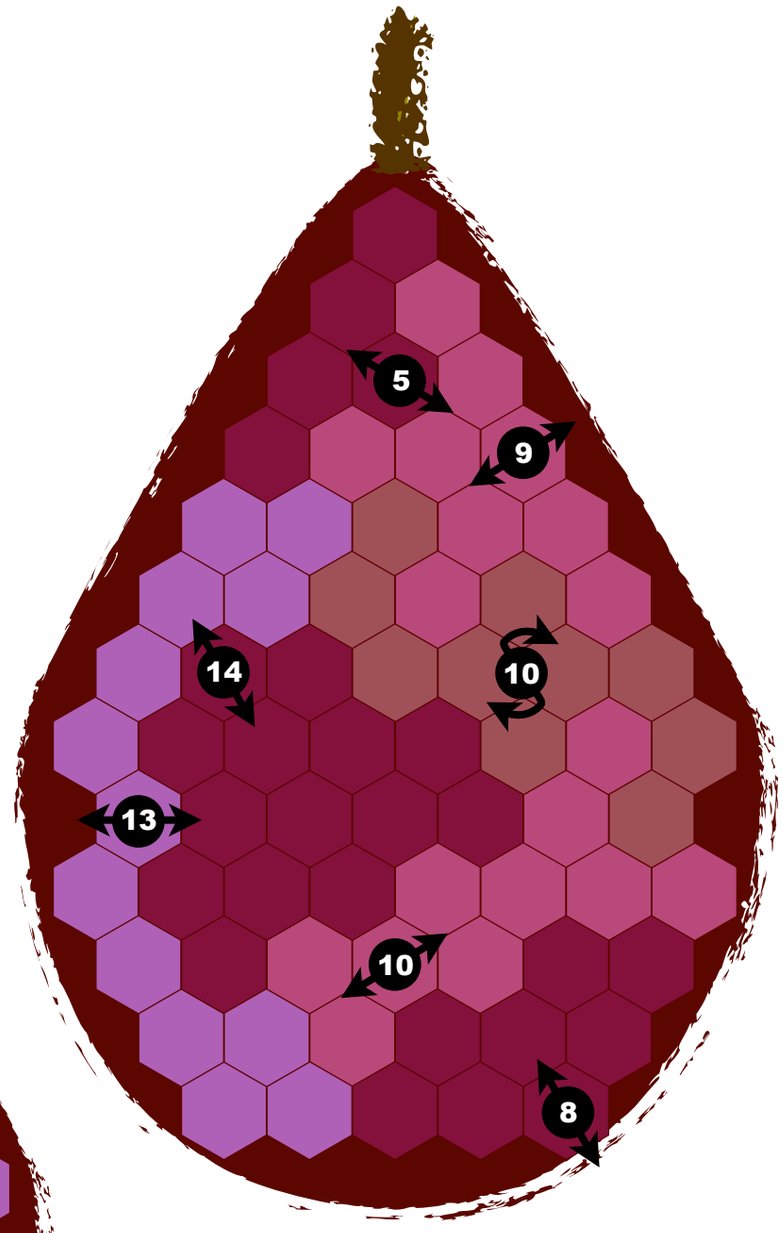
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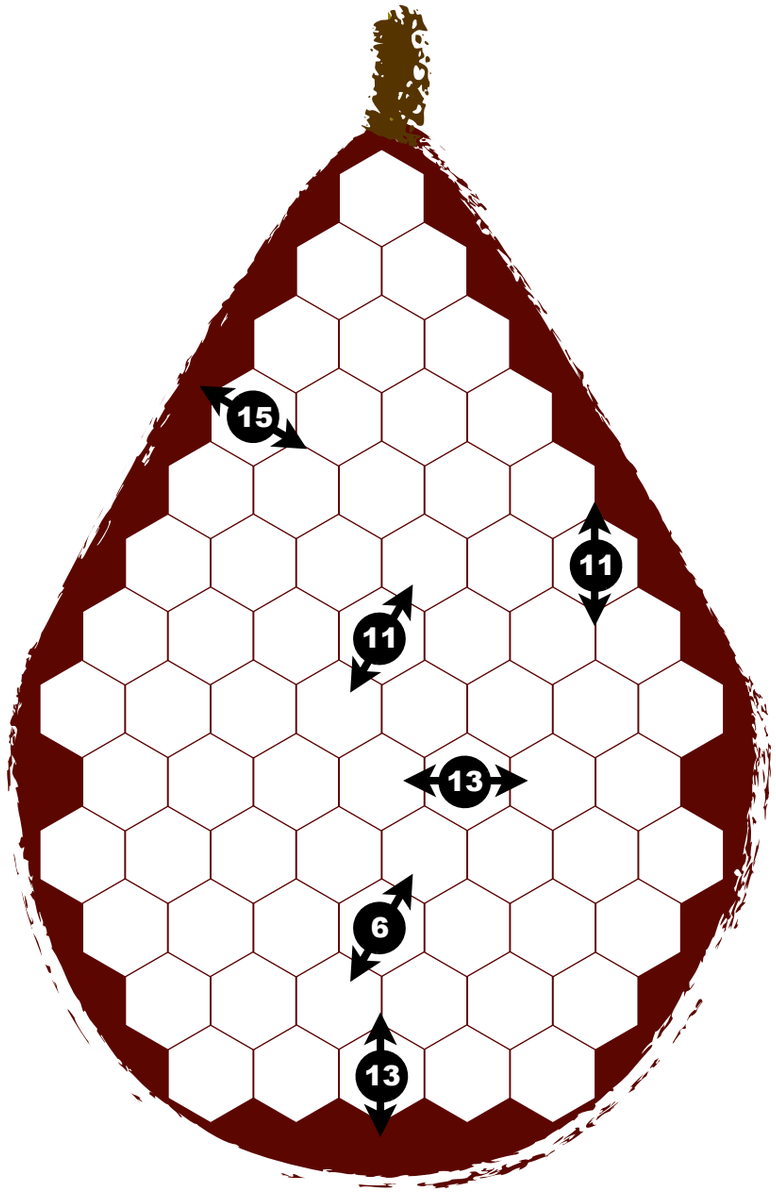
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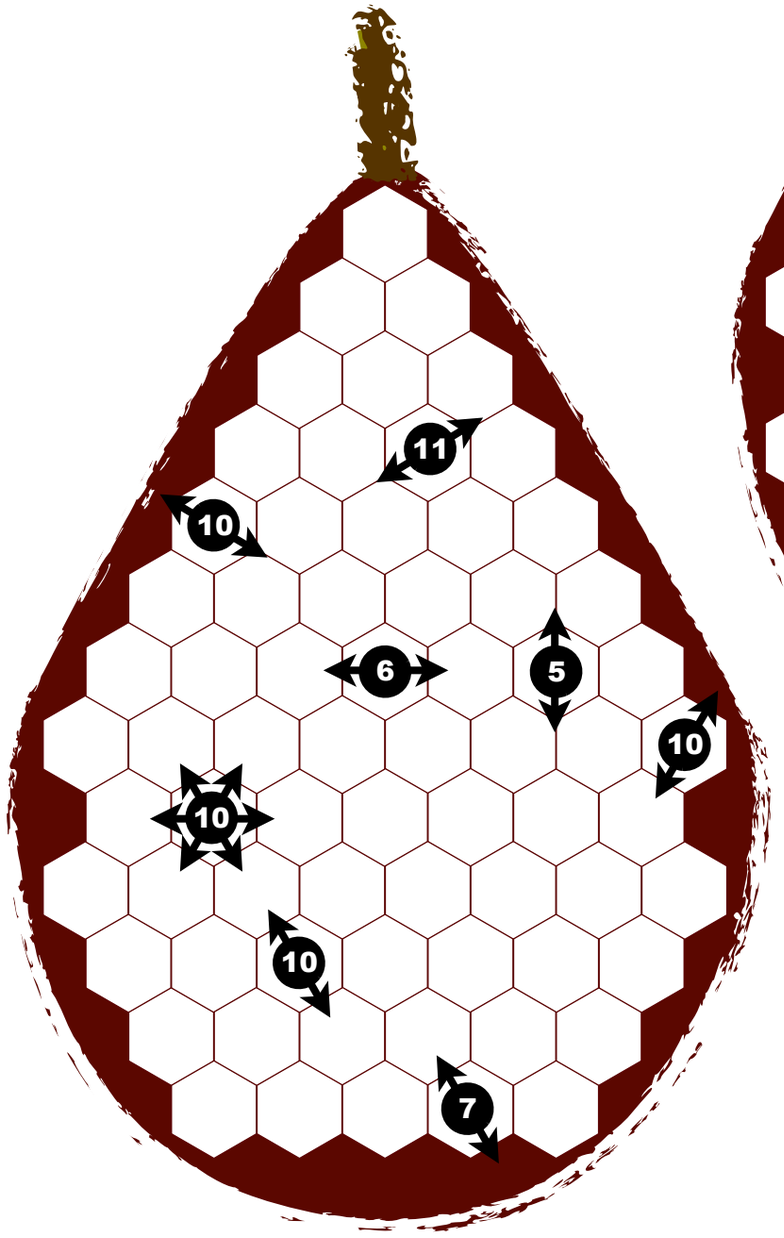
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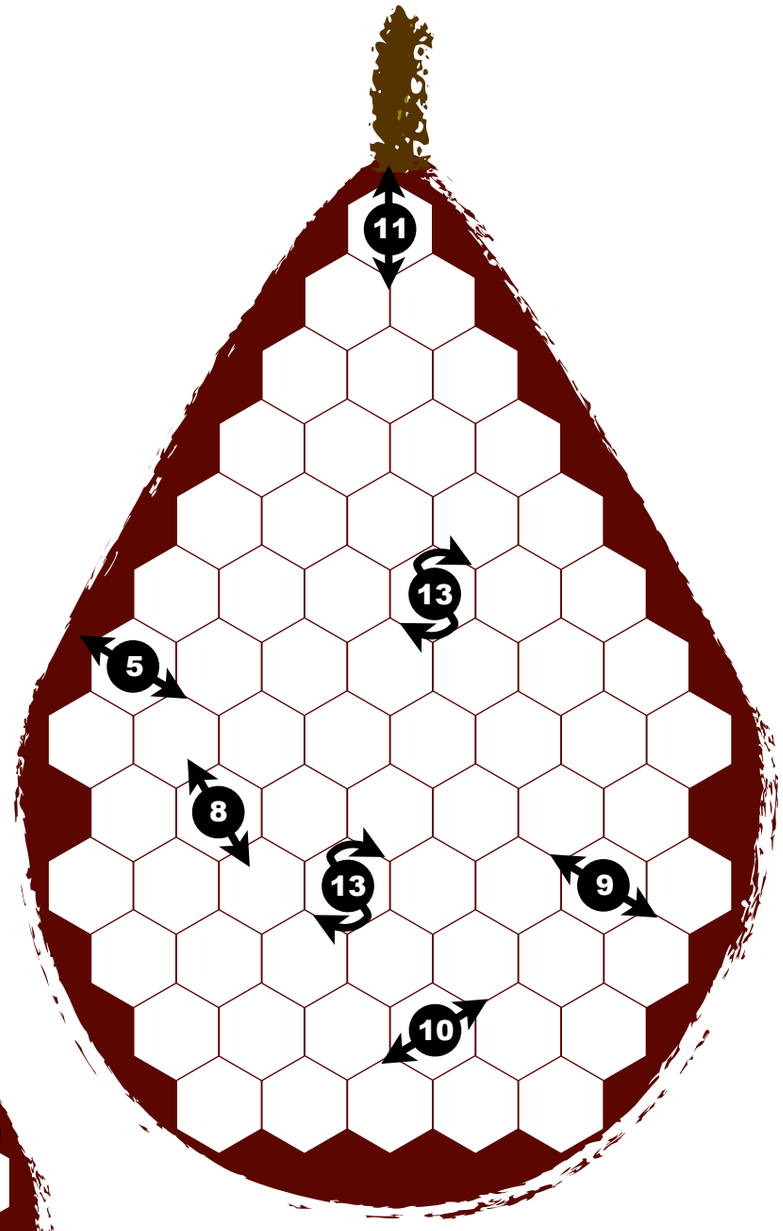
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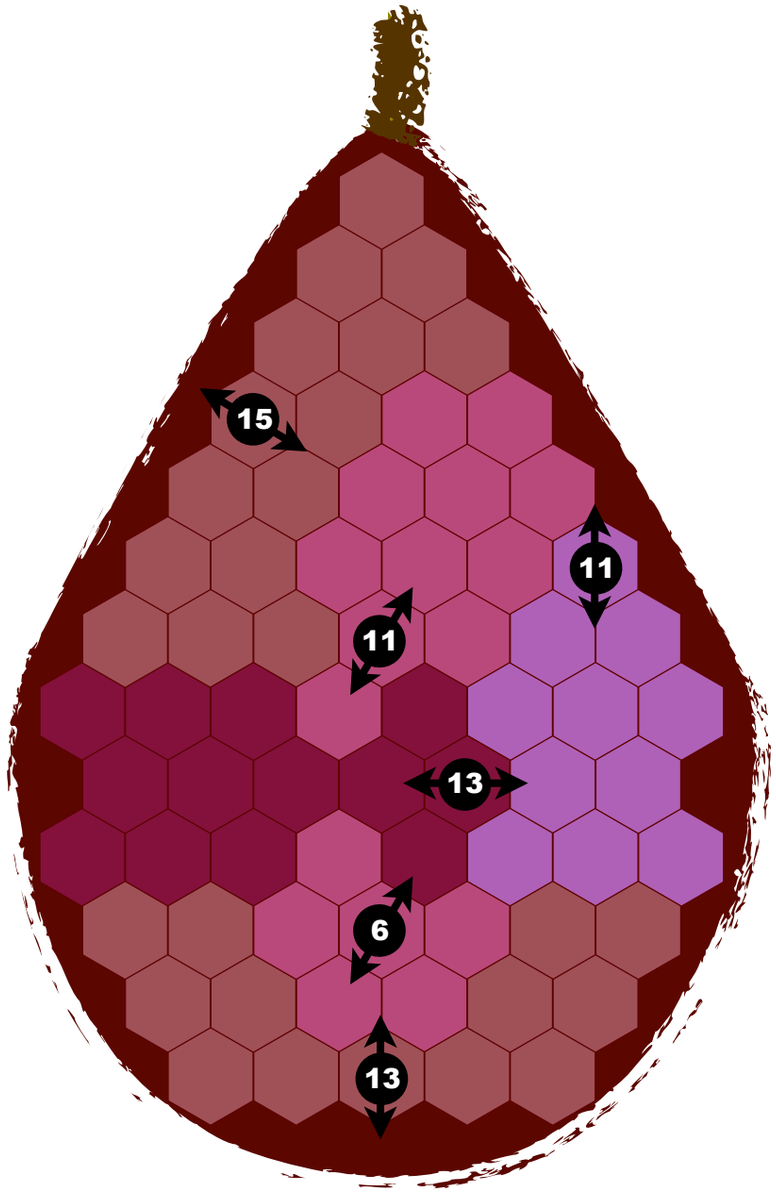
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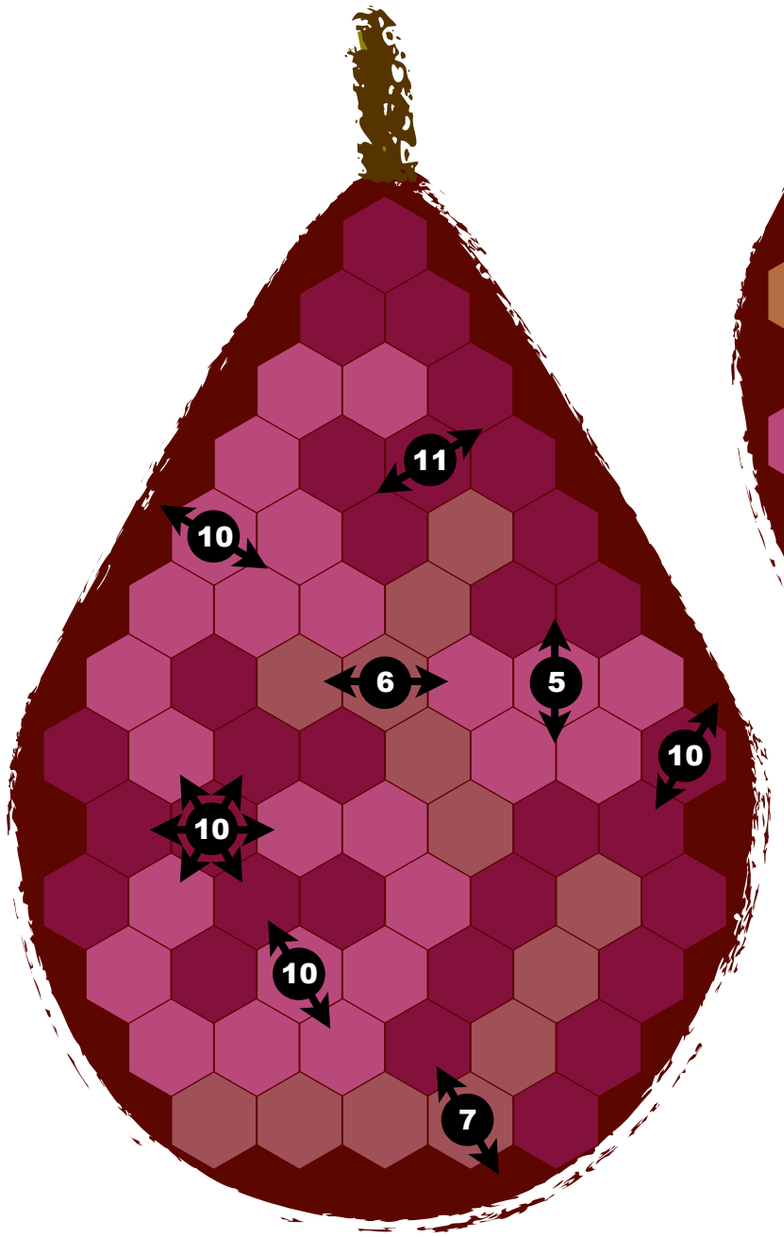
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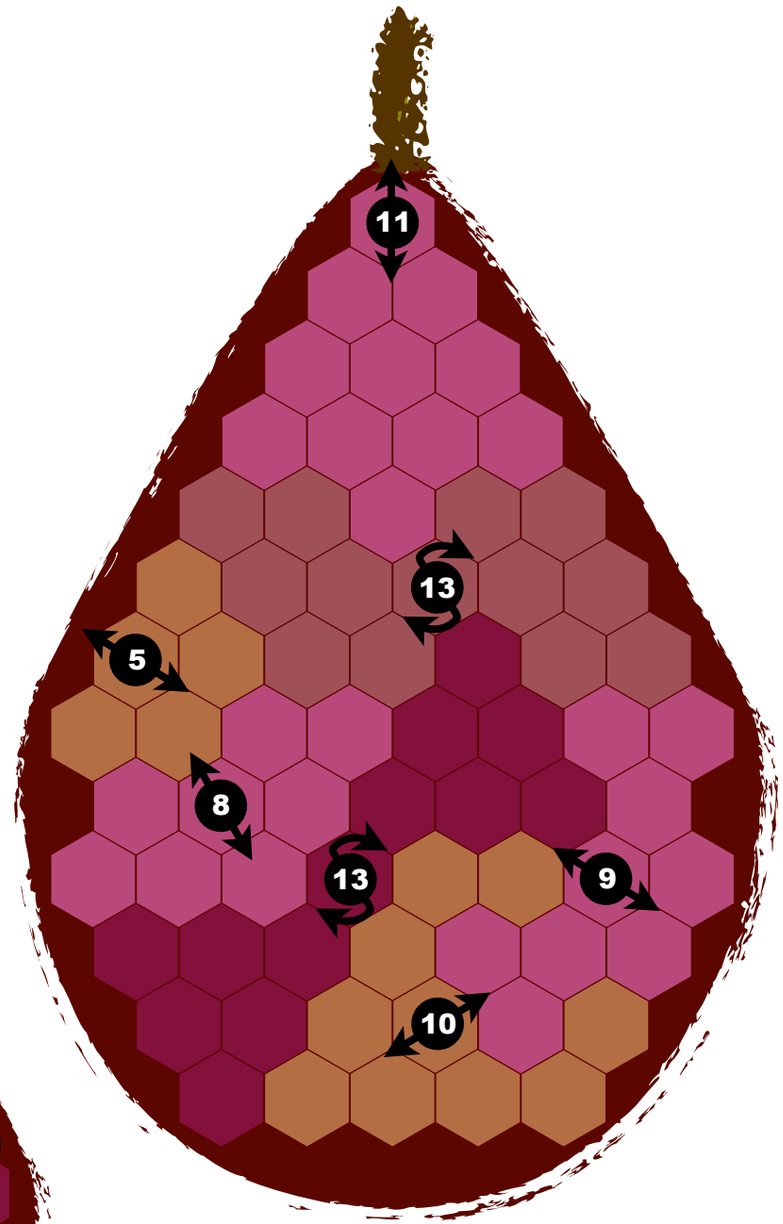
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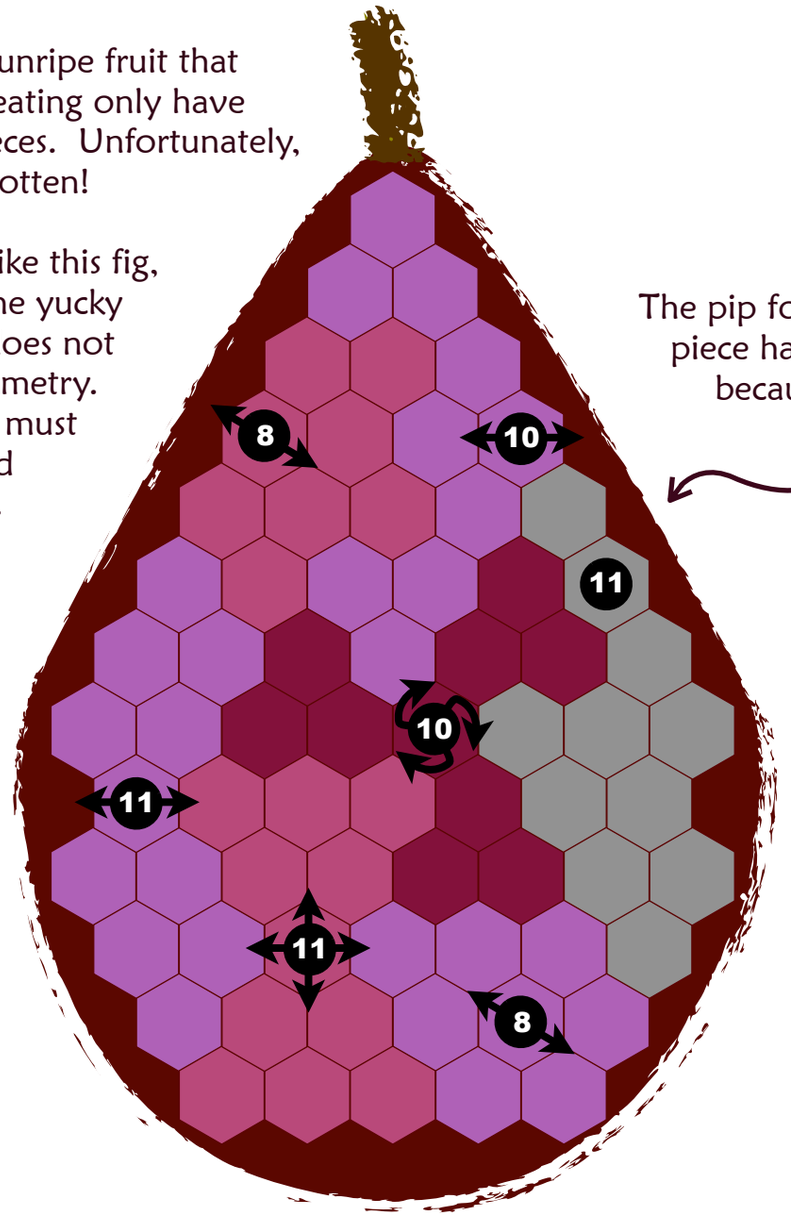
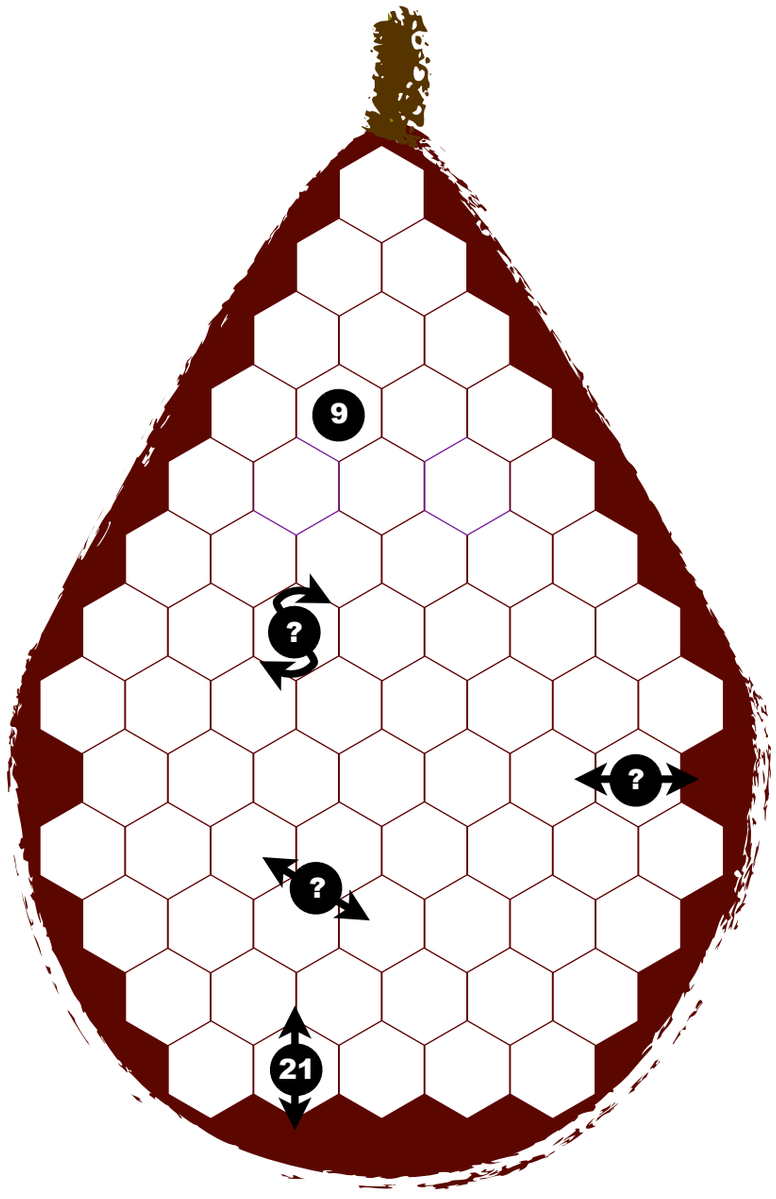
The ripe and unripe fruit that you've been eating only have symmetric pieces. Unfortunately, some fruit is rotten!

Rotten fruit, like this fig, has at least one yucky piece which does not have any symmetry. Rotten pieces must be cut out and thrown away.



The pip for the rotten piece has no arrows because it has no symmetry.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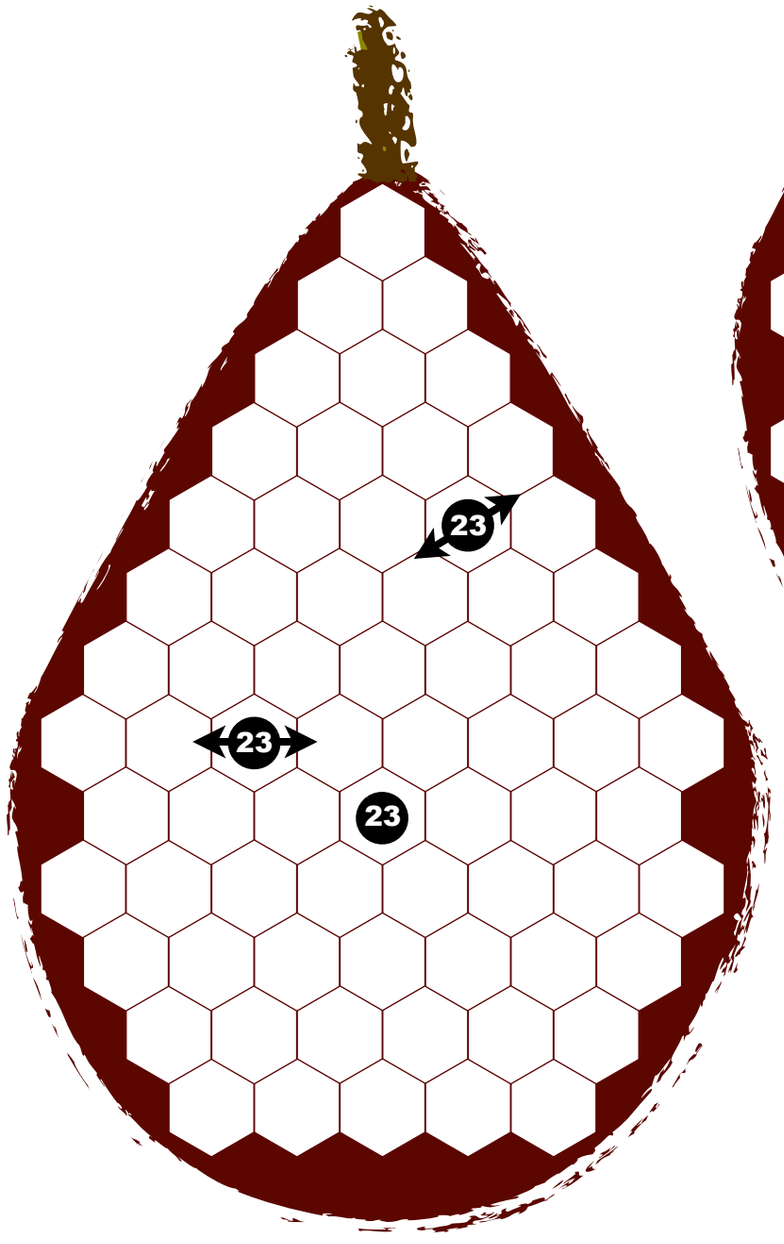
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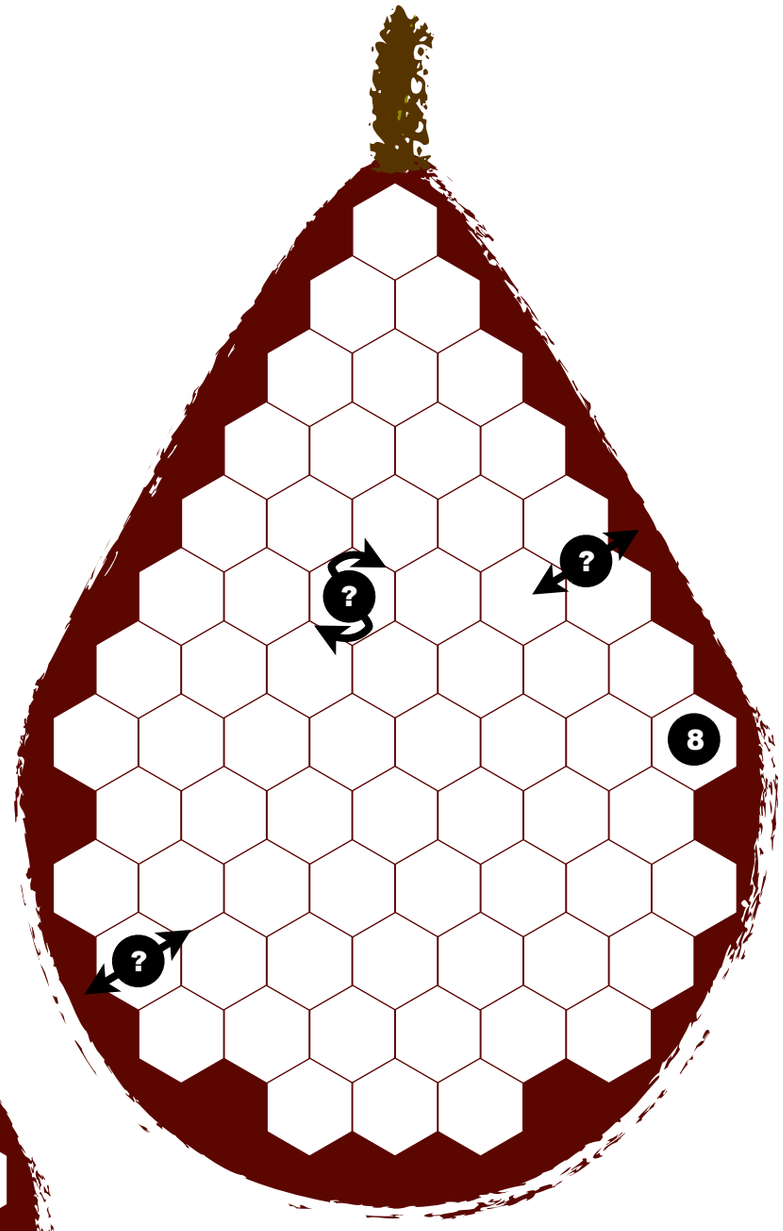
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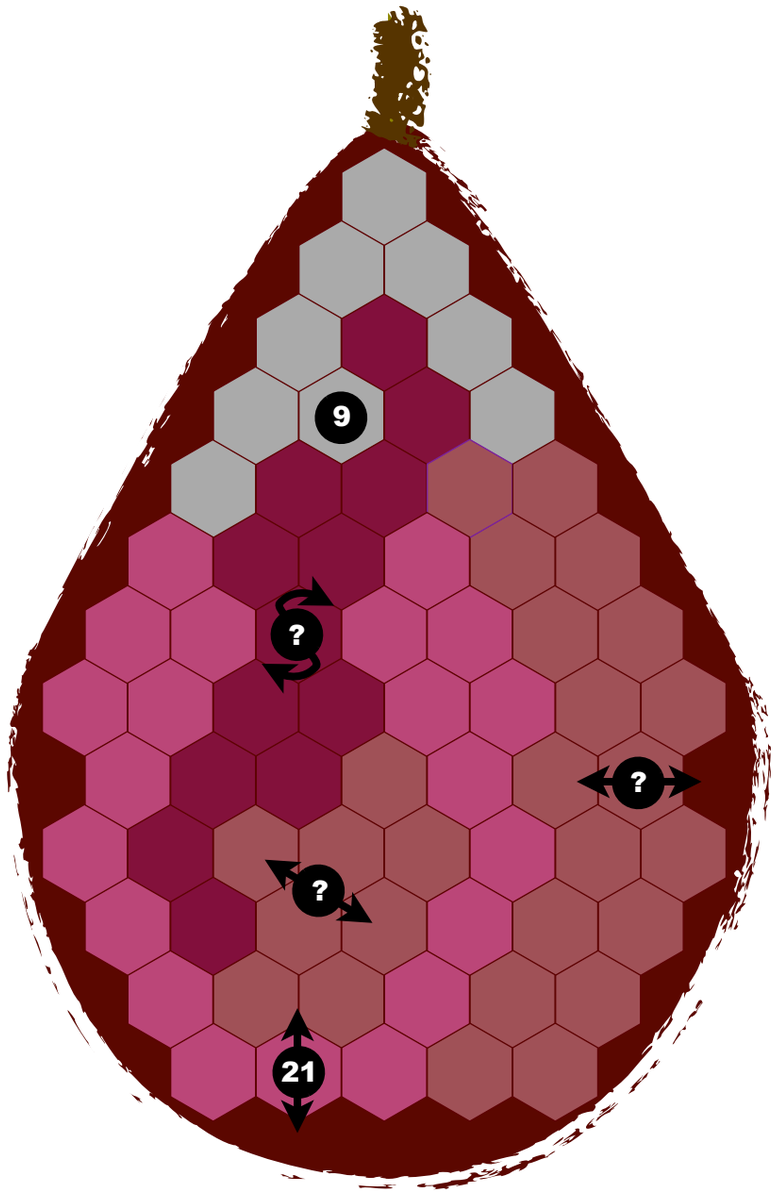
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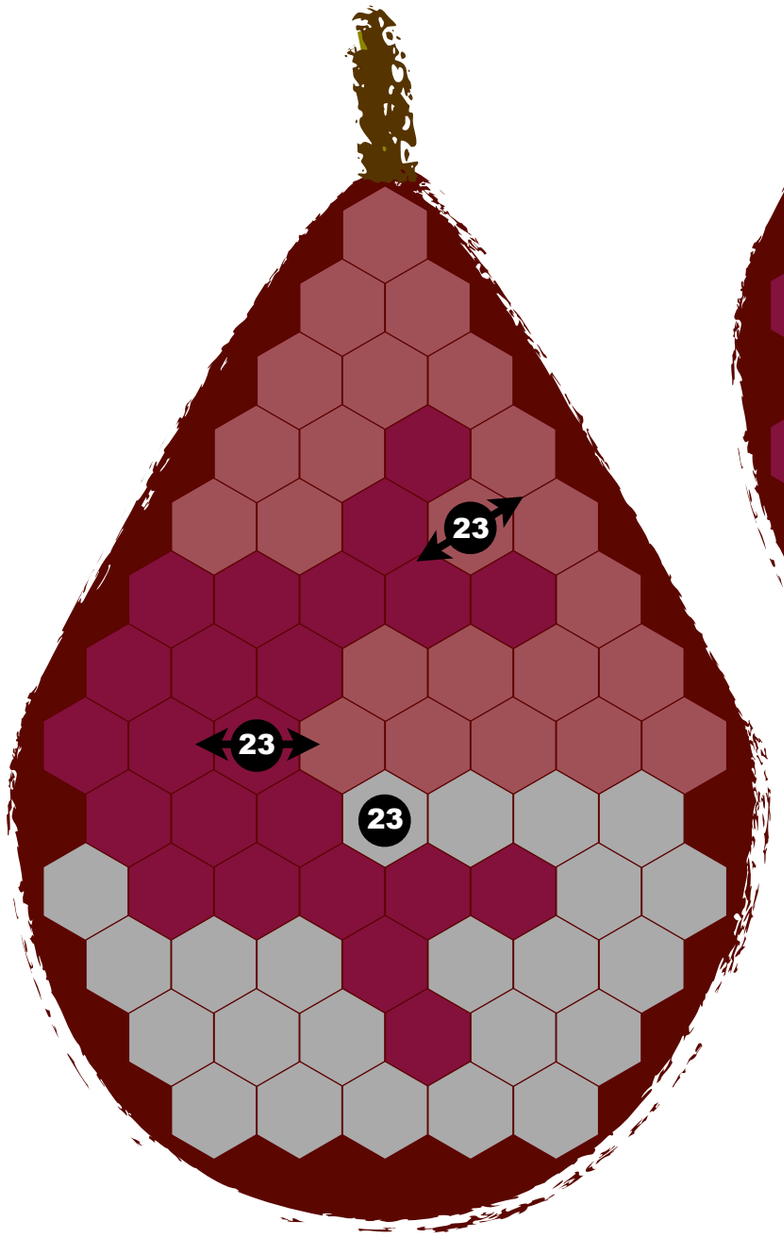
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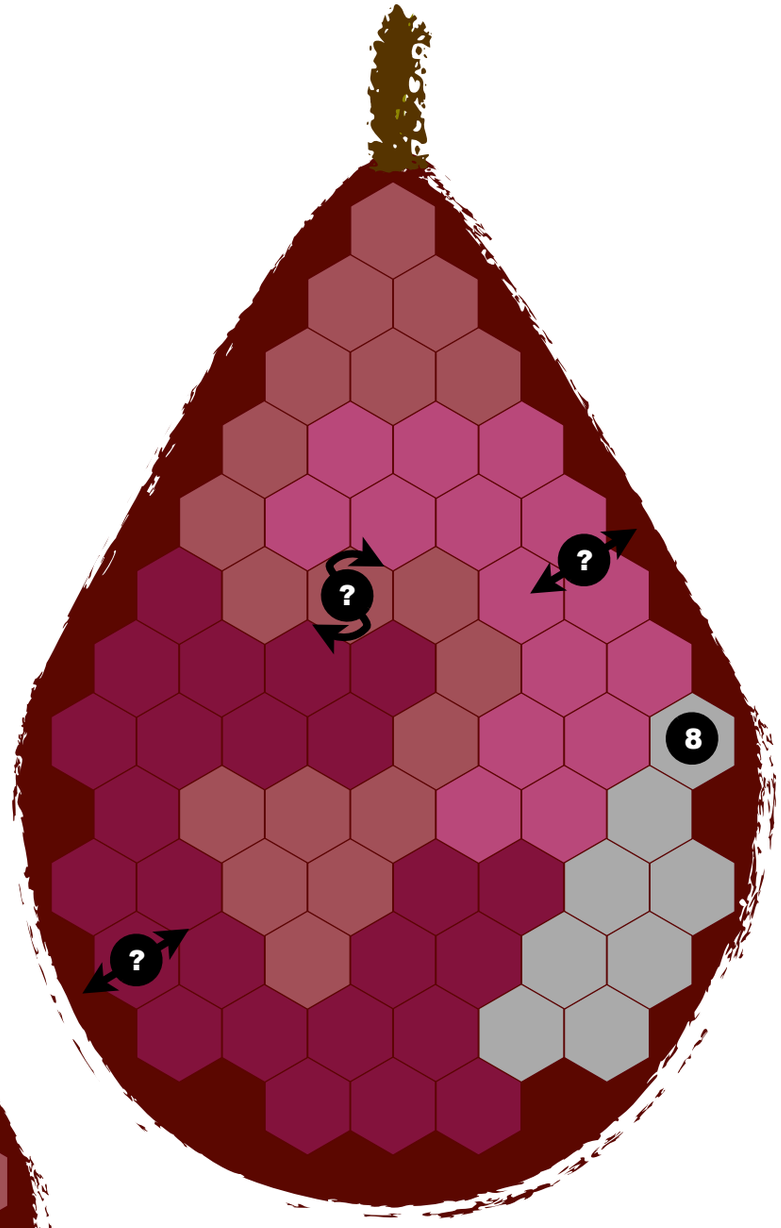
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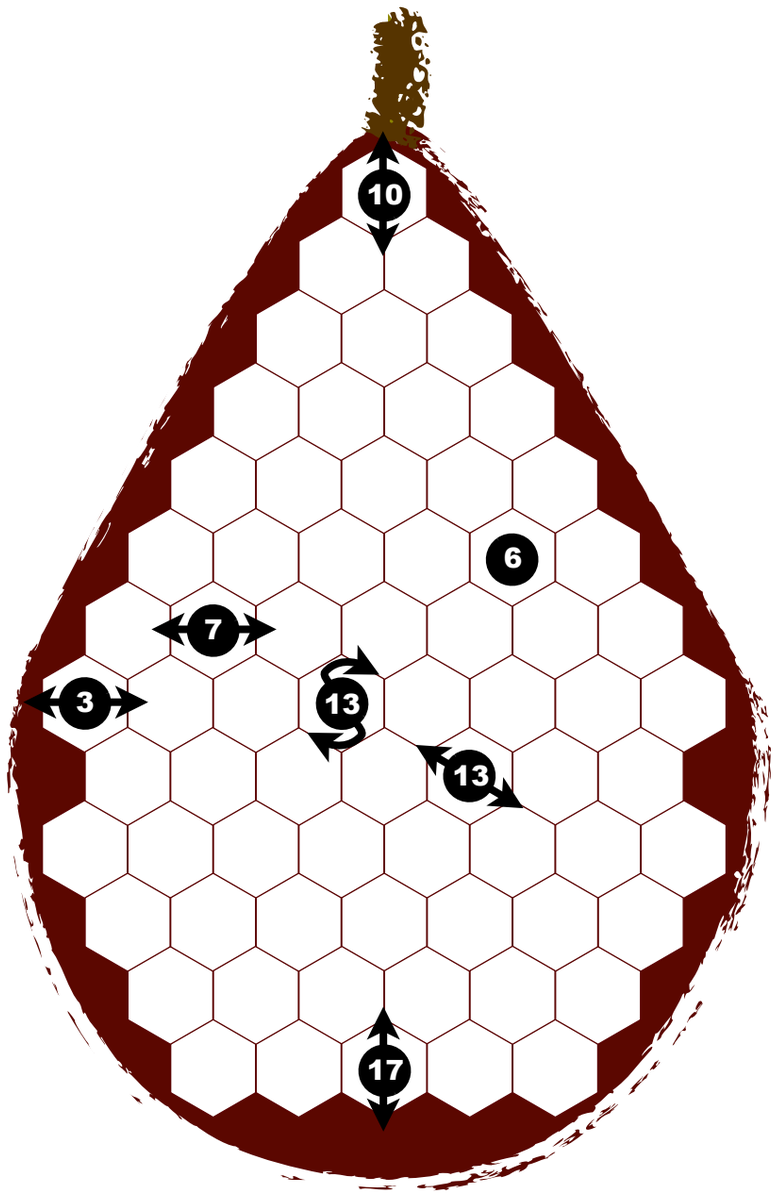
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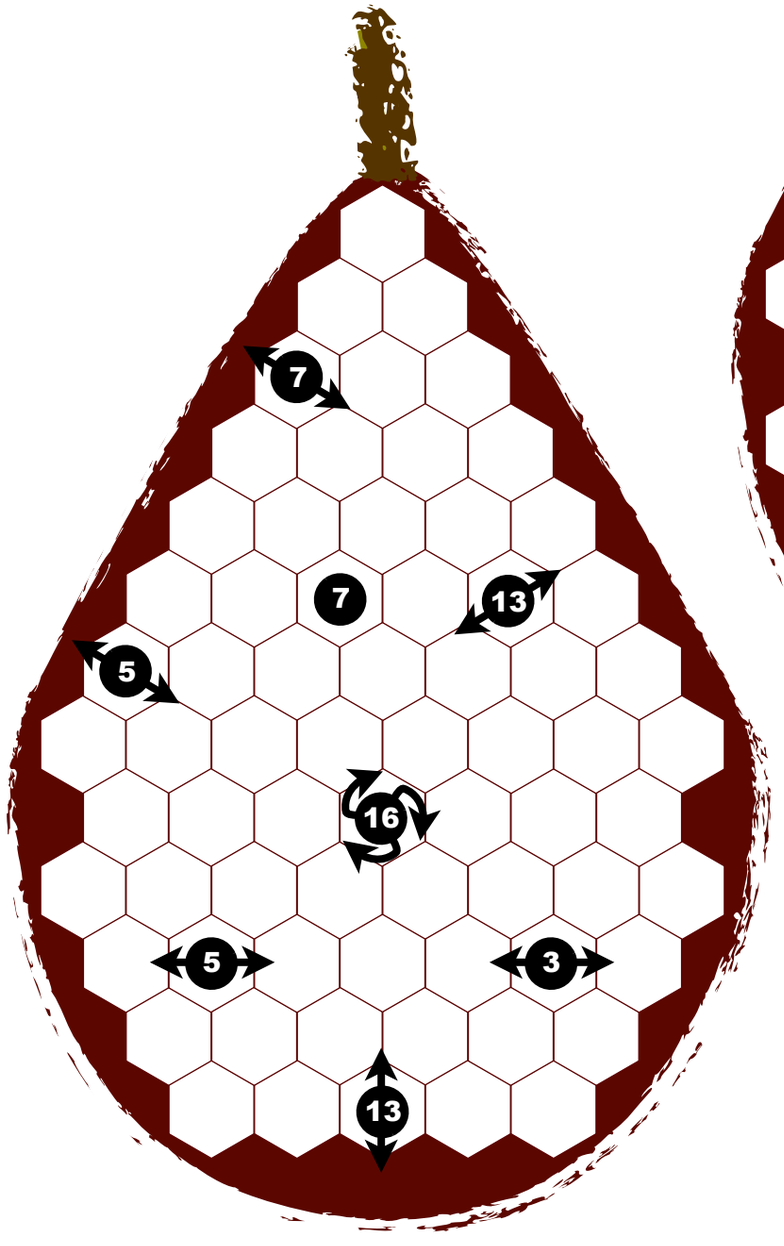
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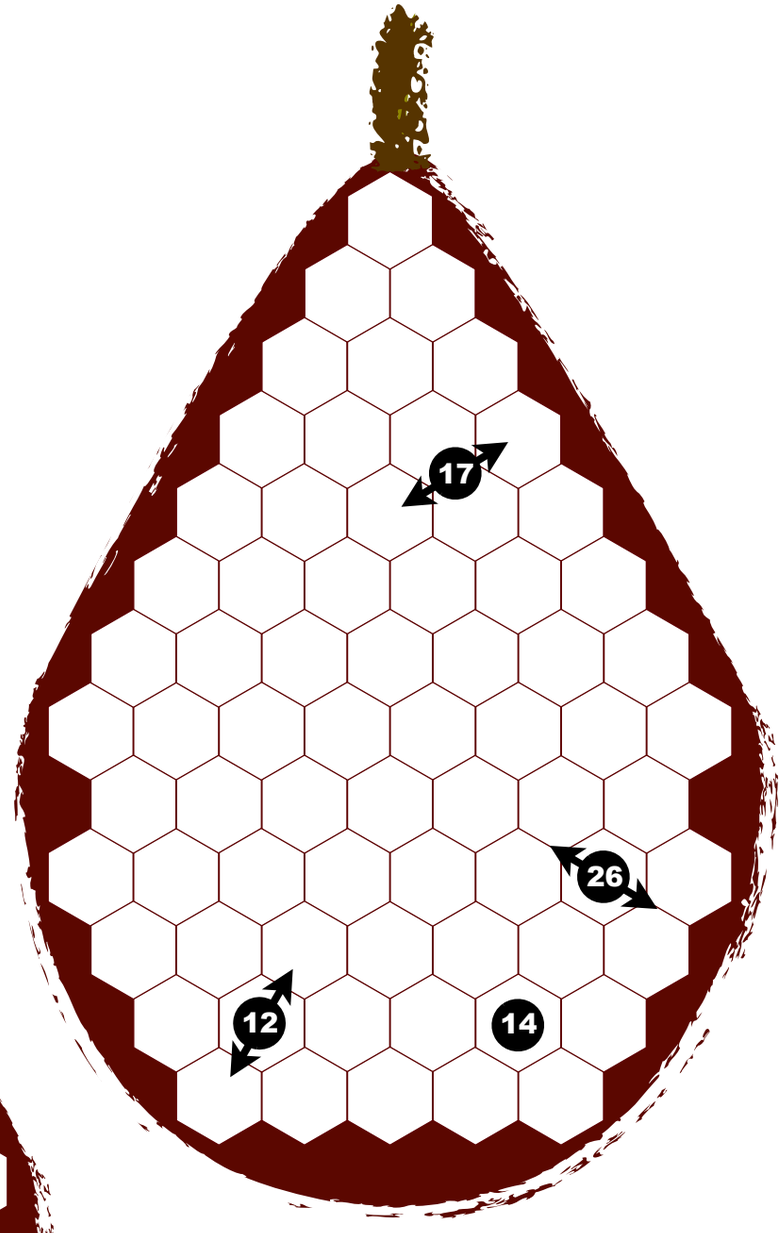
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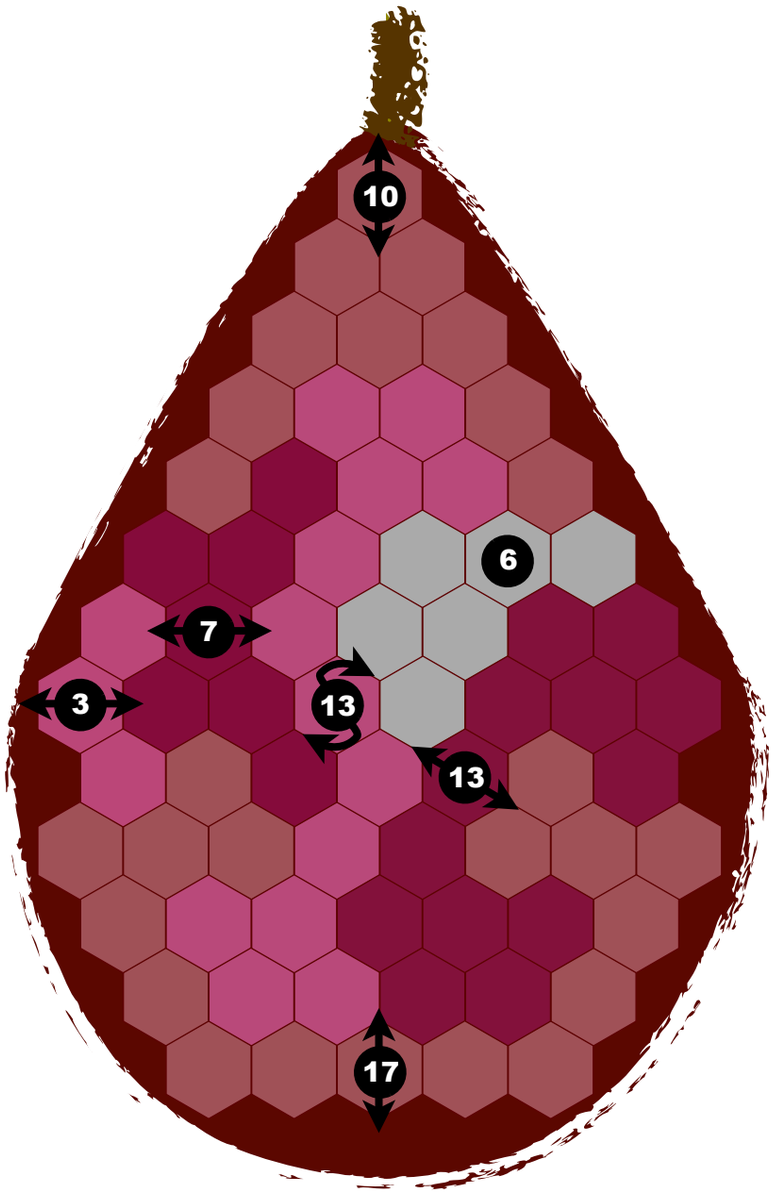
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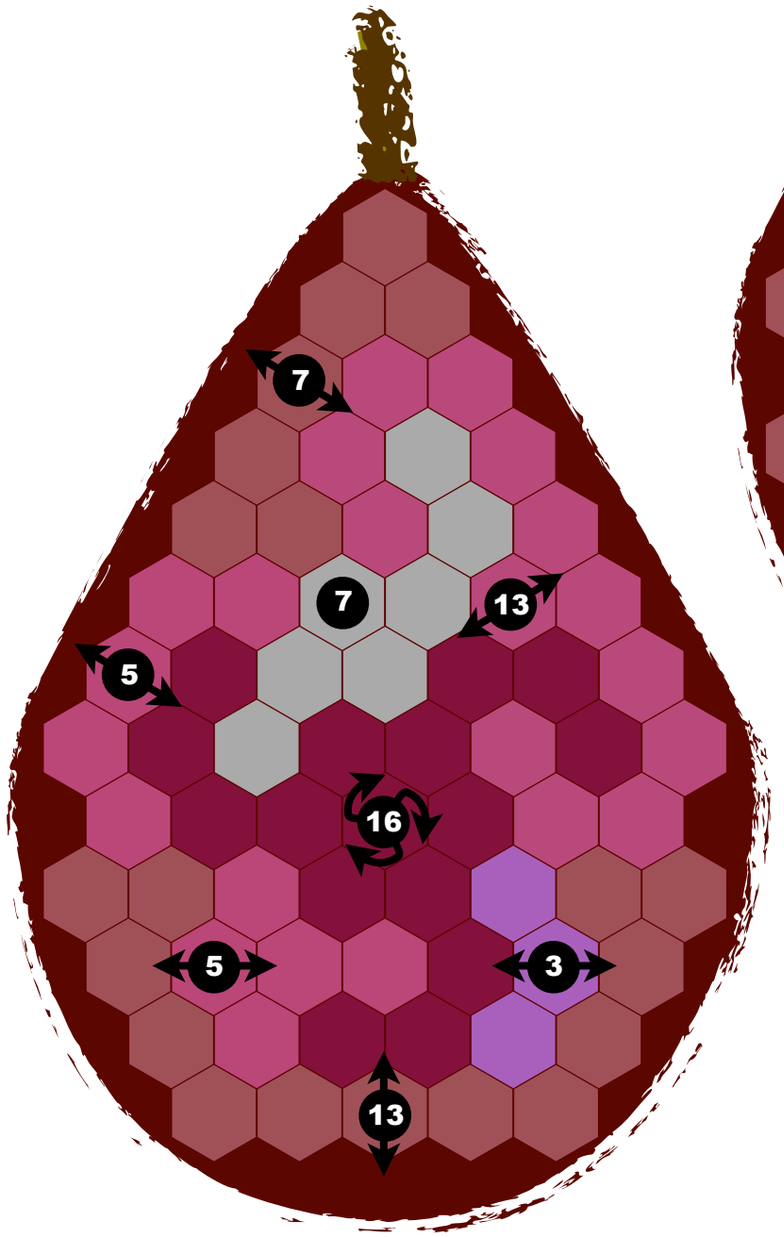
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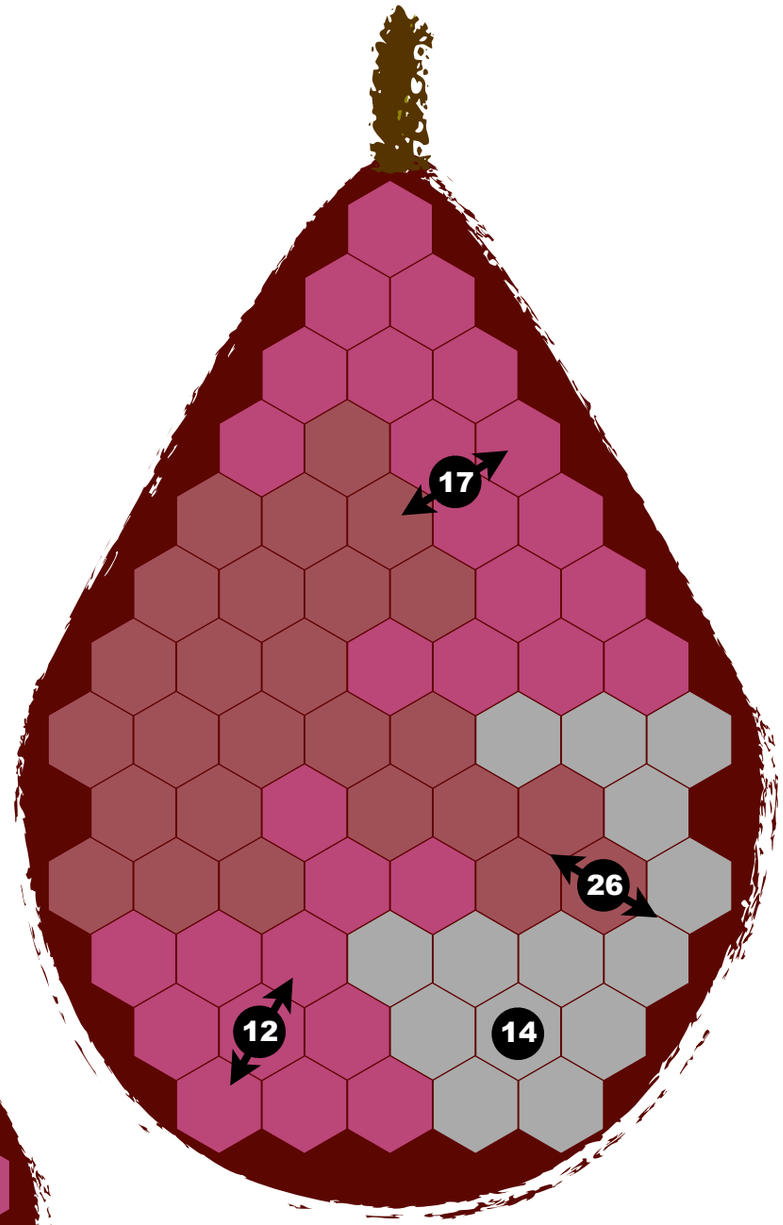
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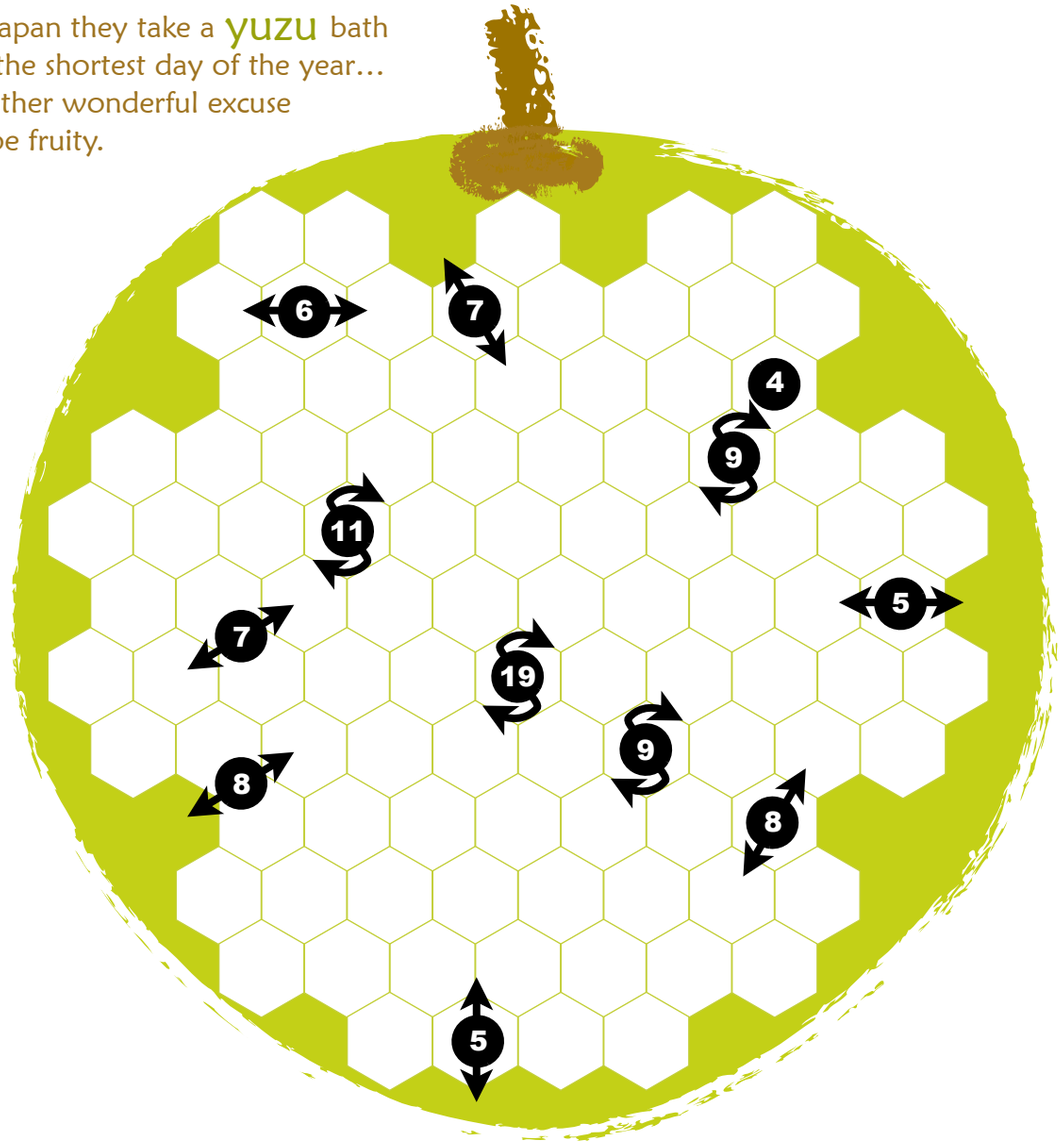
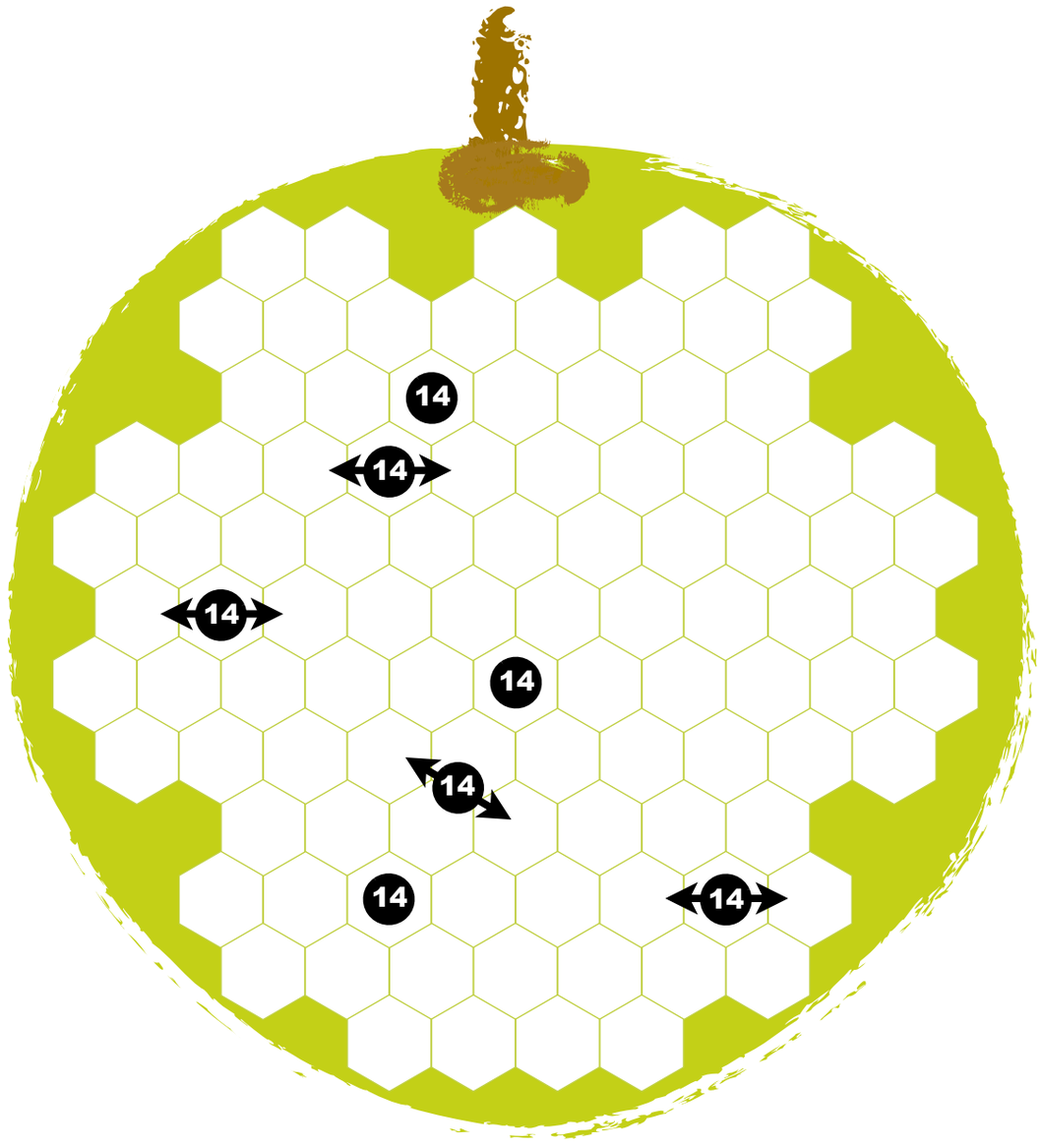
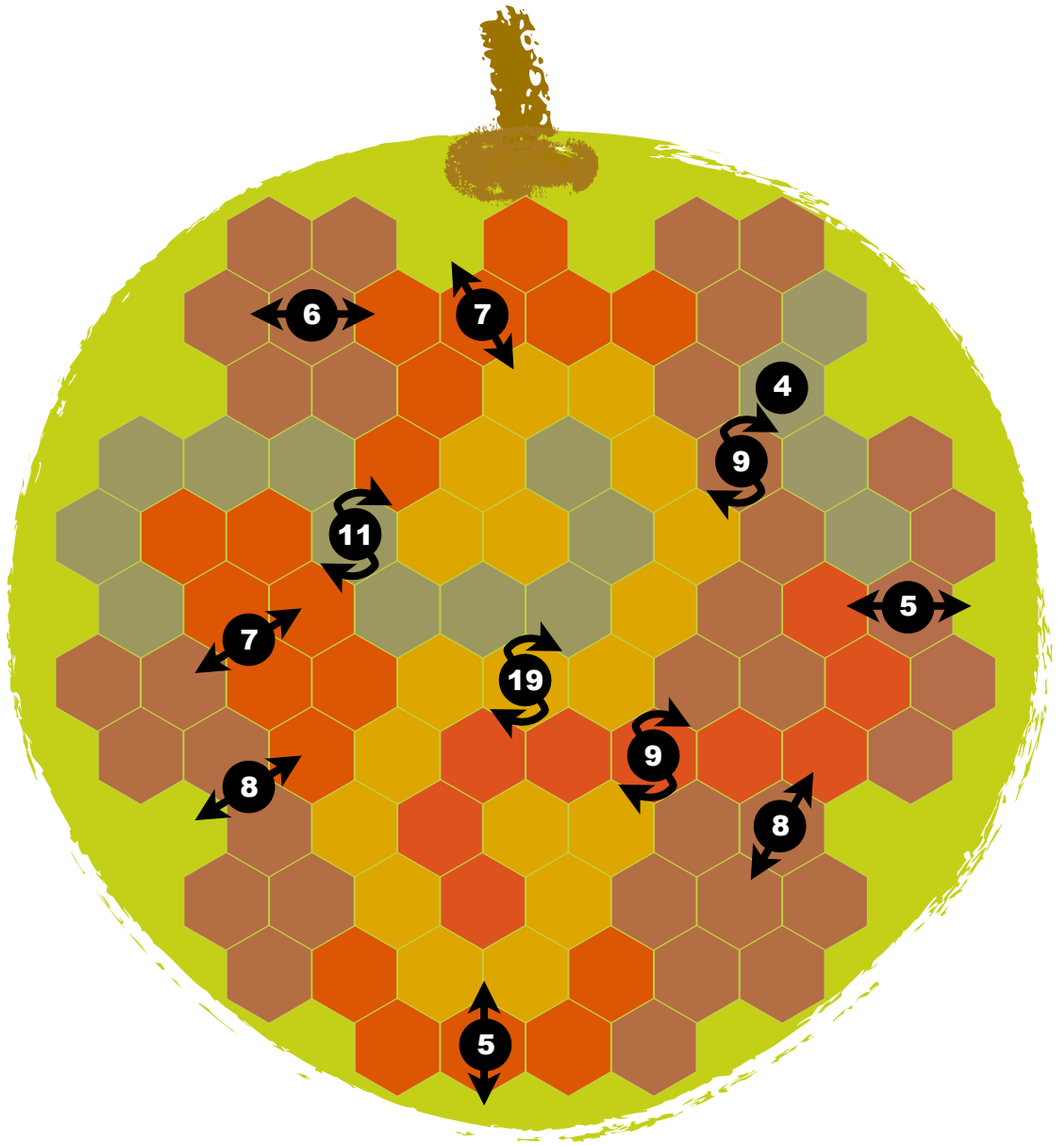
In Japan they take a yuzu bath on the shortest day of the year… another wonderful excuse to be fruity.



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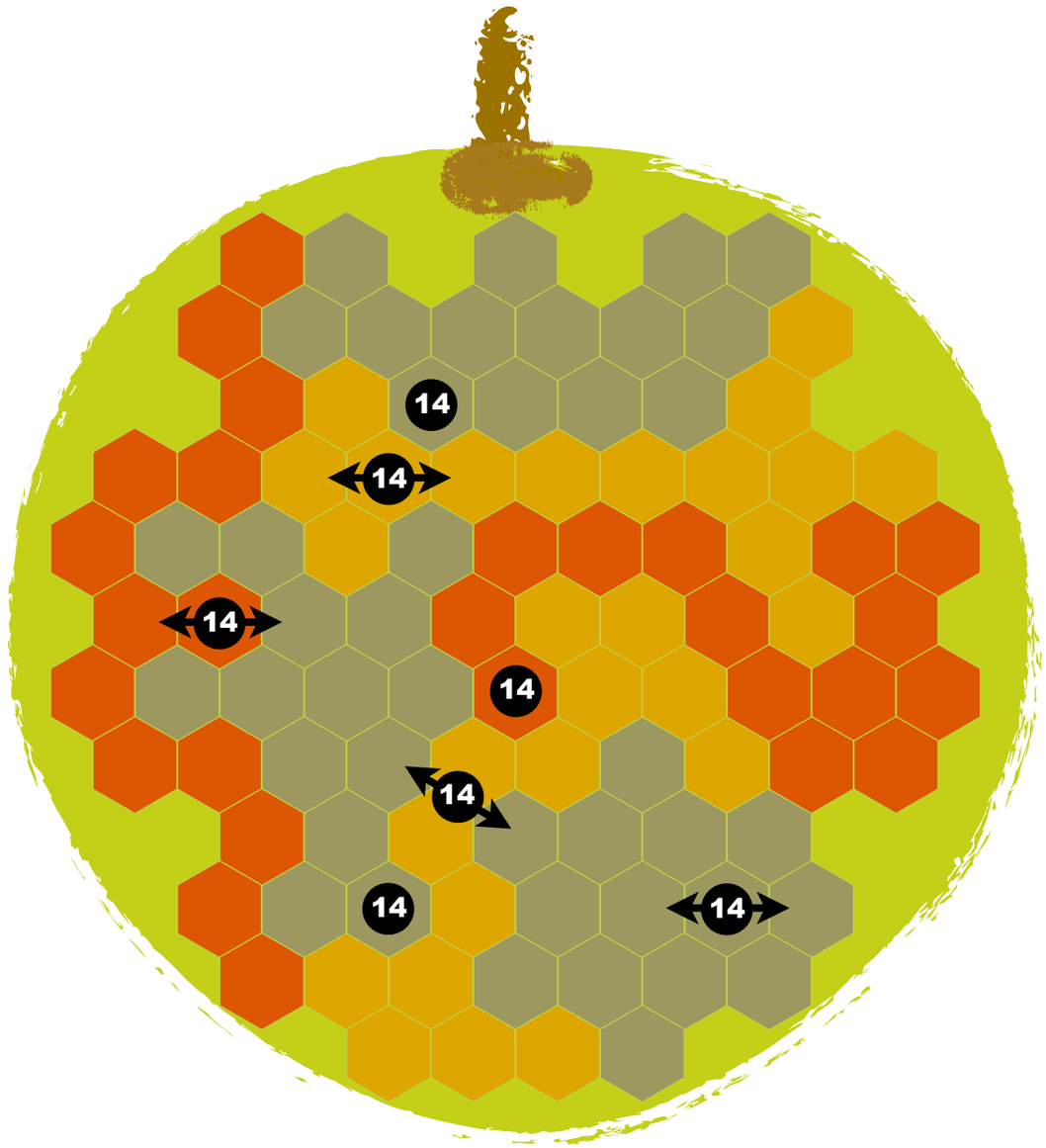
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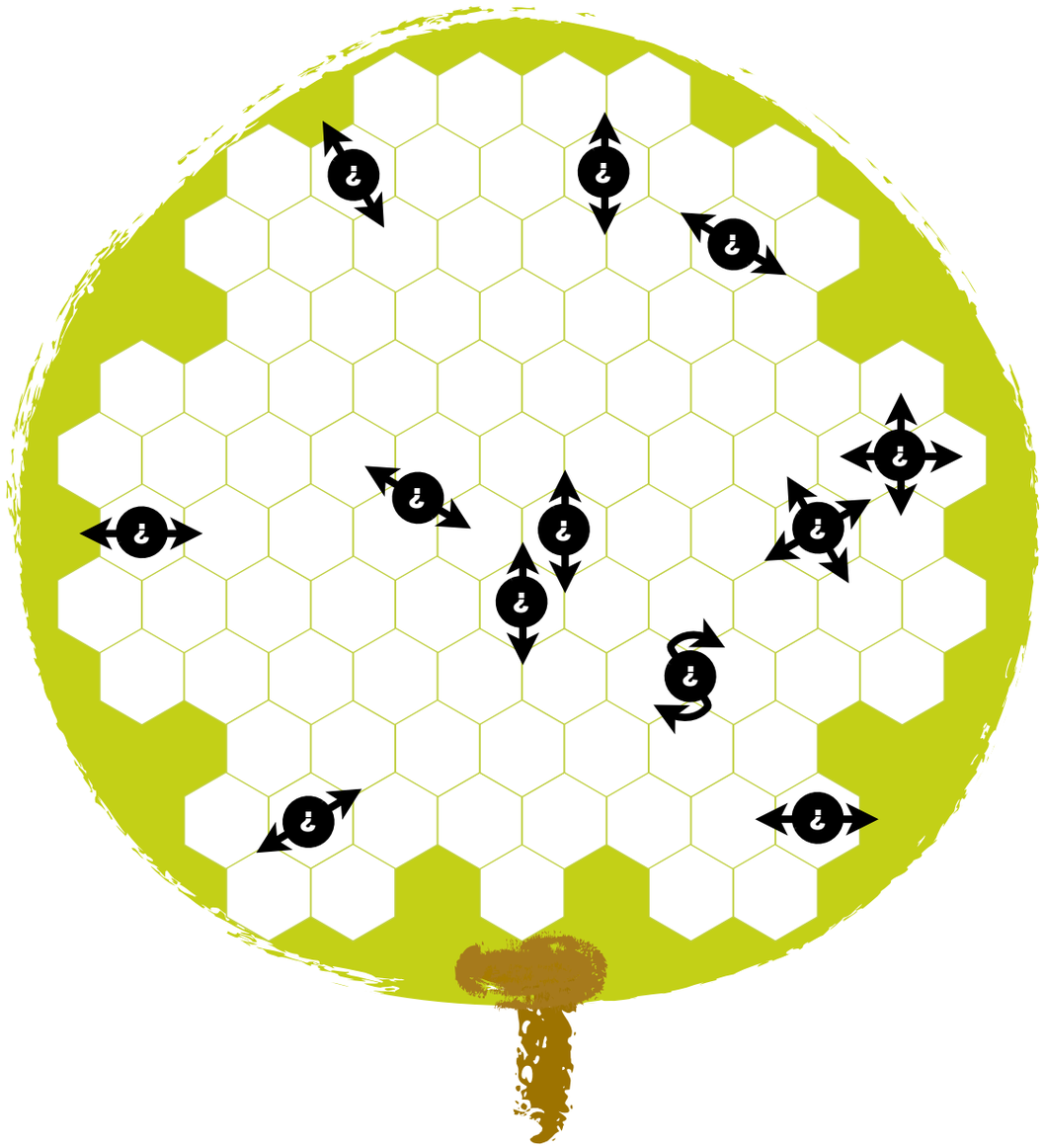
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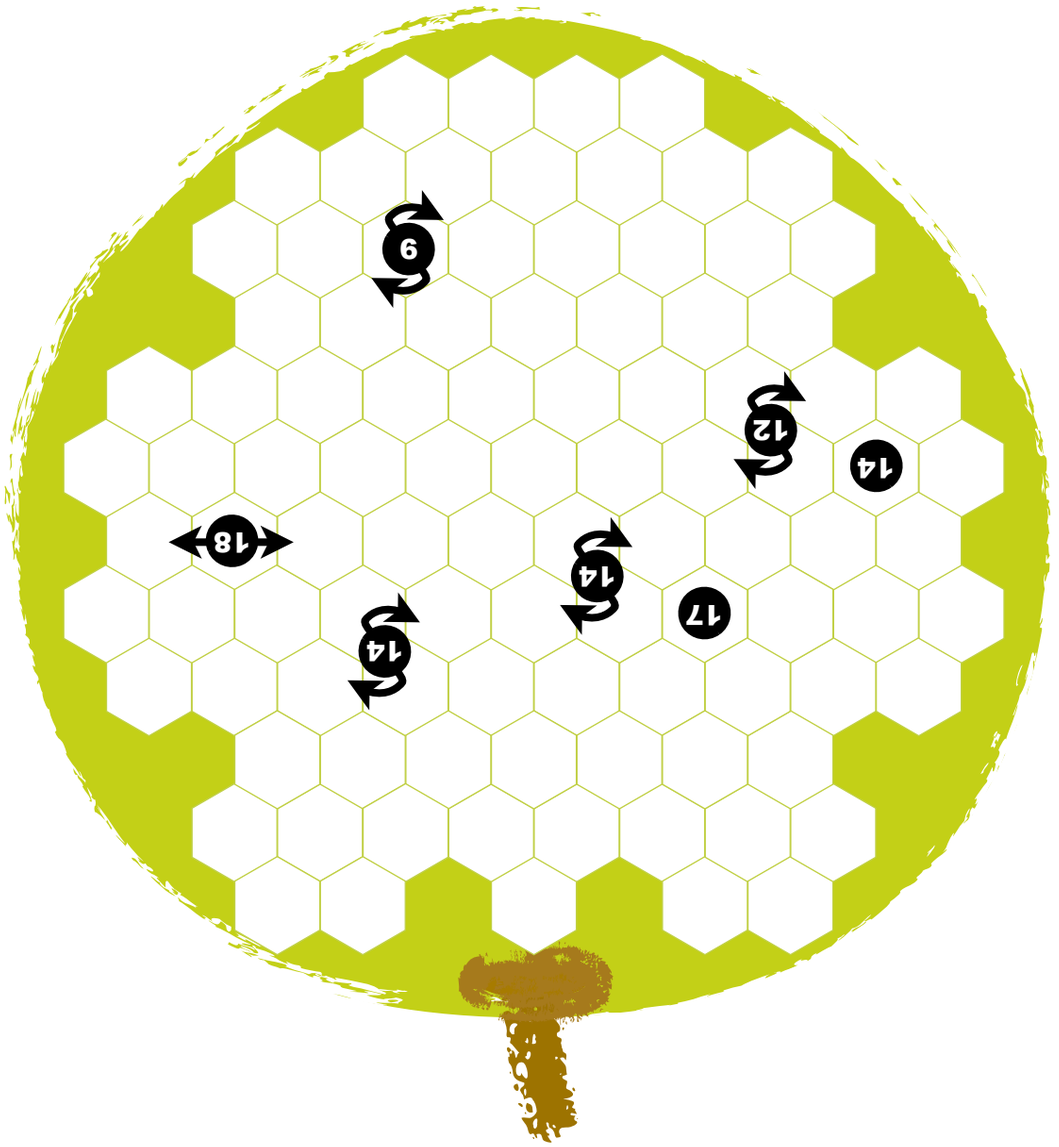
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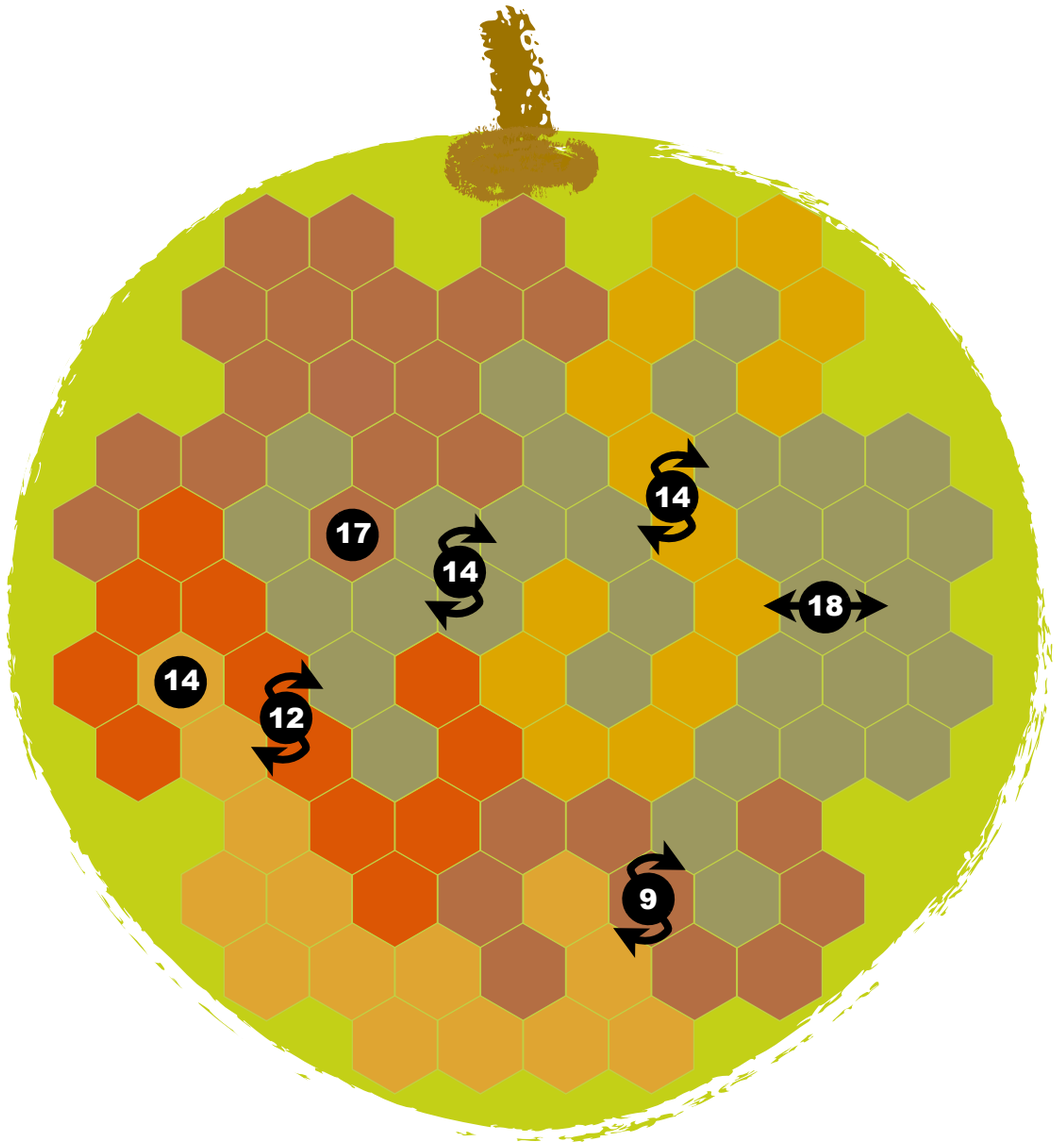
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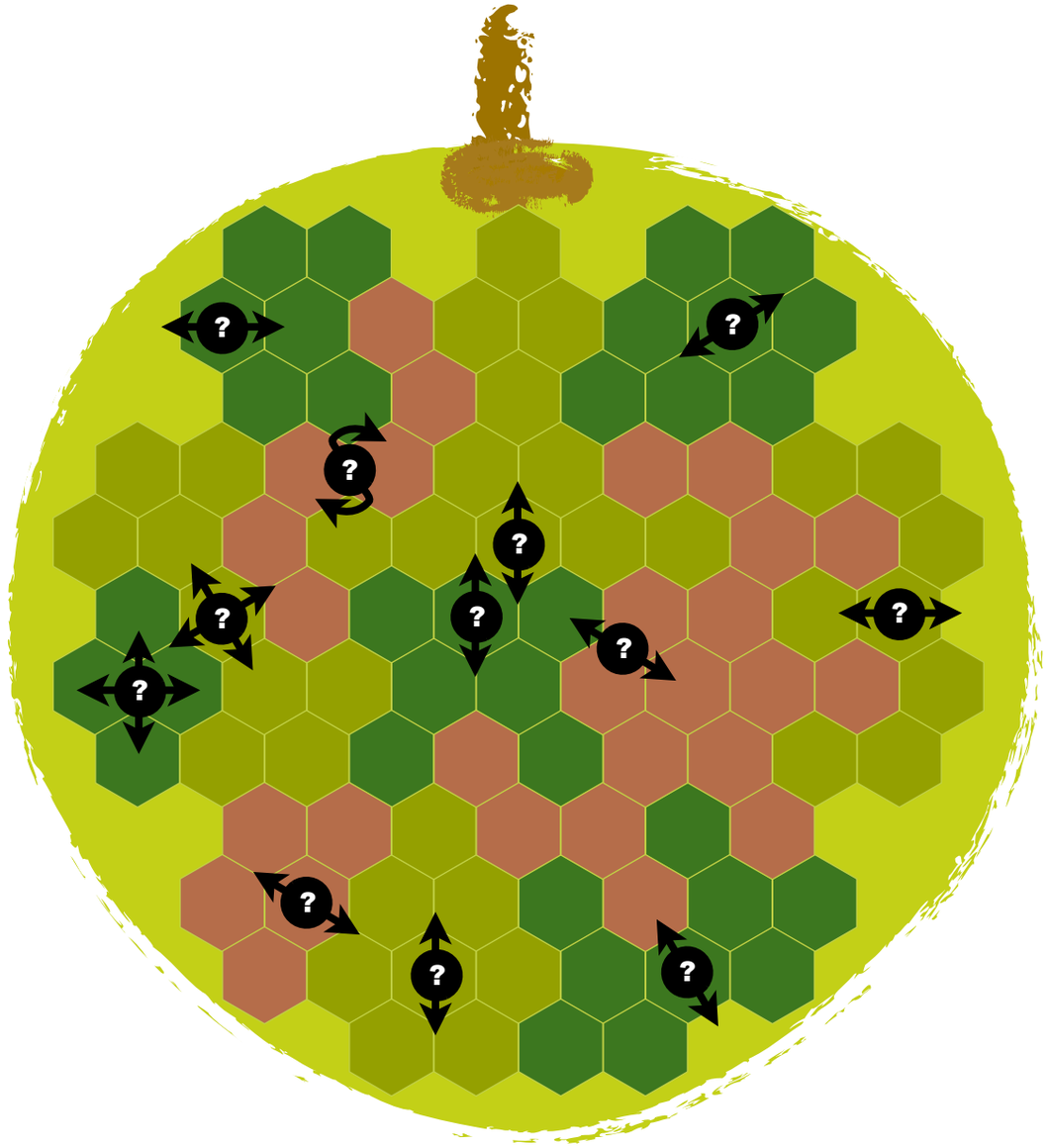
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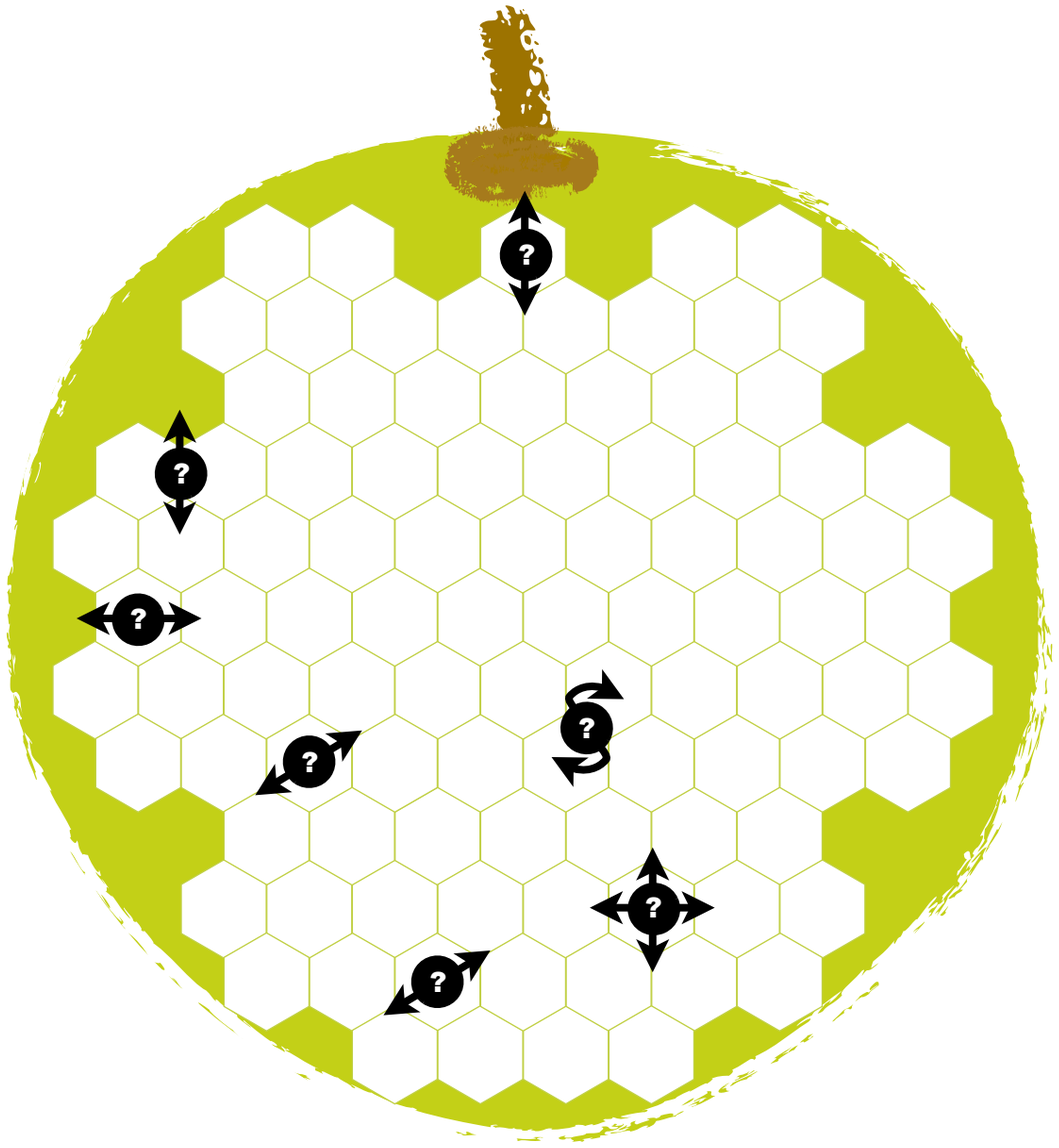

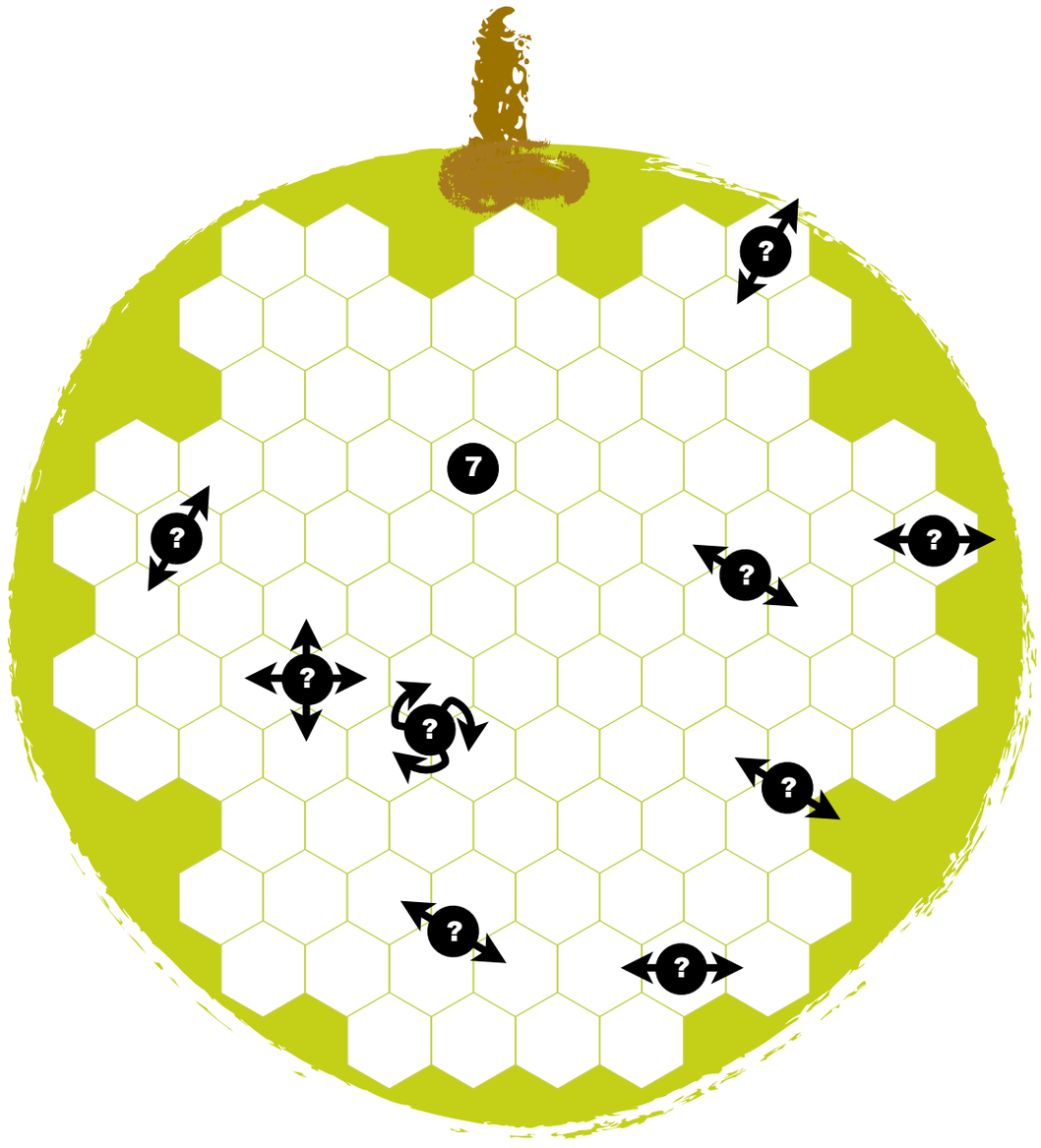
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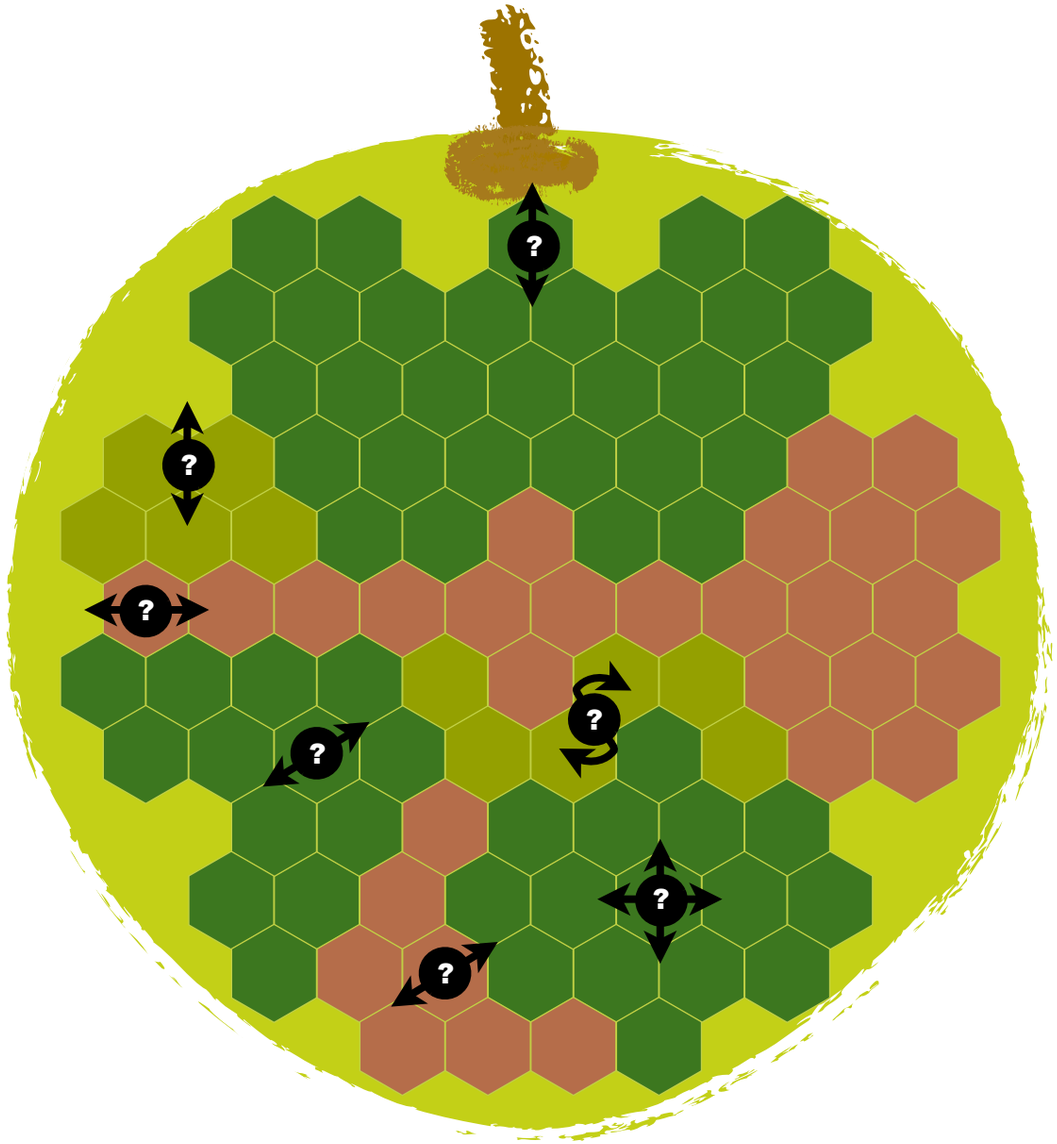
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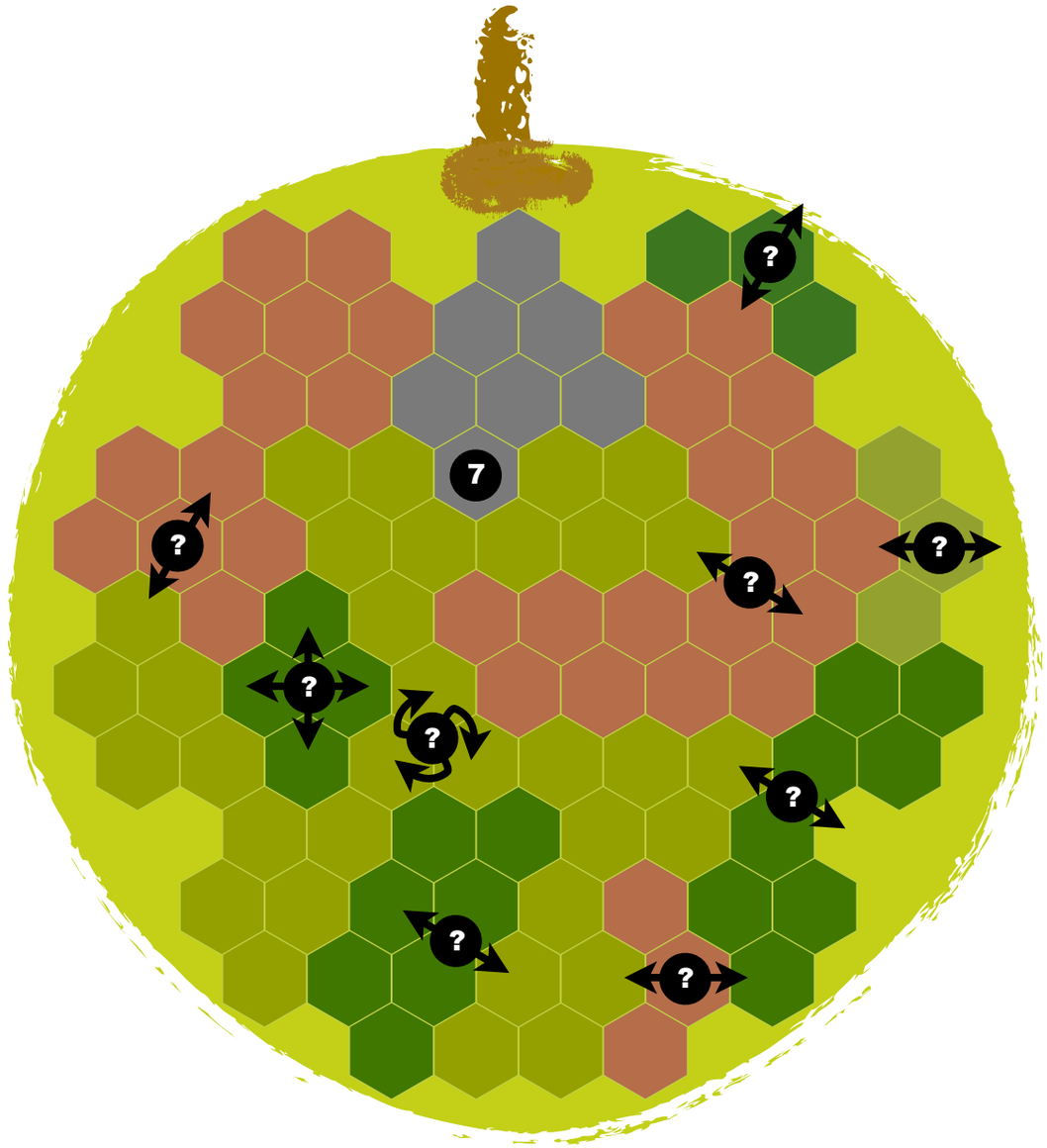
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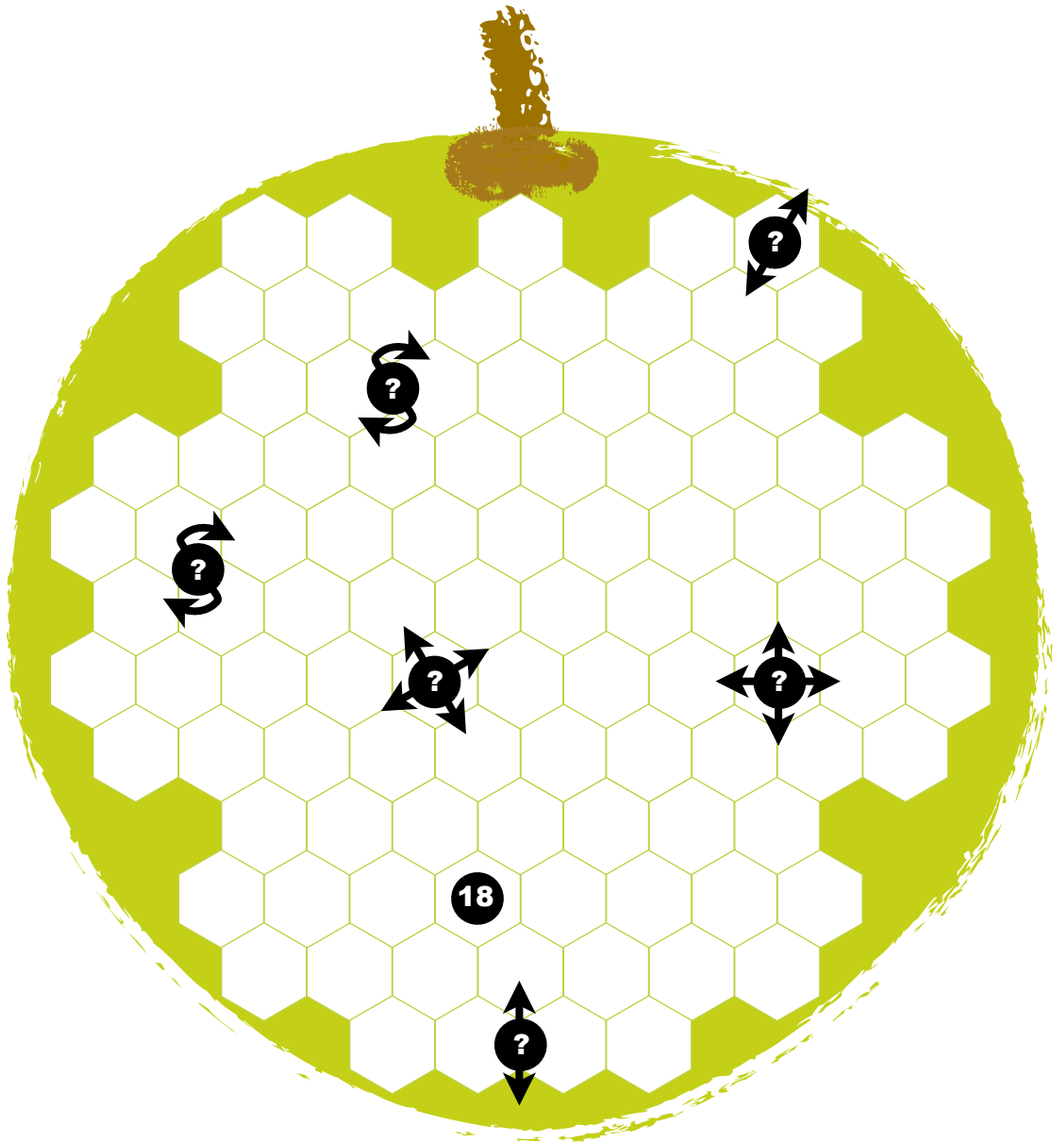
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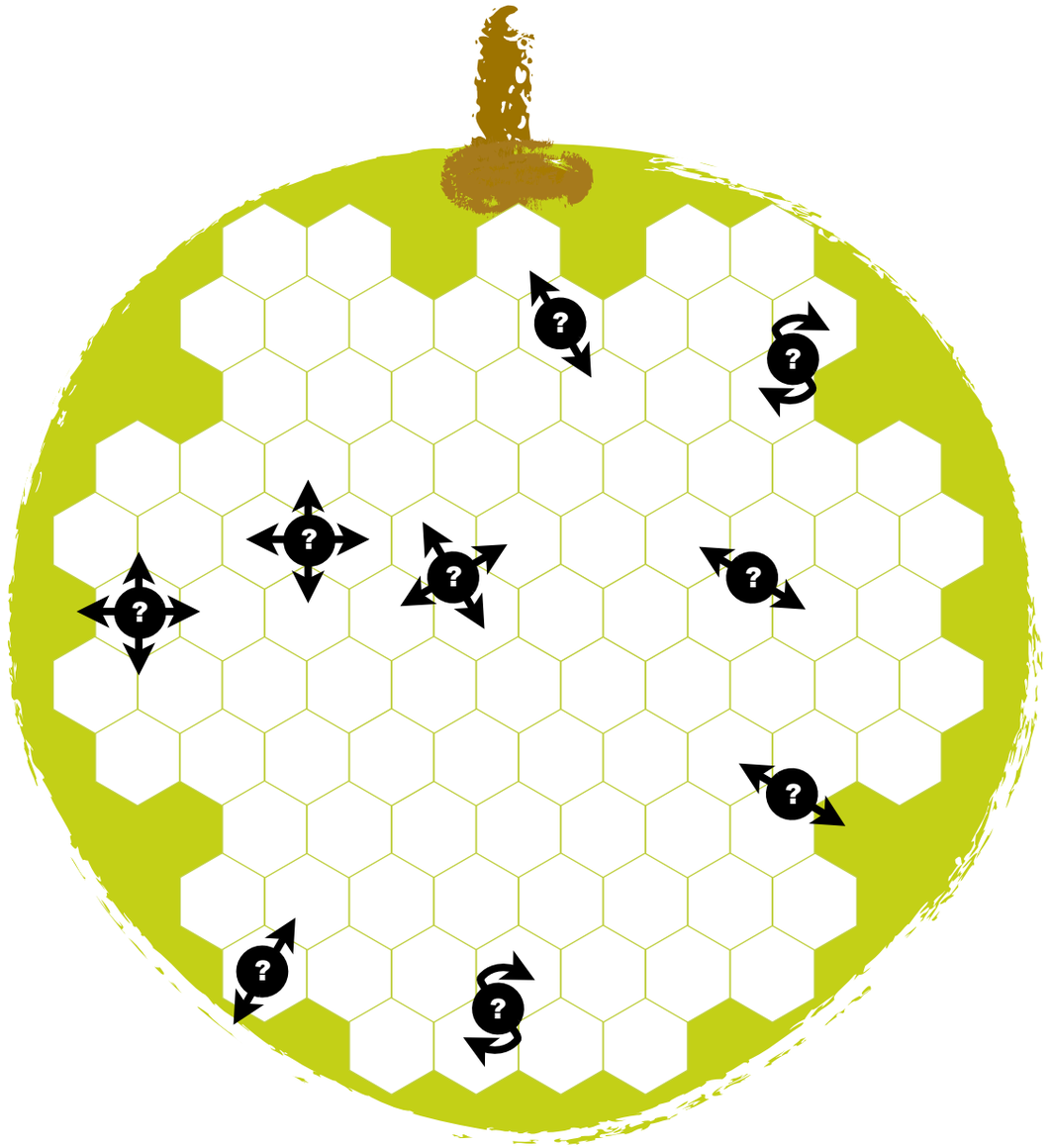
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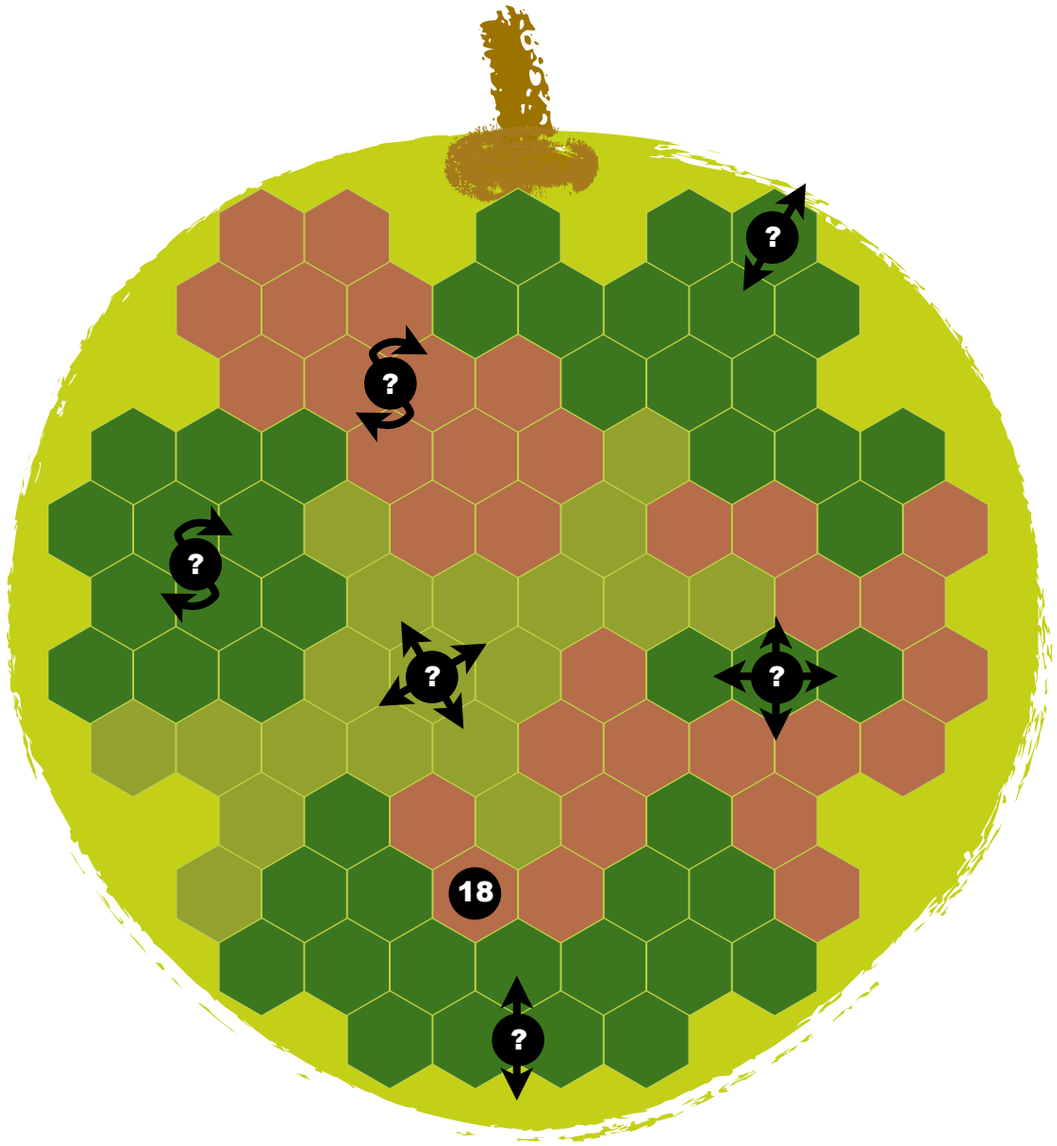
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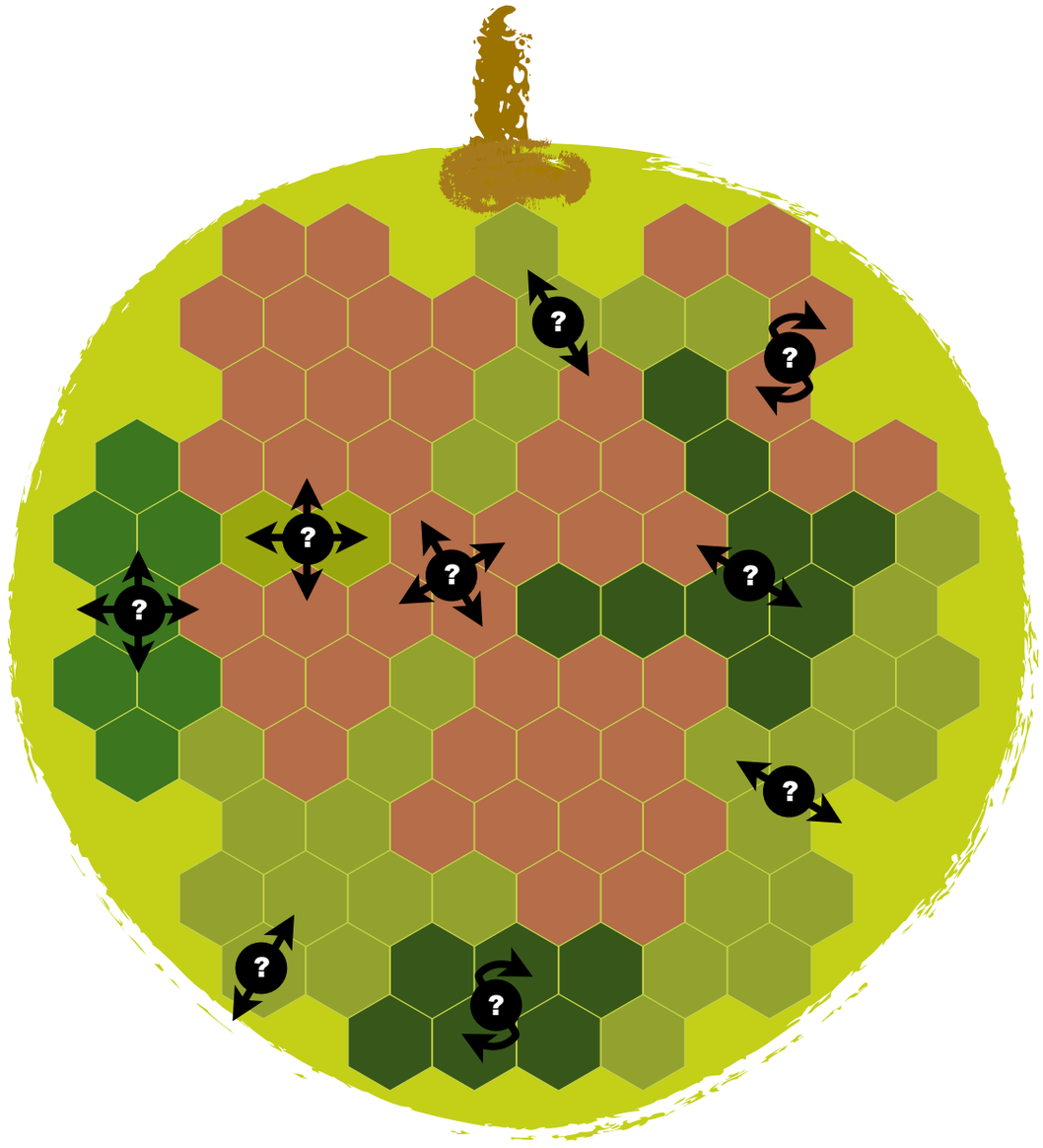
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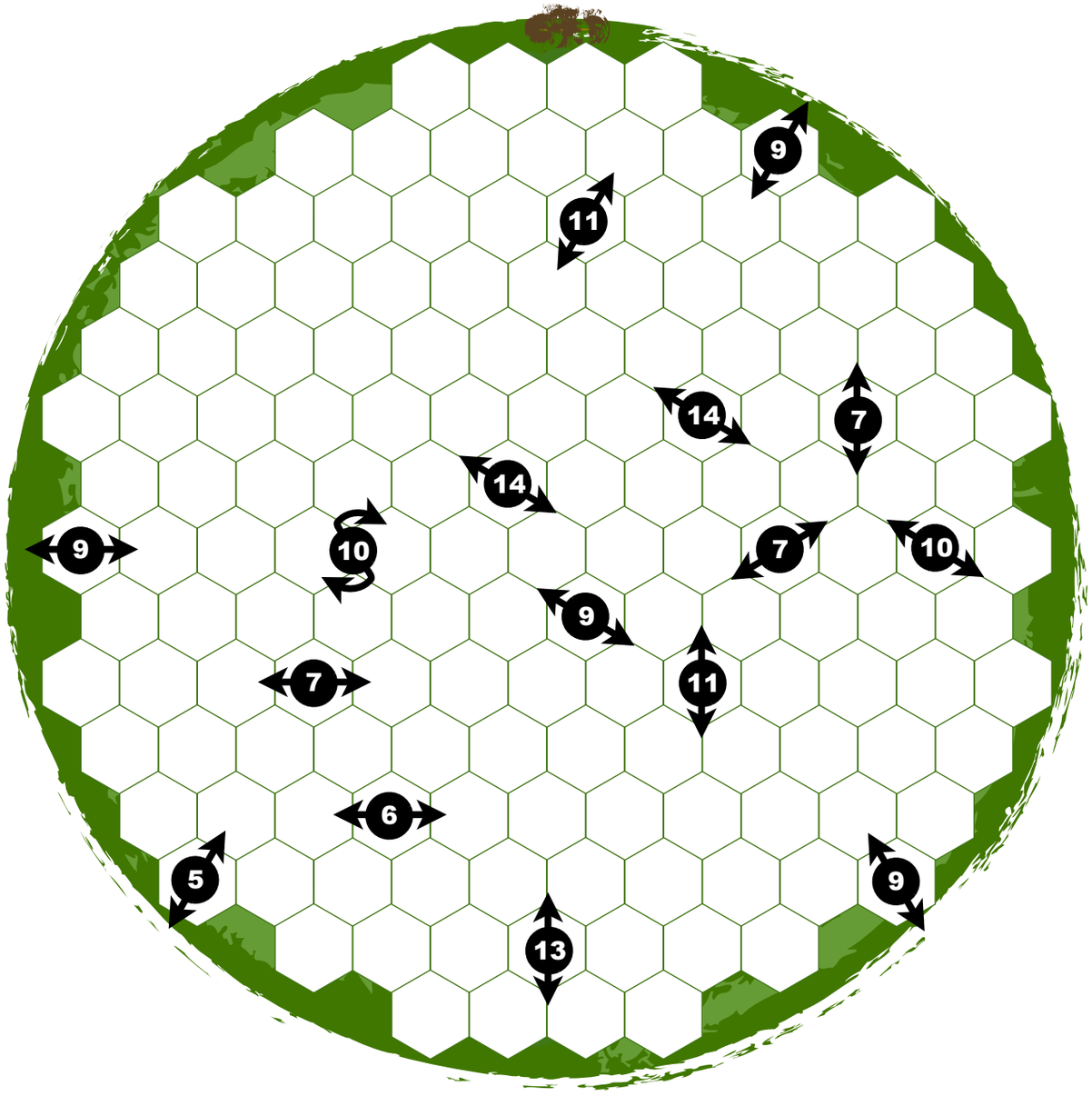
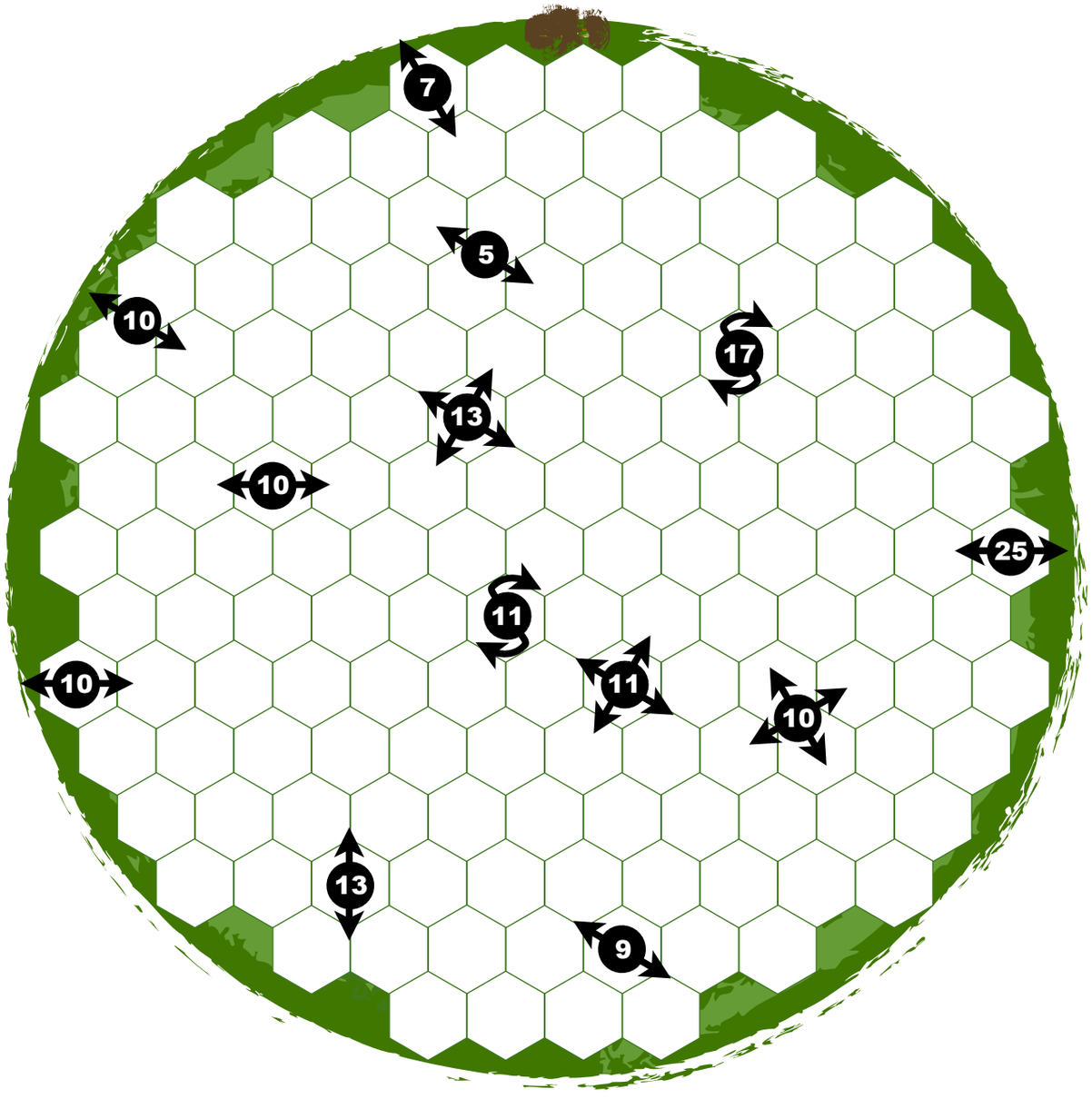
Melon is a luxury food in Japan.
Eat it and live in poverty.



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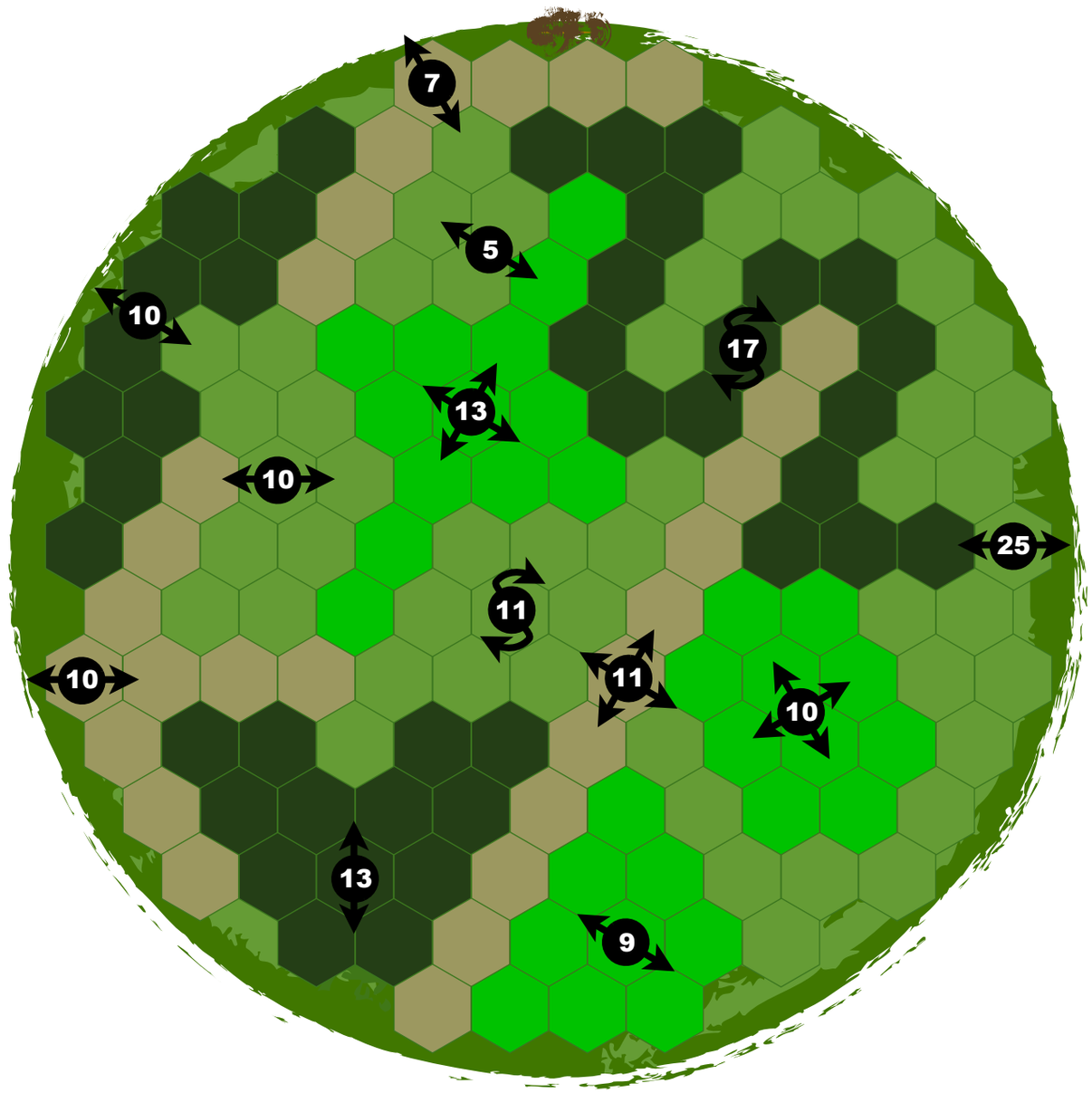
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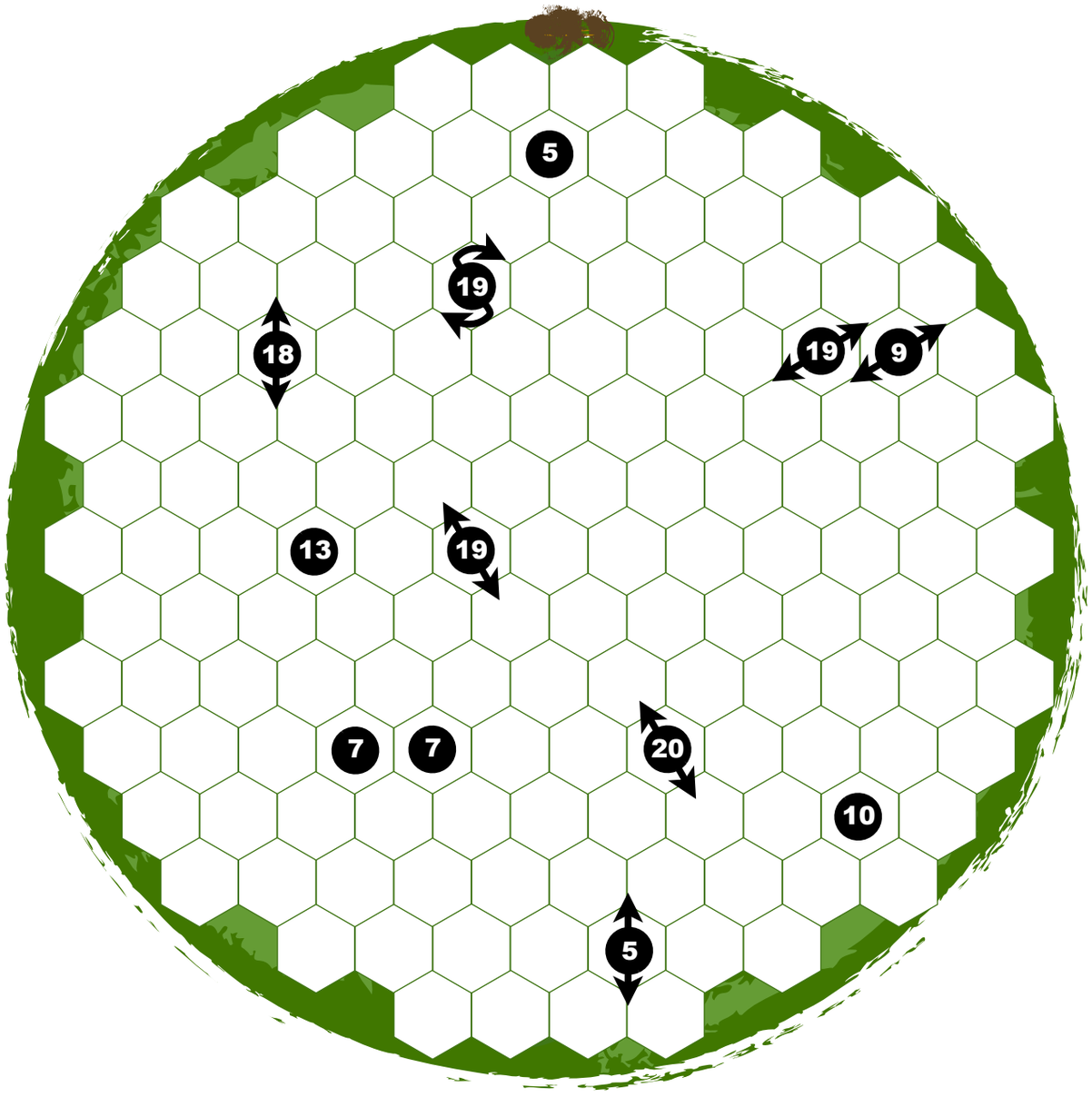
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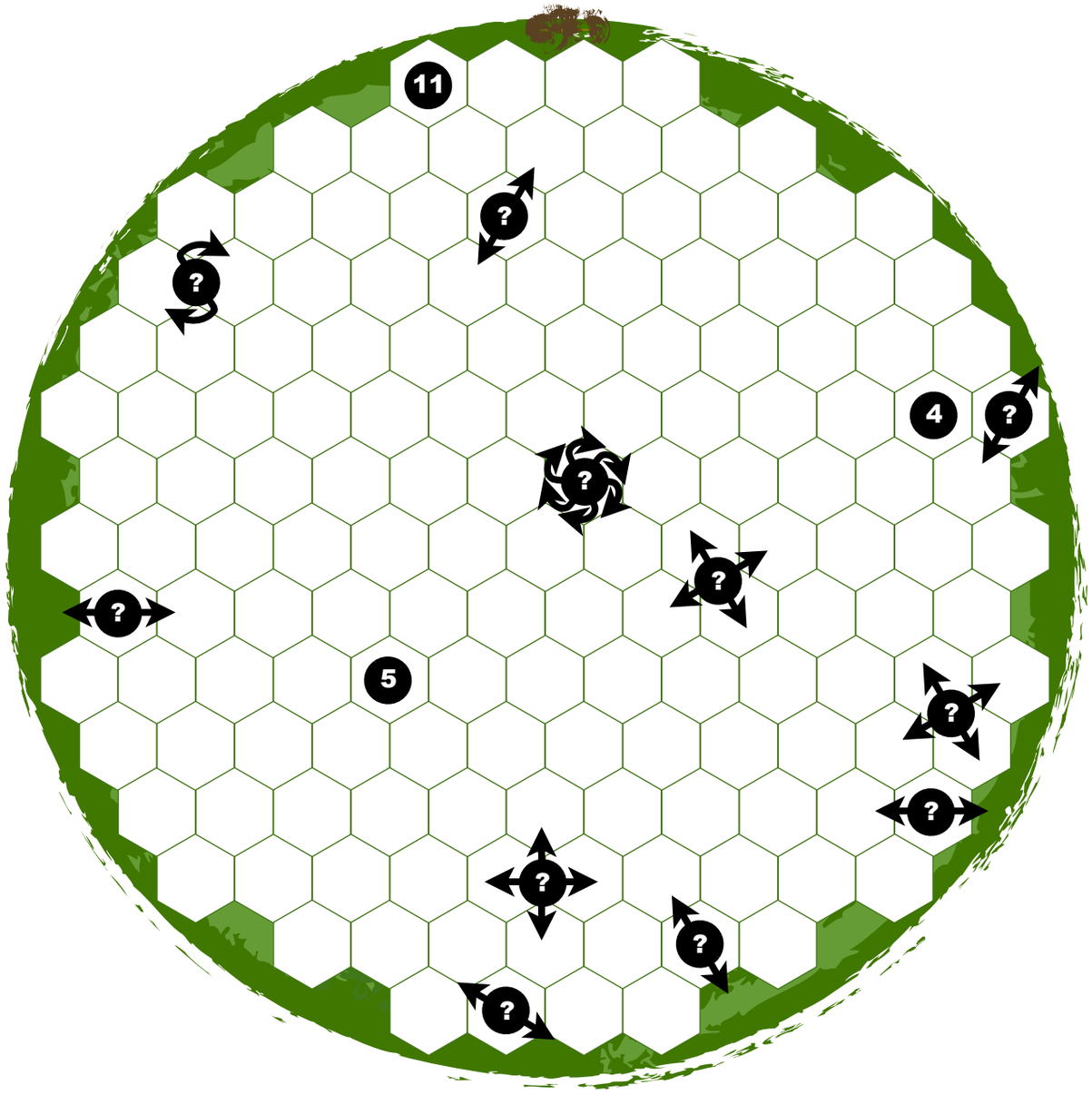
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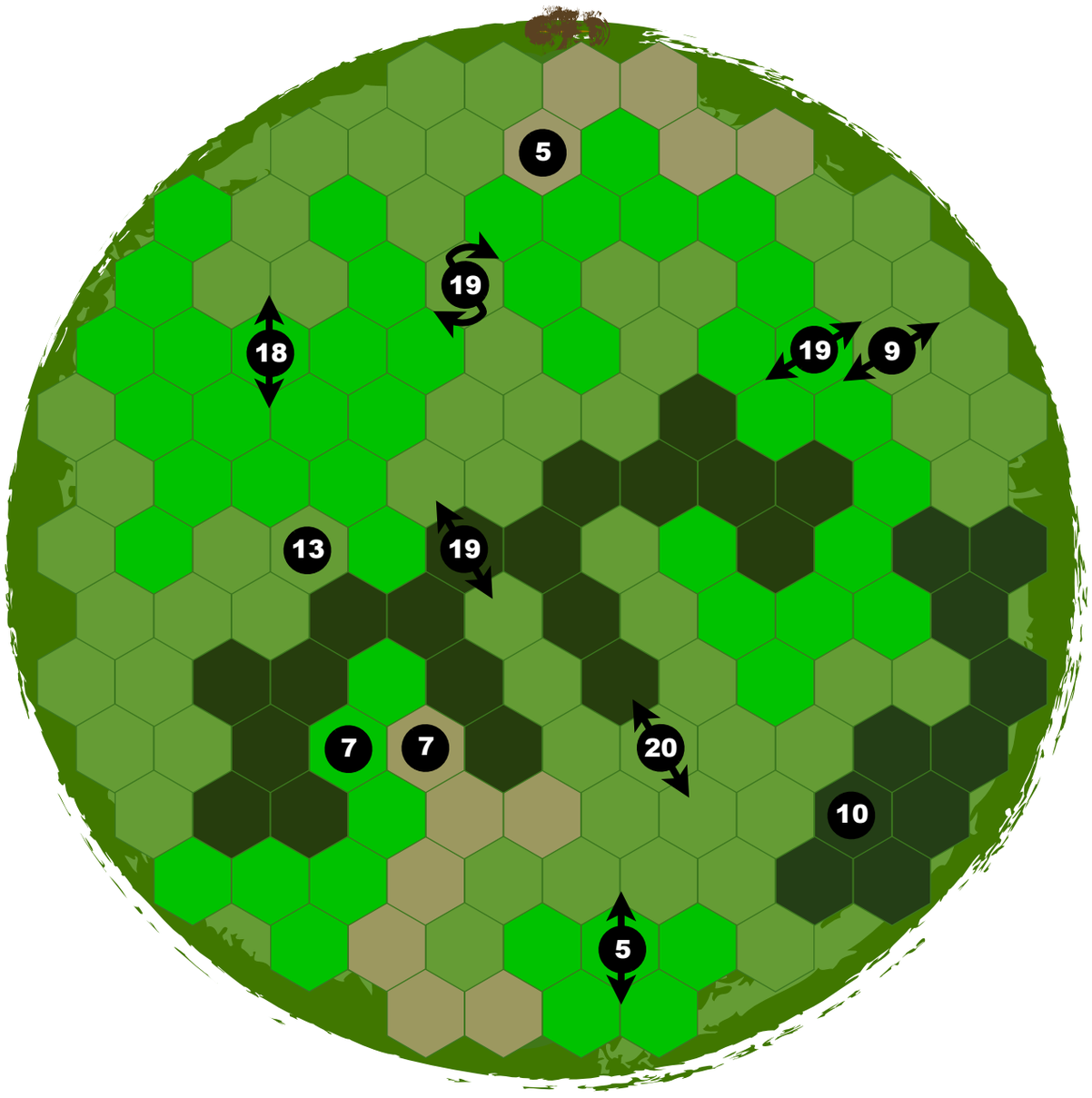
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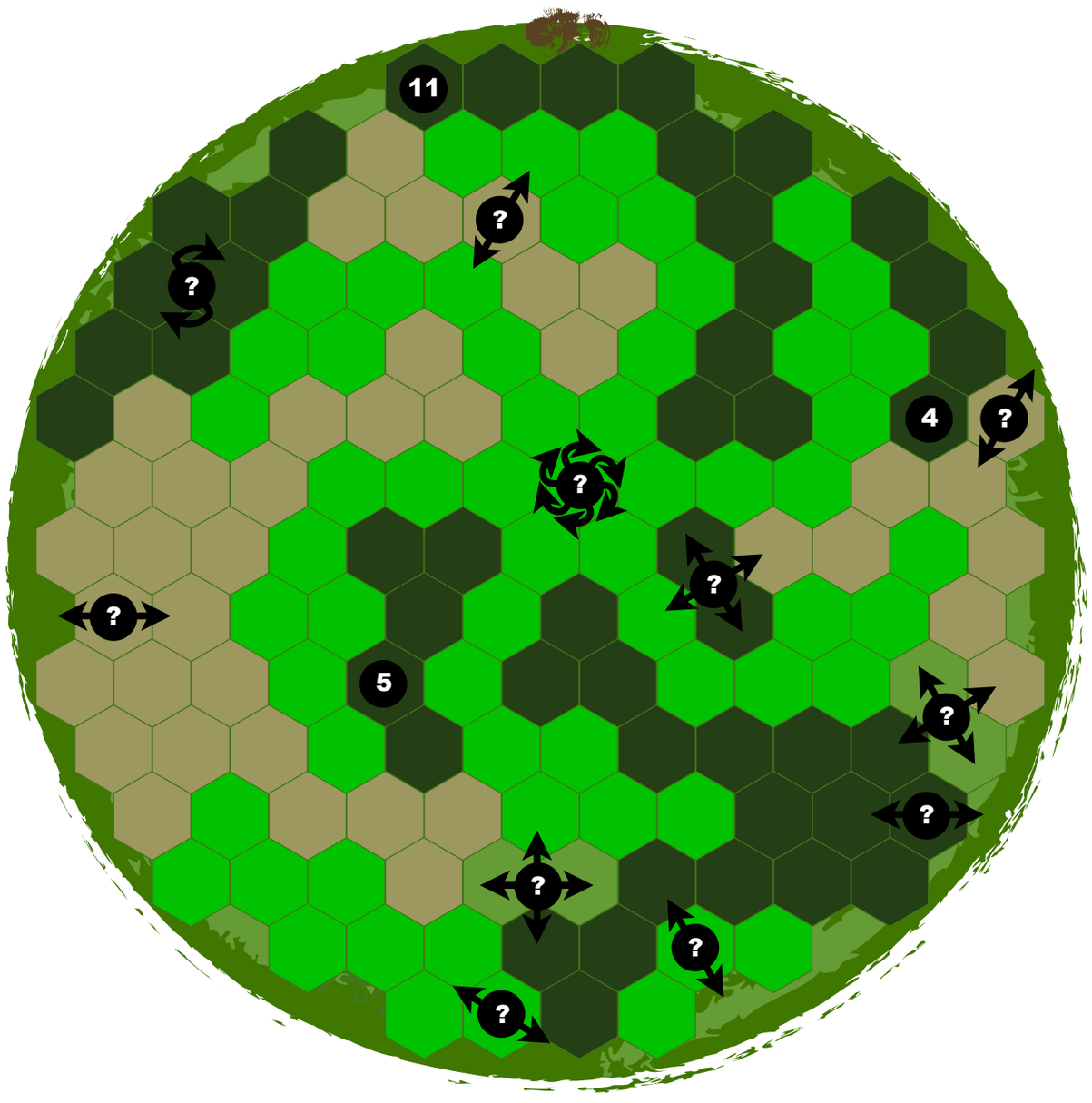
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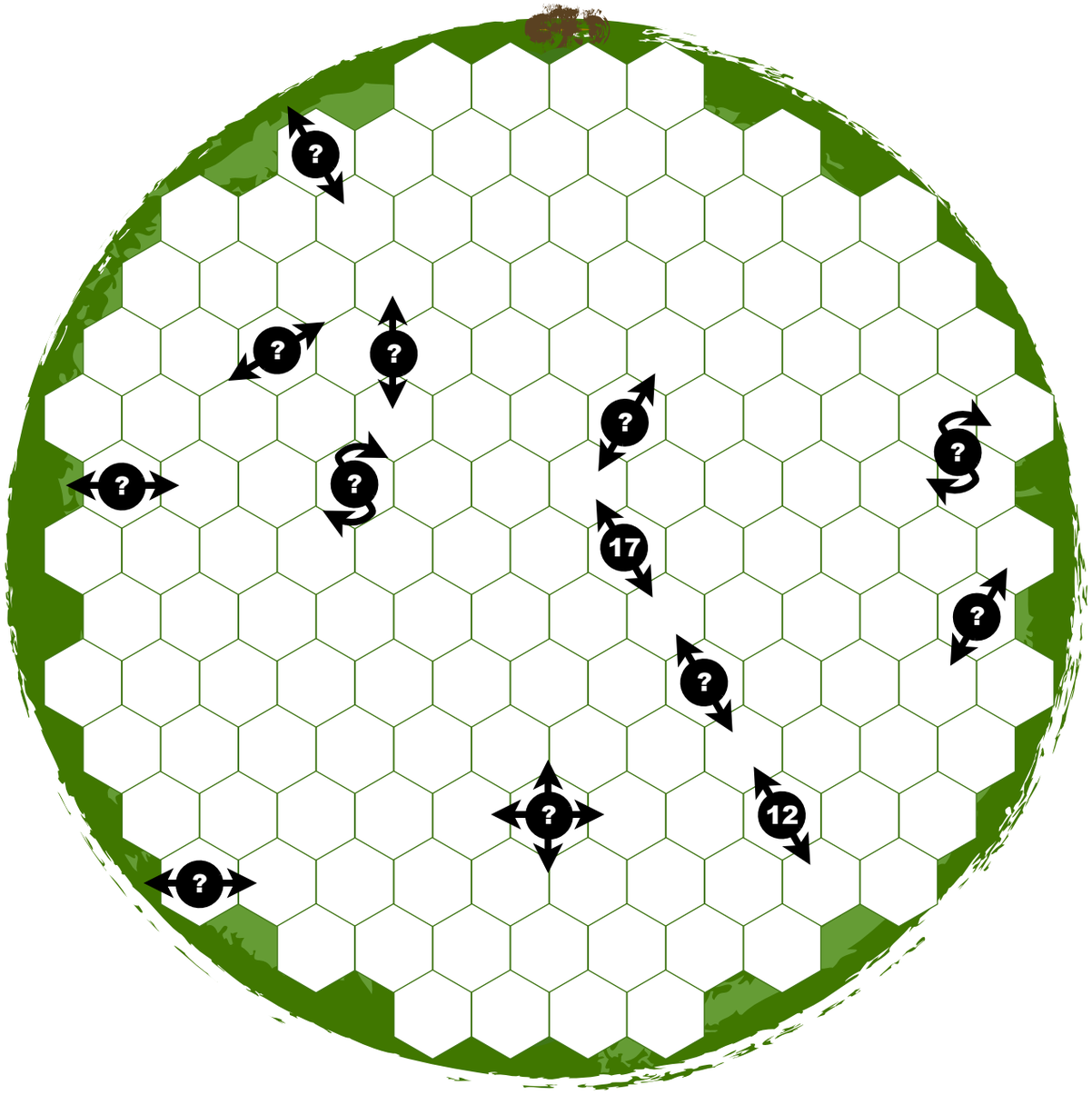
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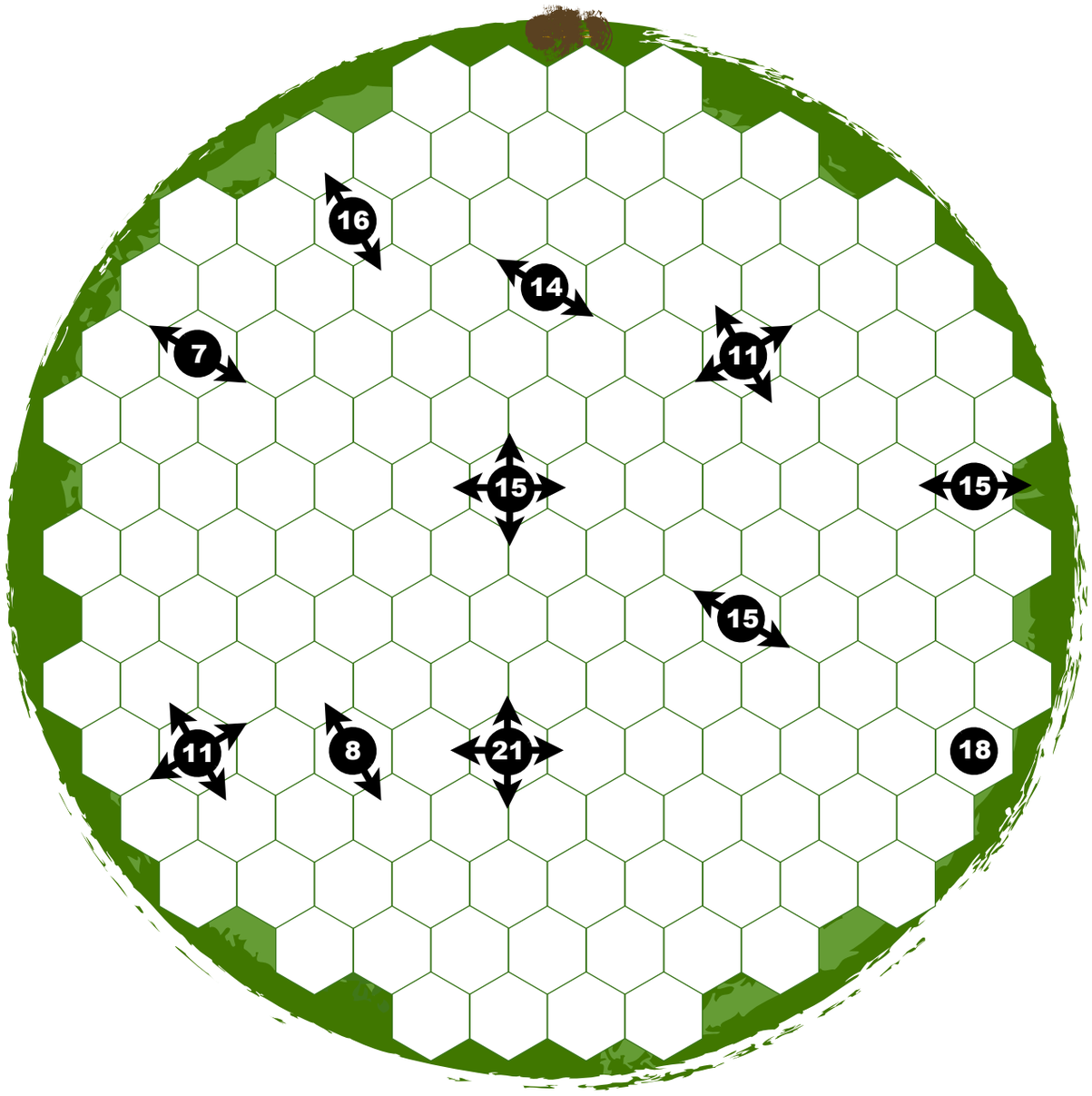
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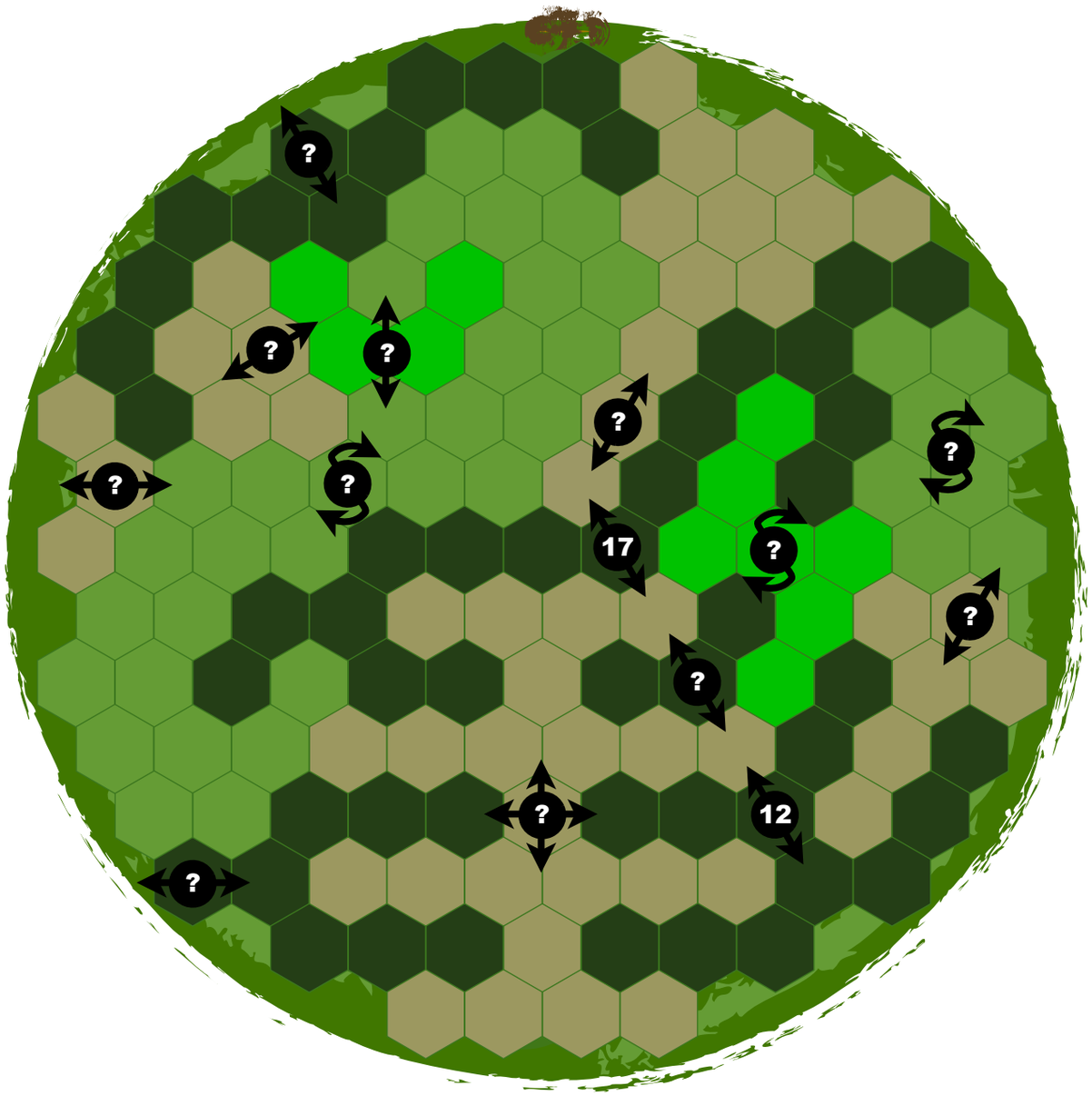
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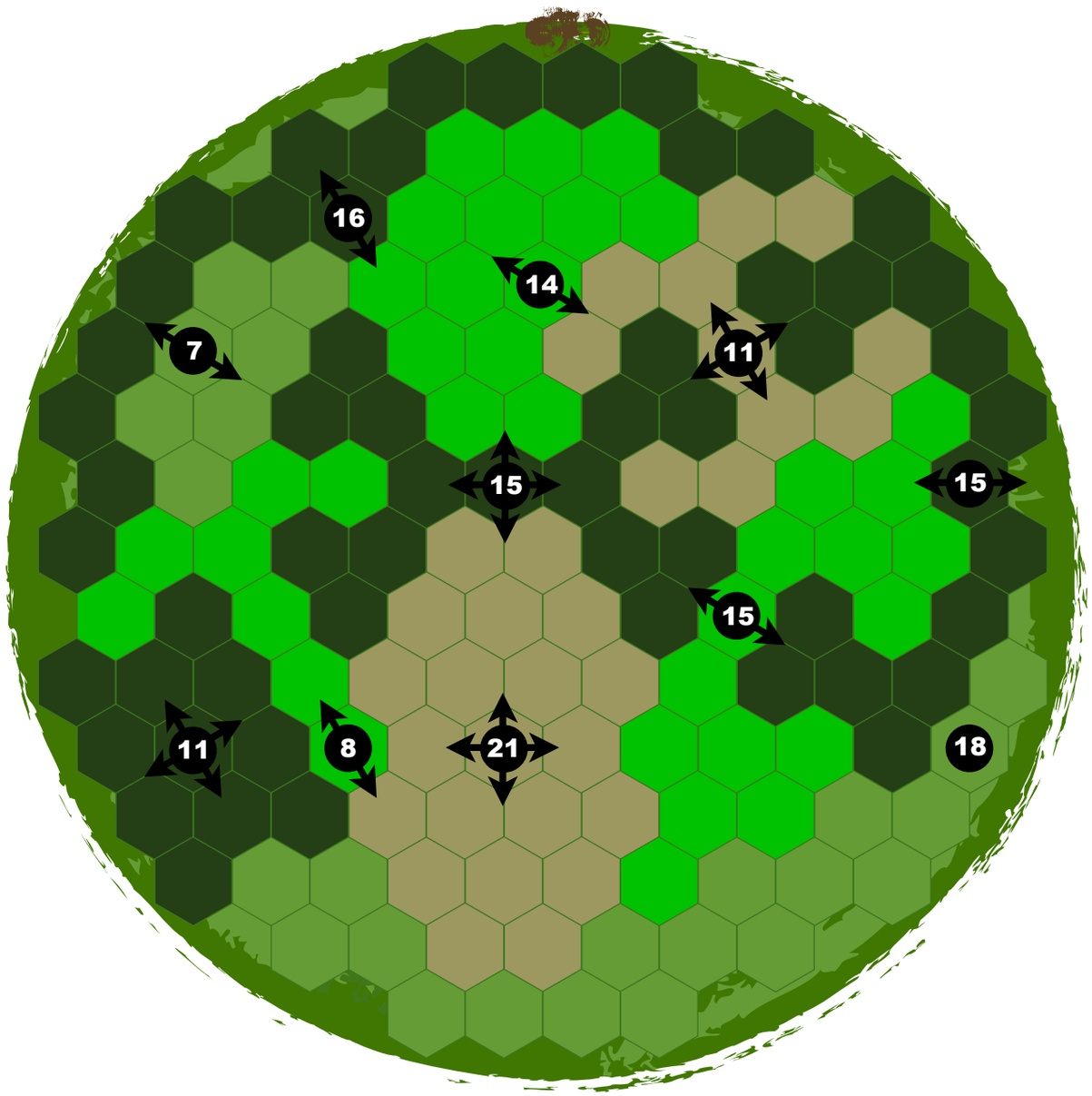
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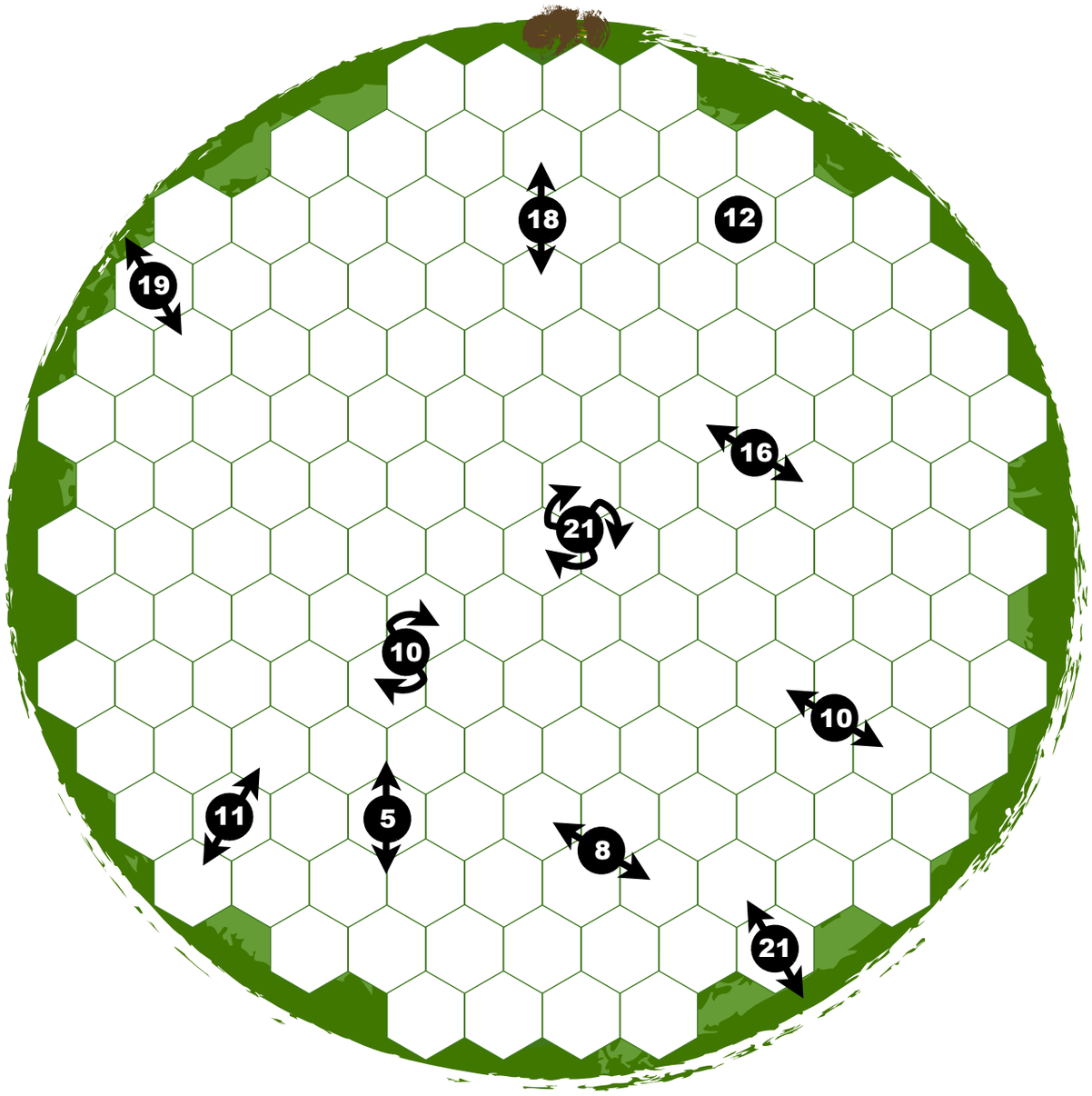
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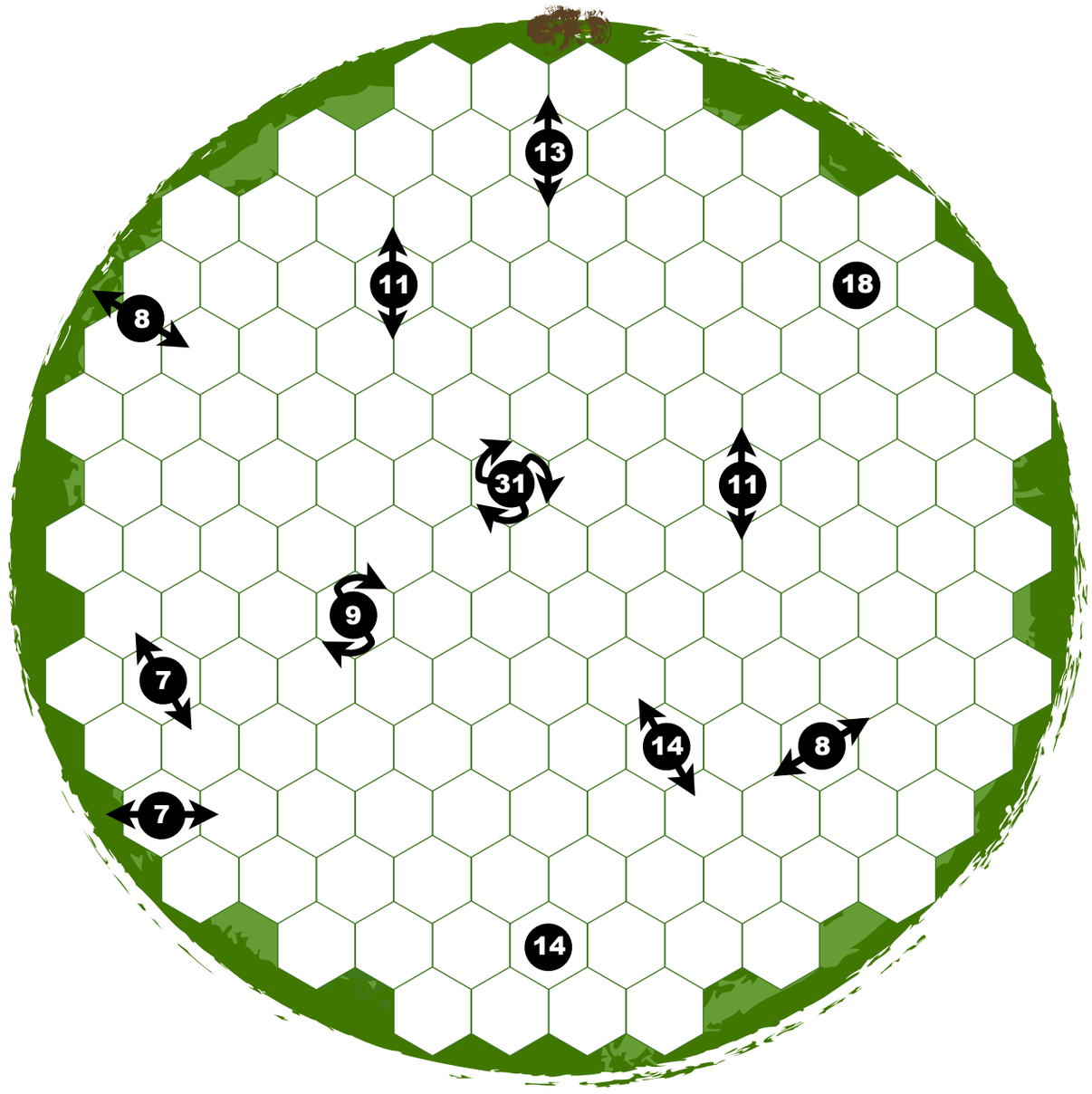
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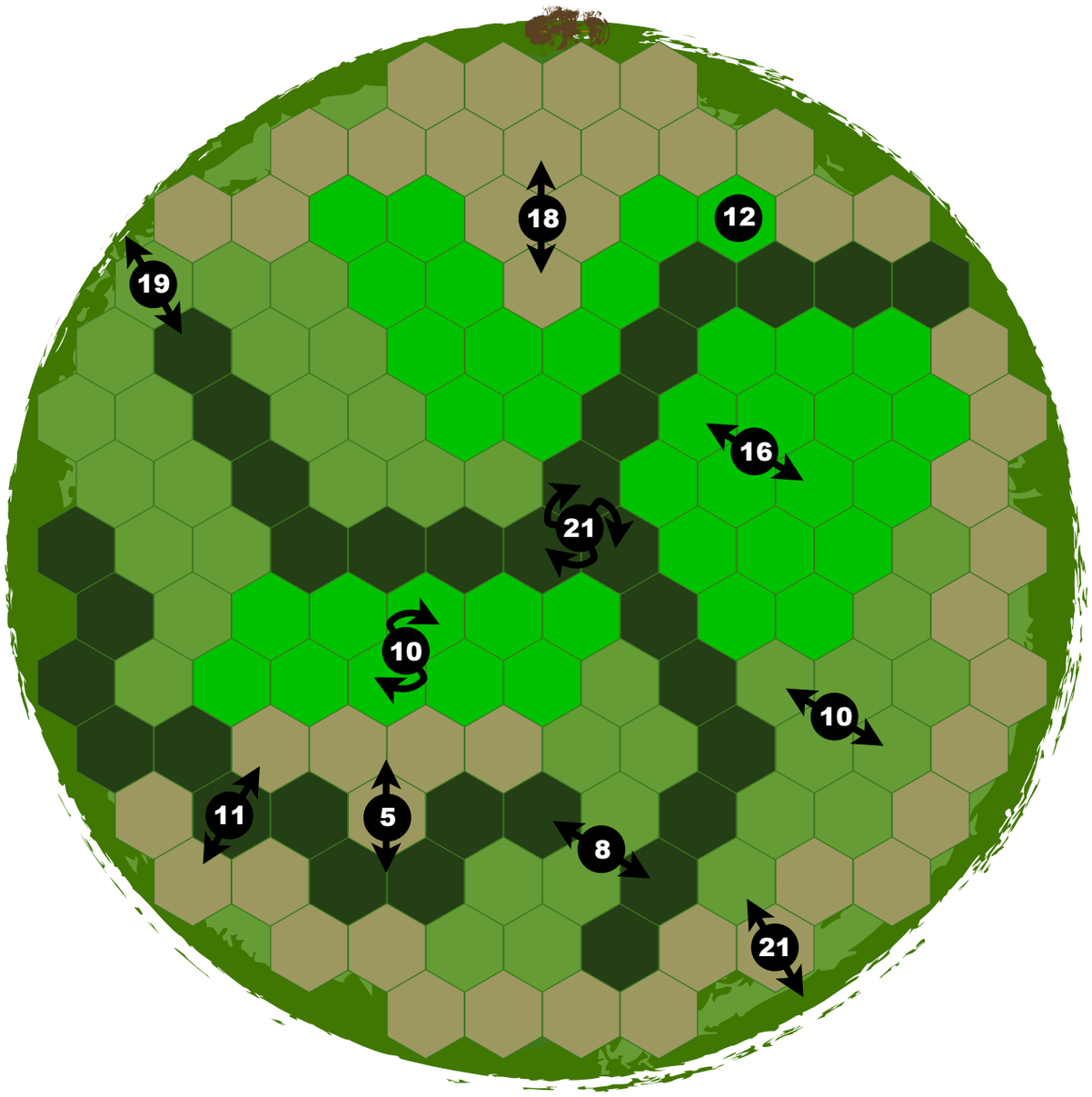
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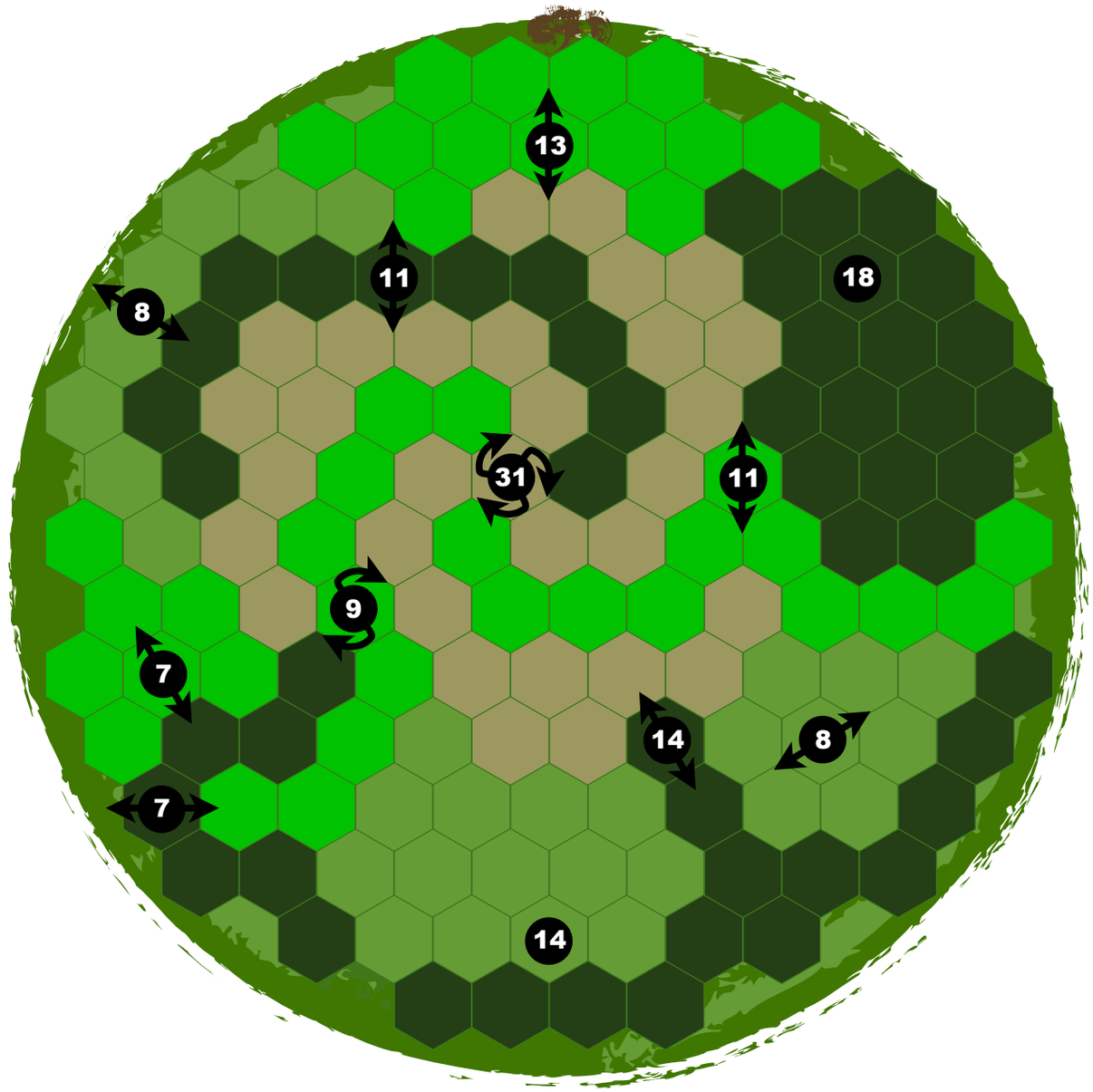
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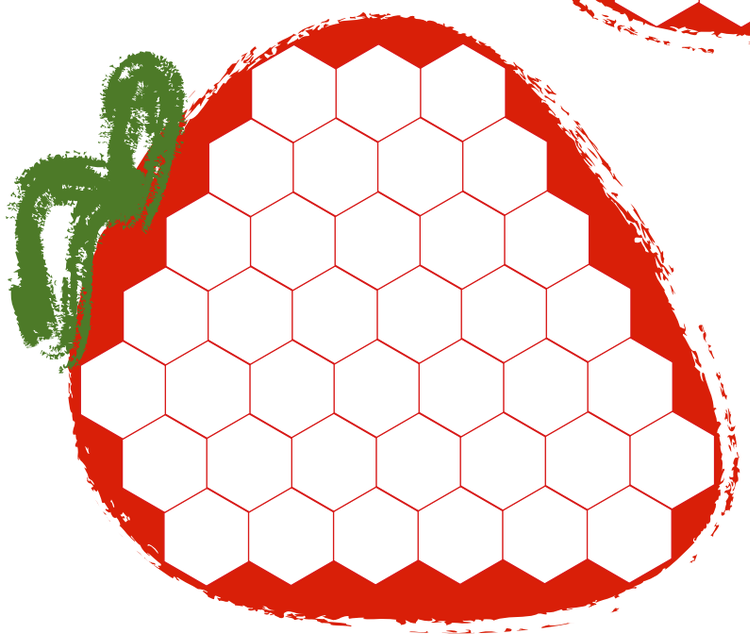

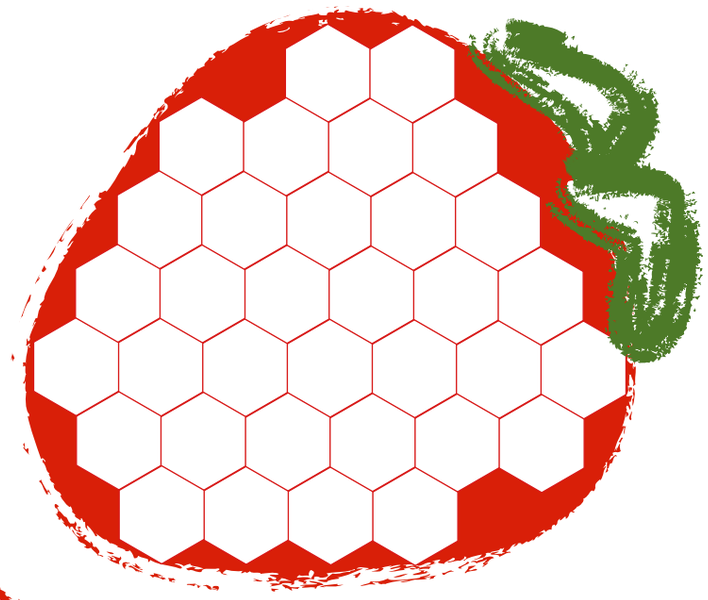

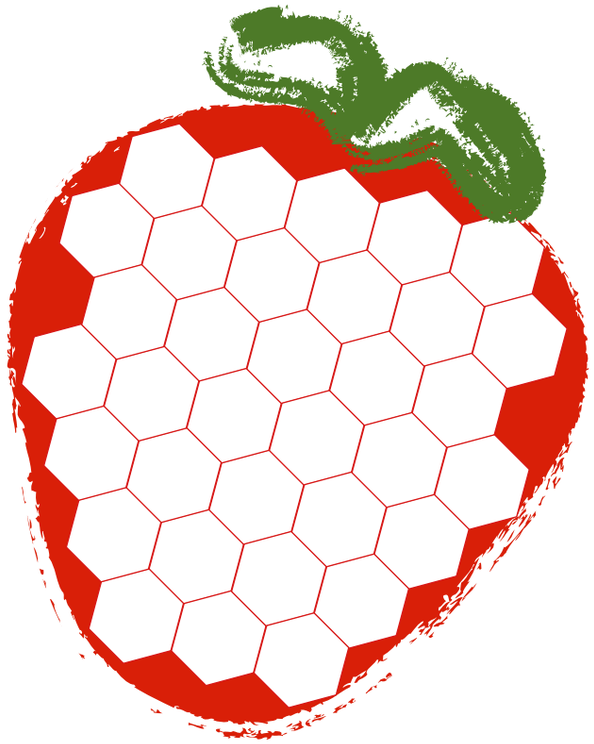

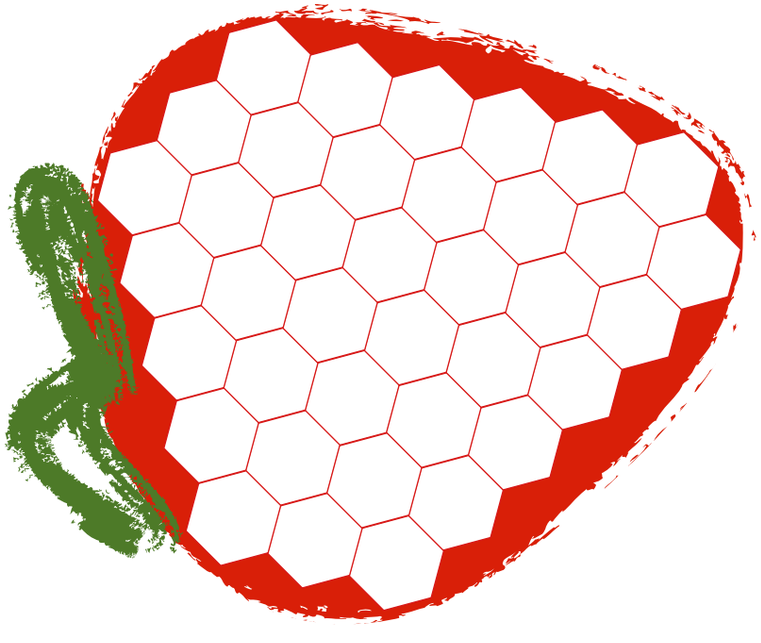

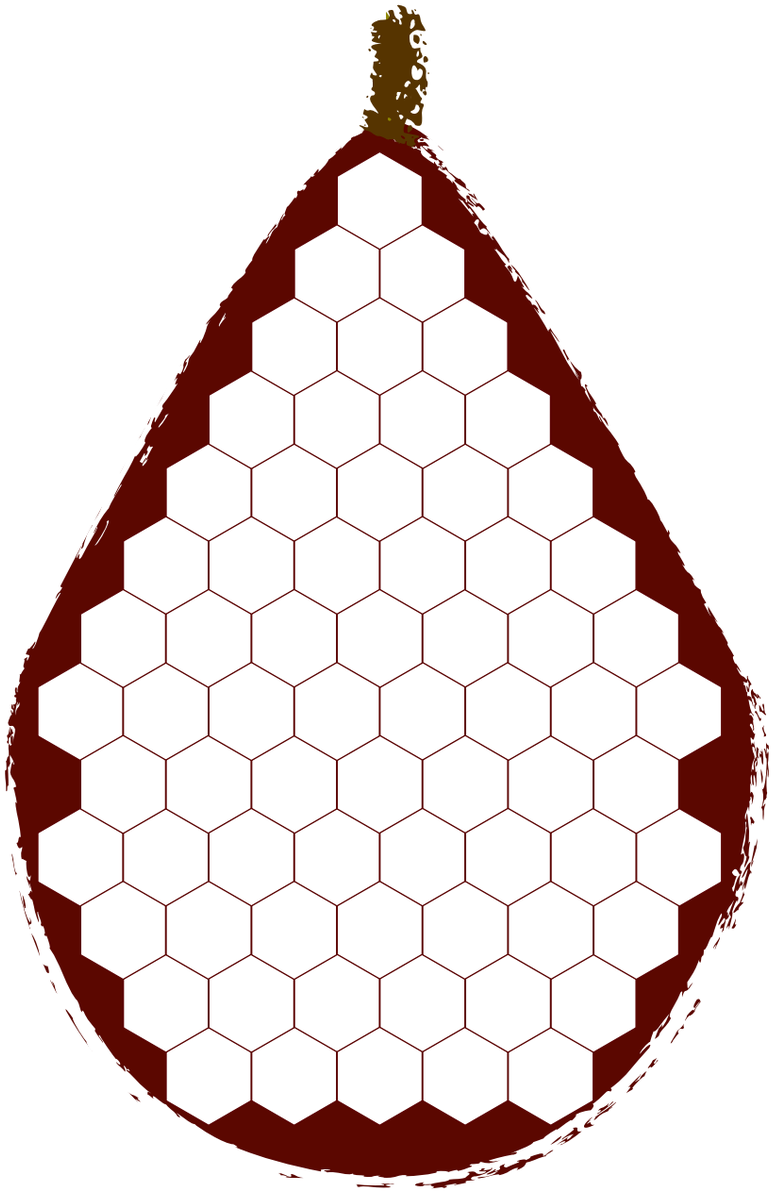
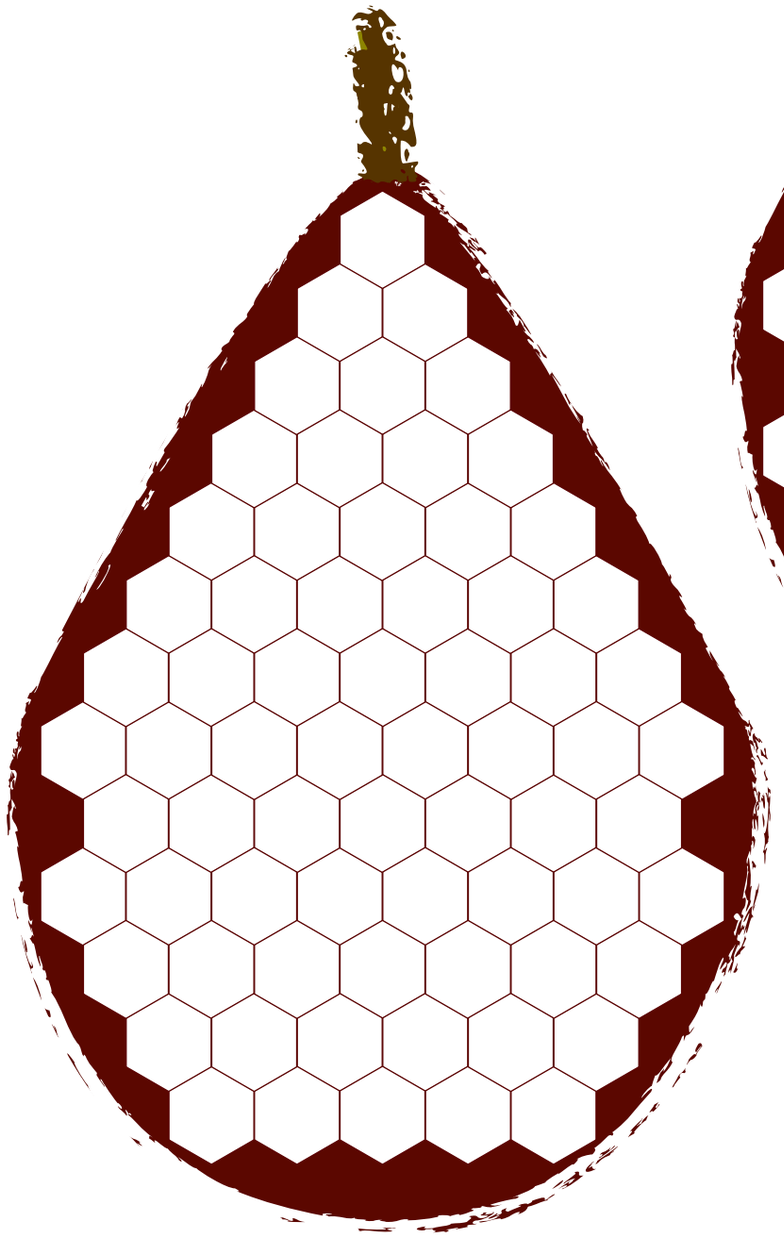
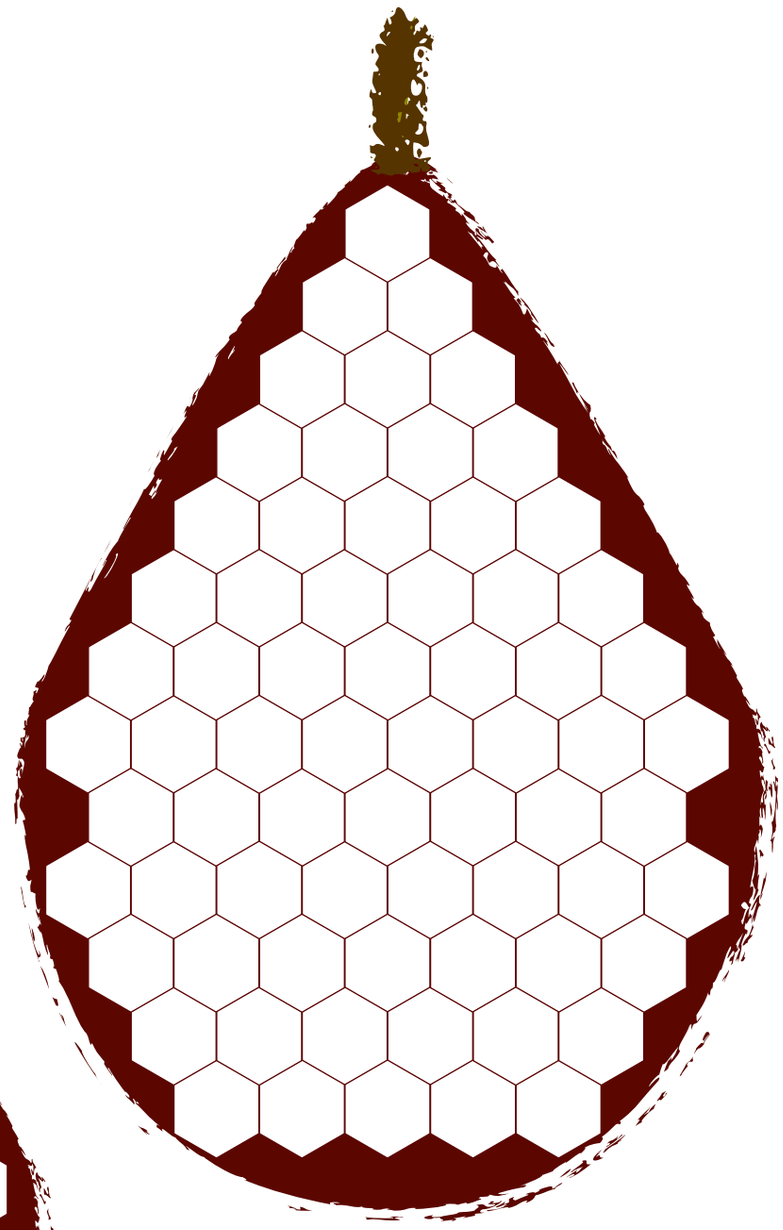

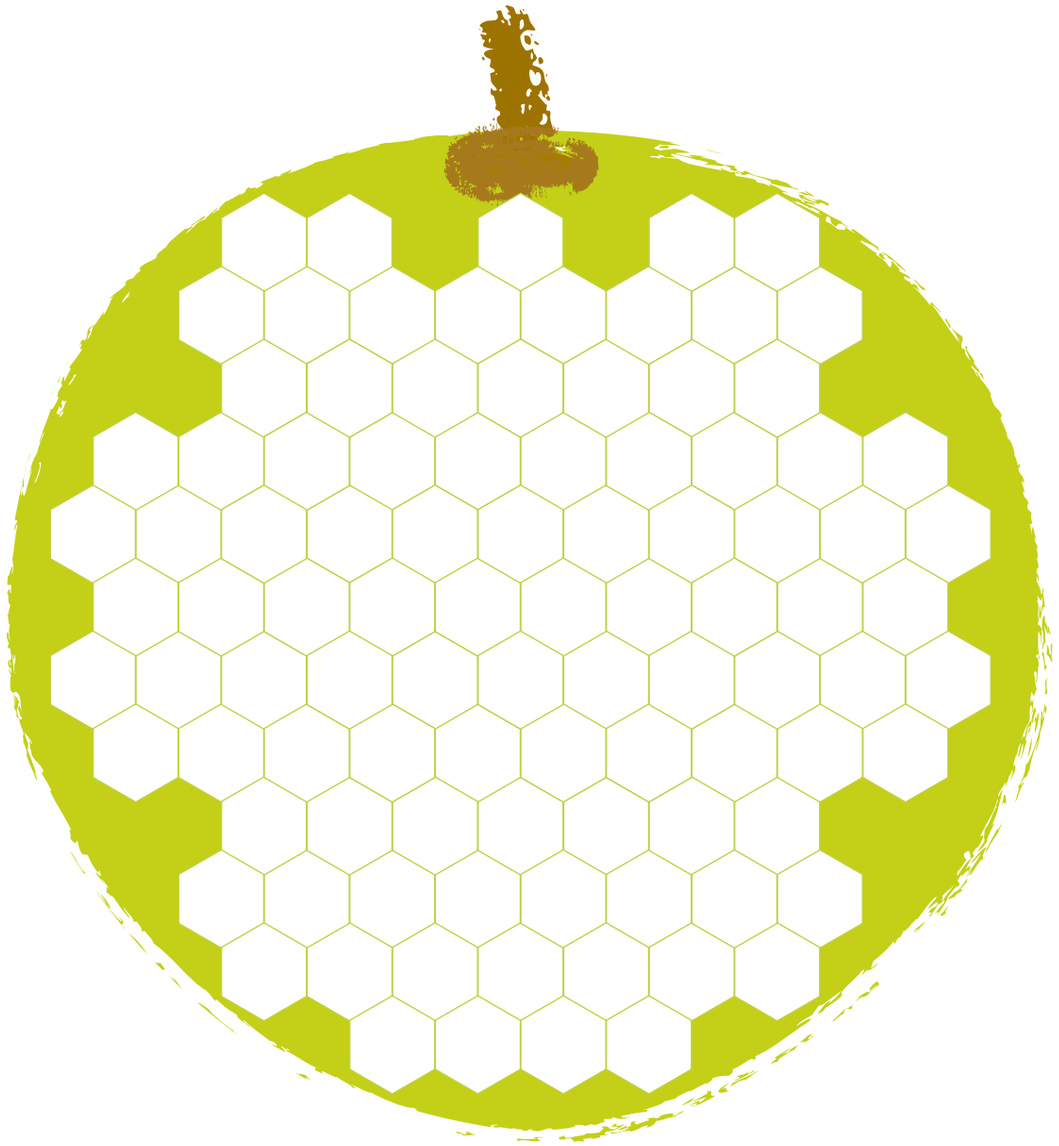

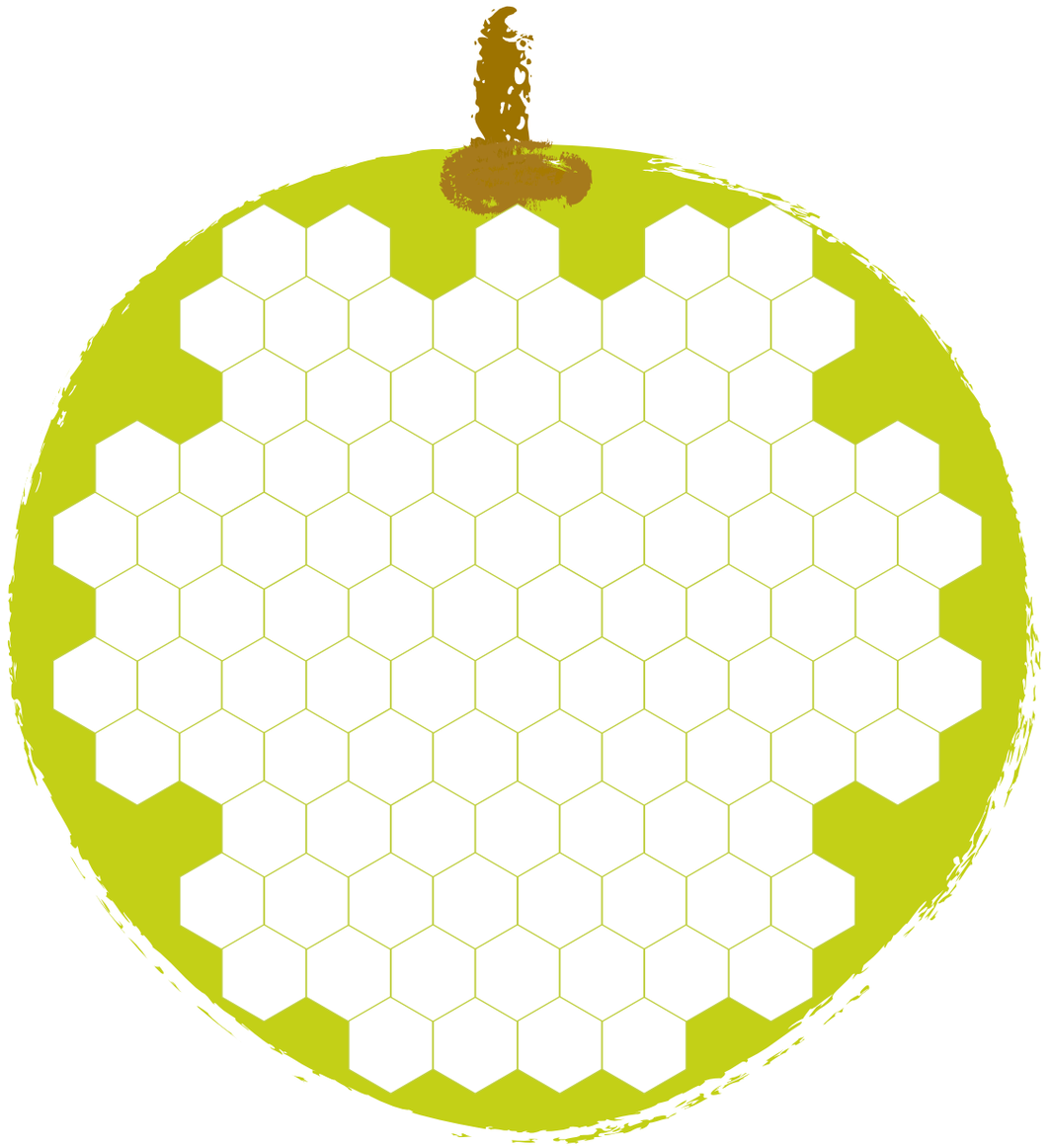

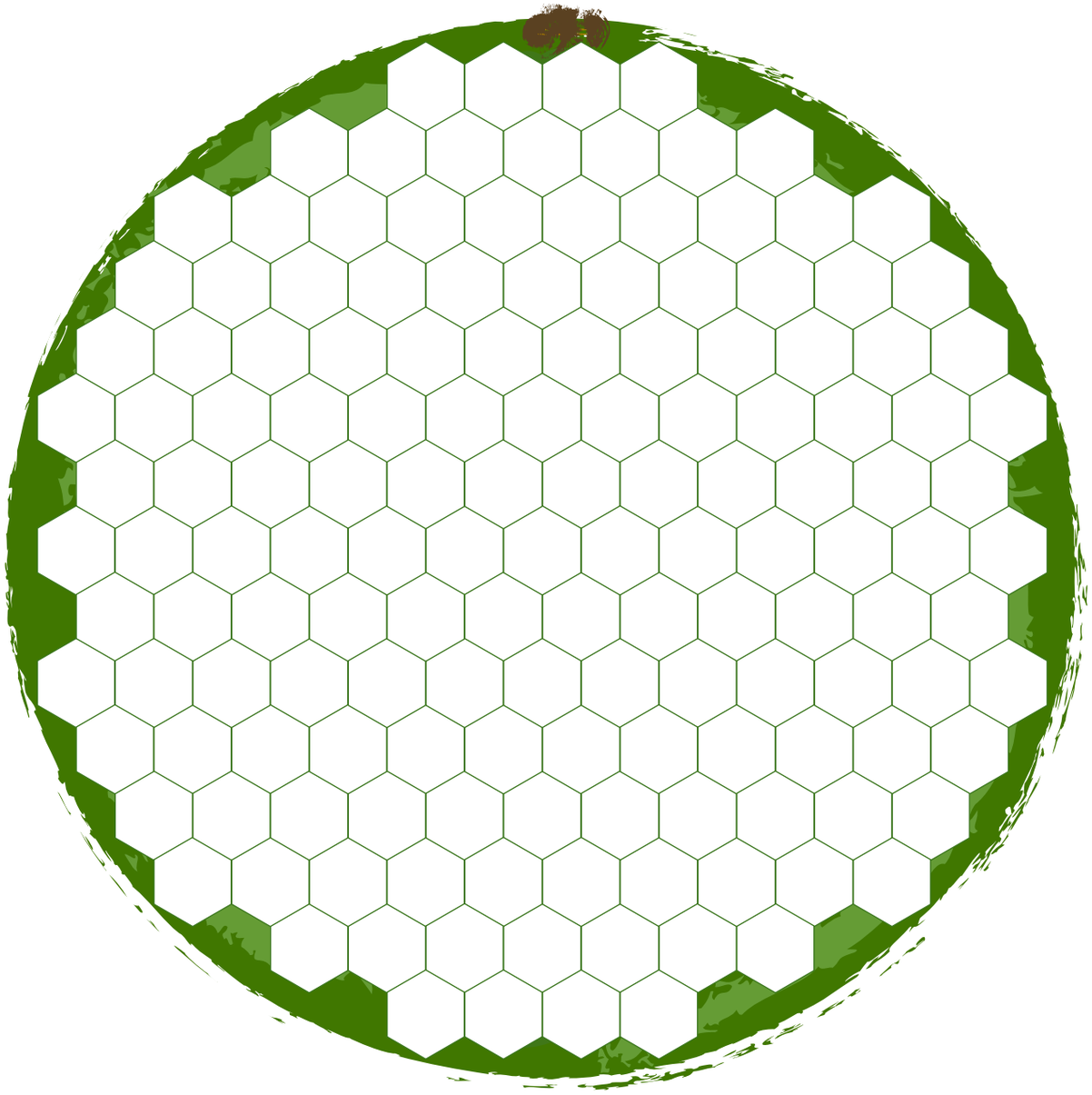
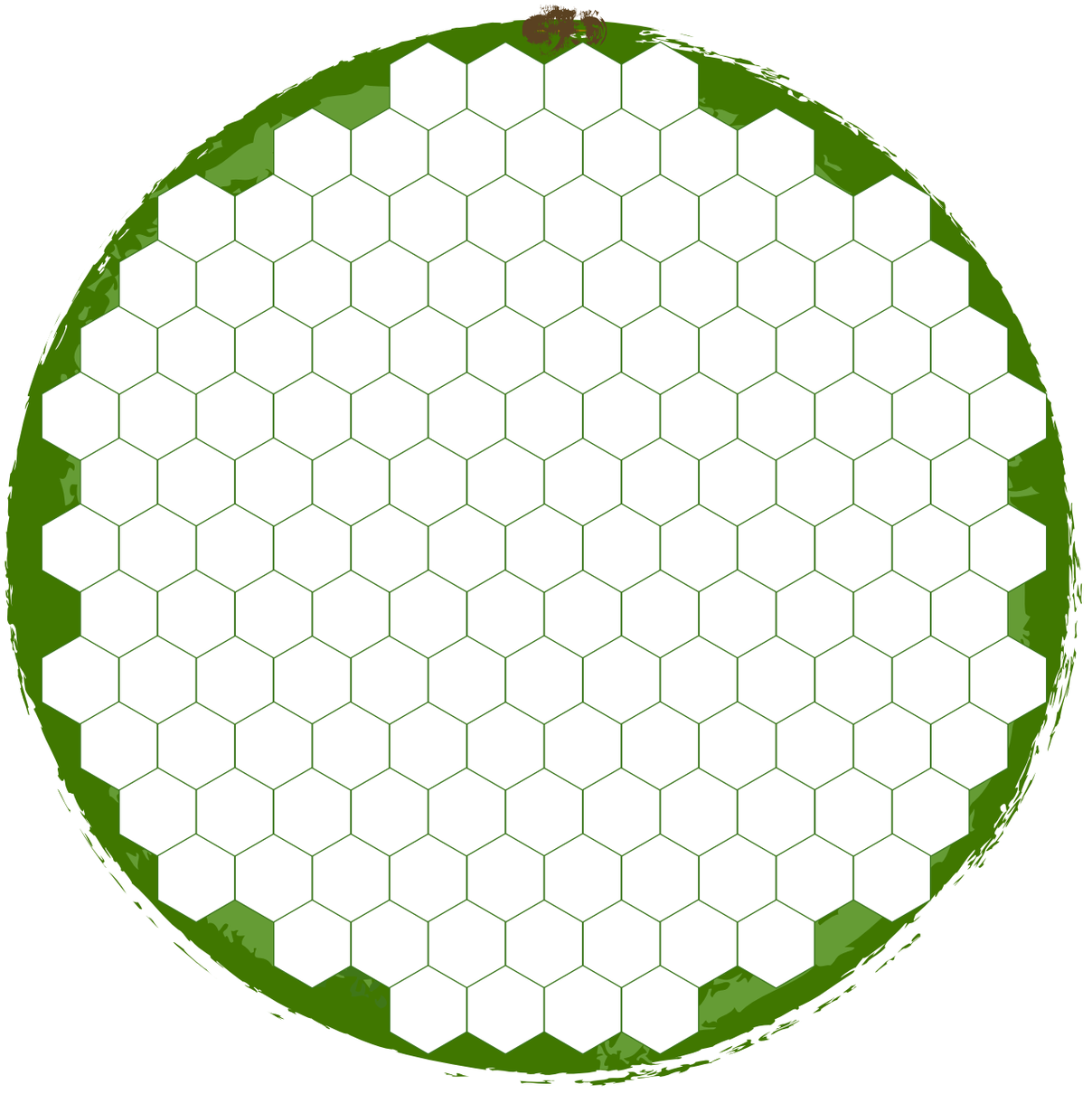

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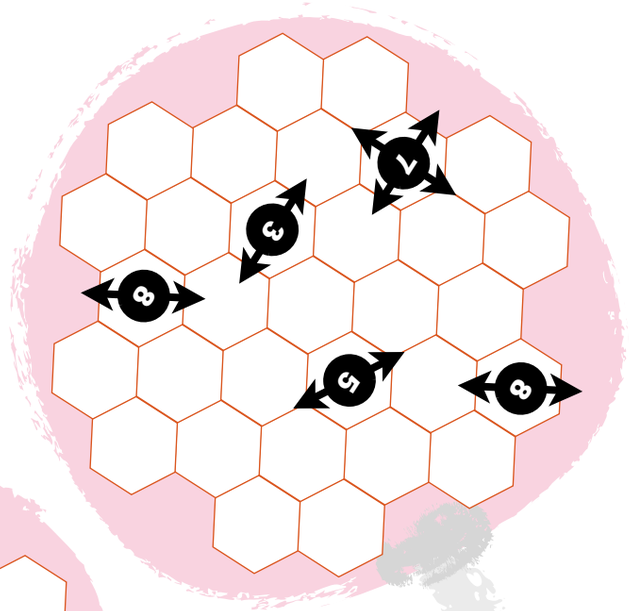

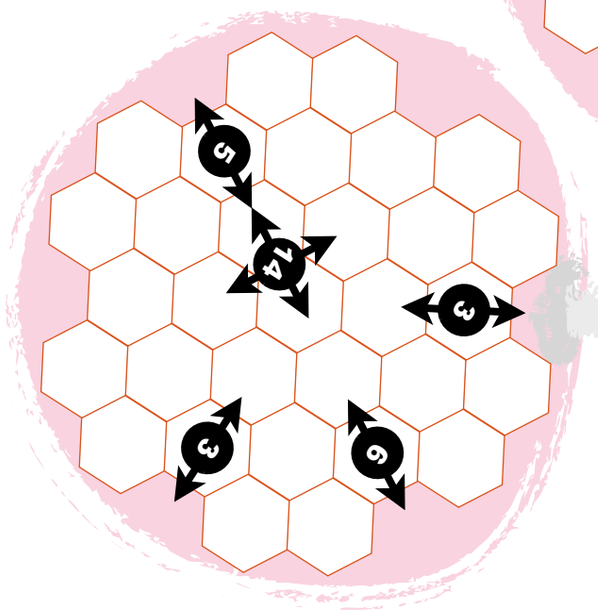

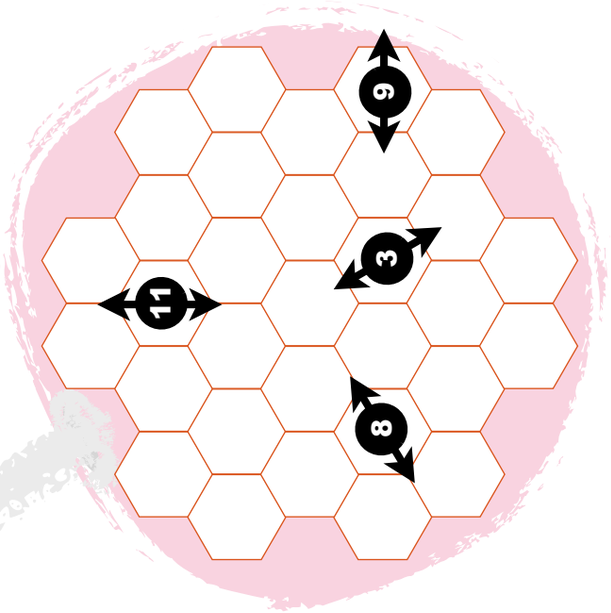

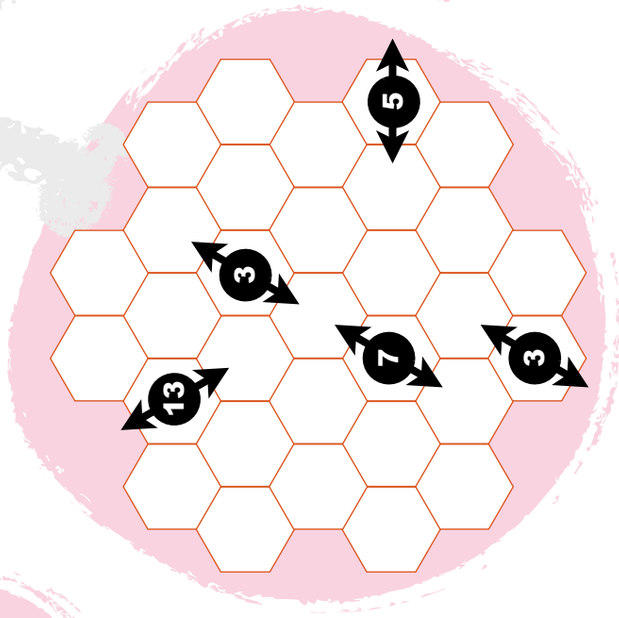

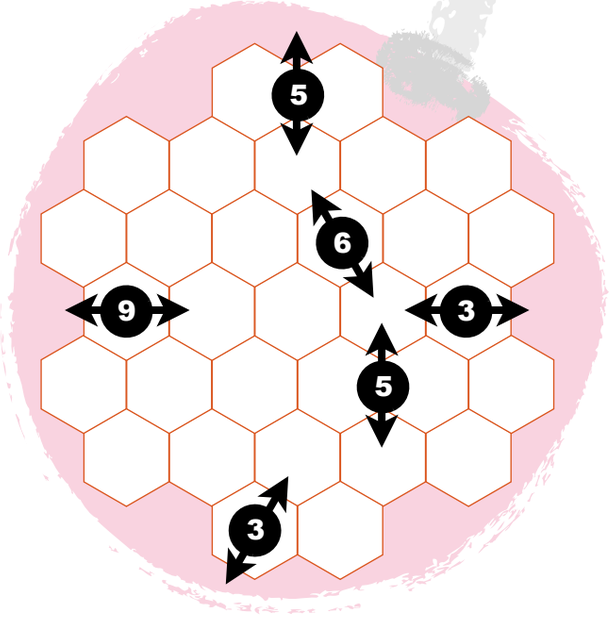

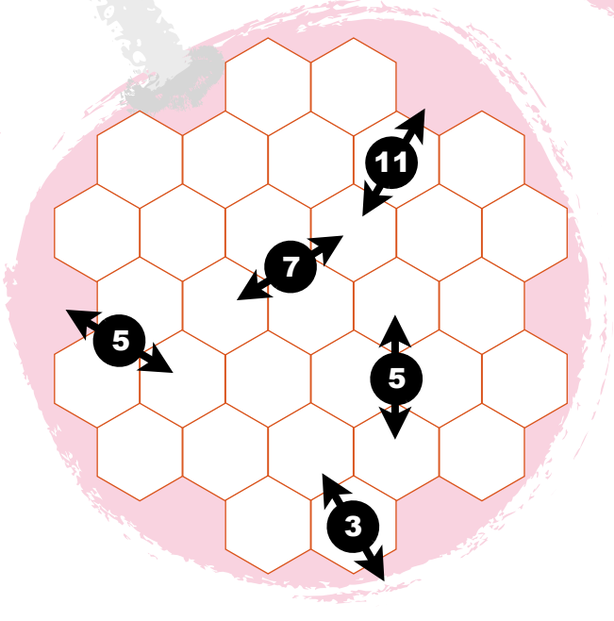

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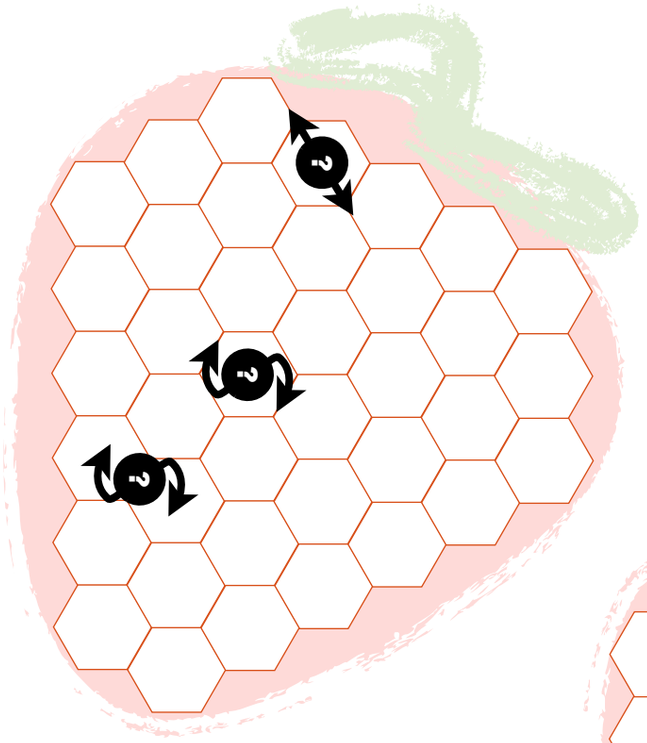

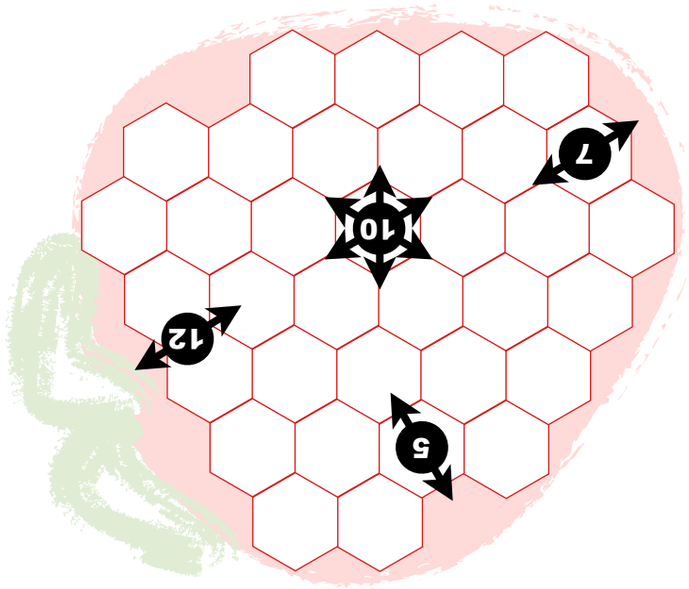

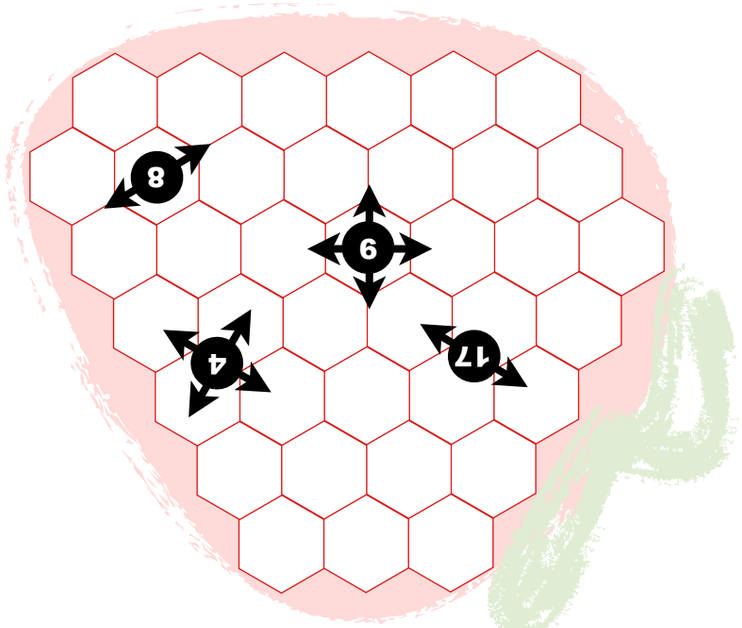

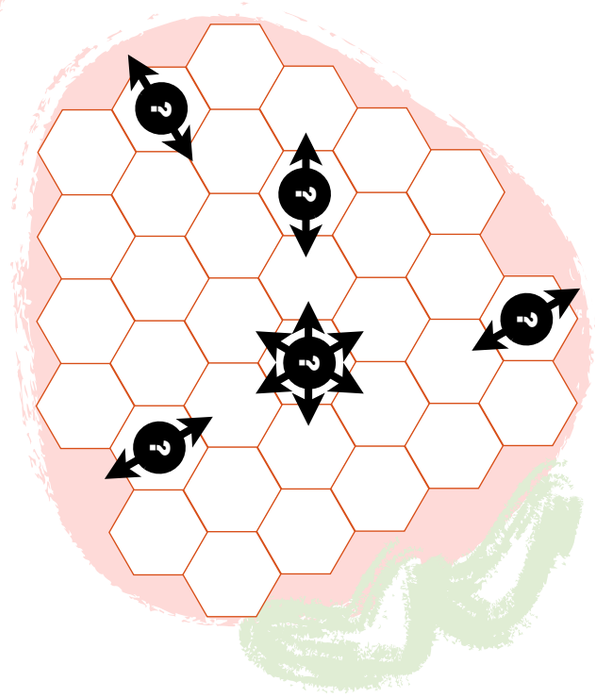

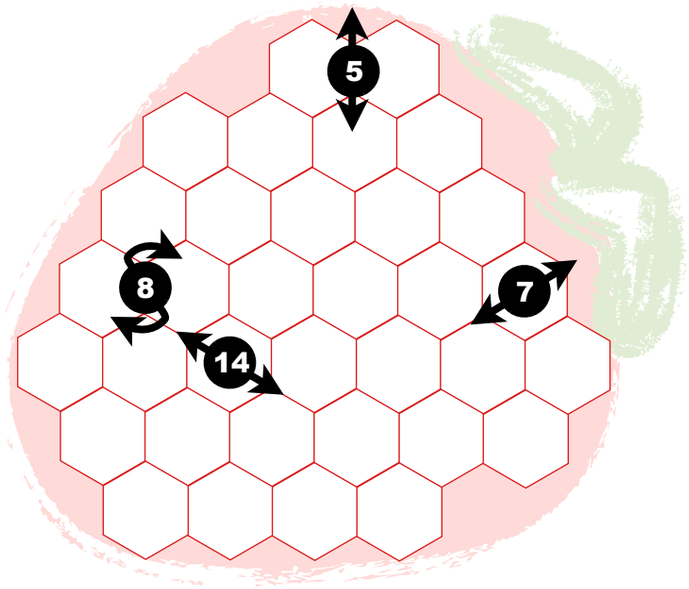

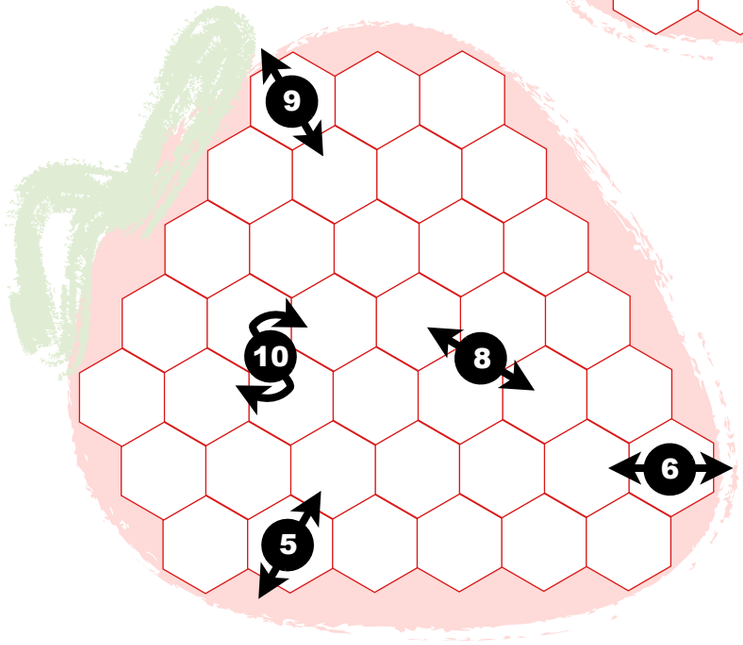

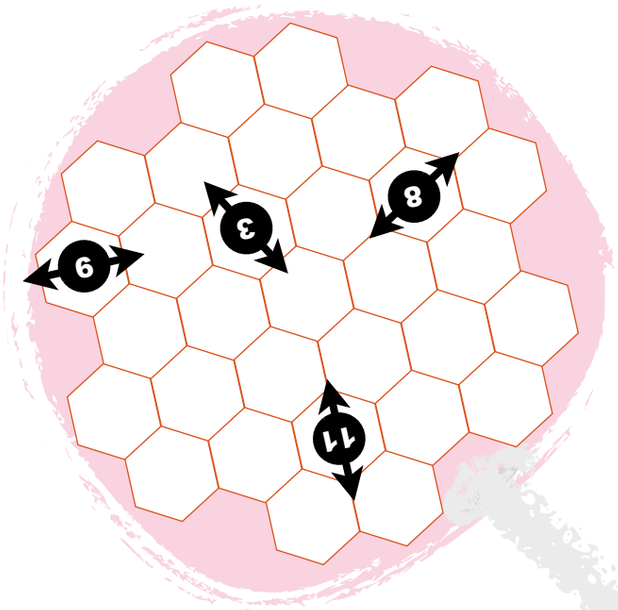
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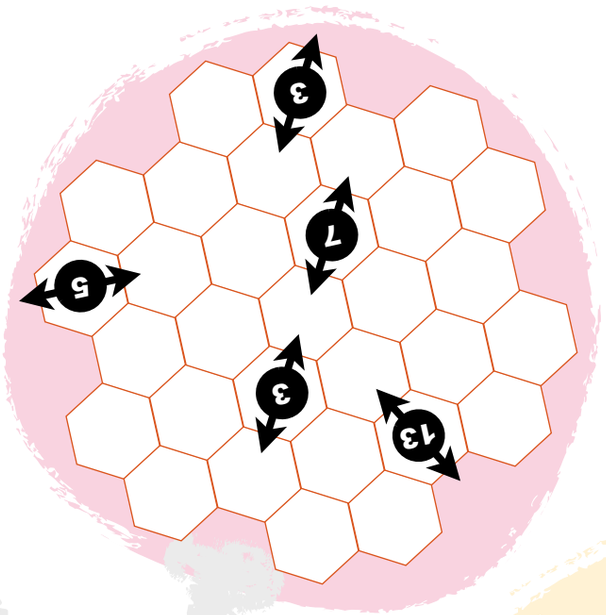
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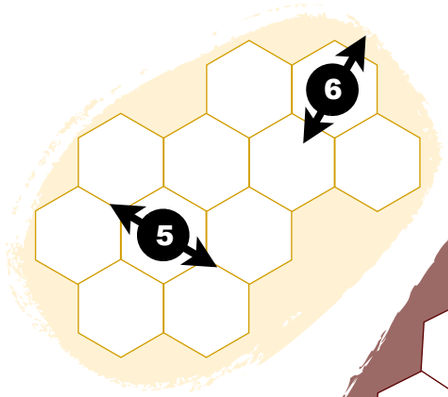
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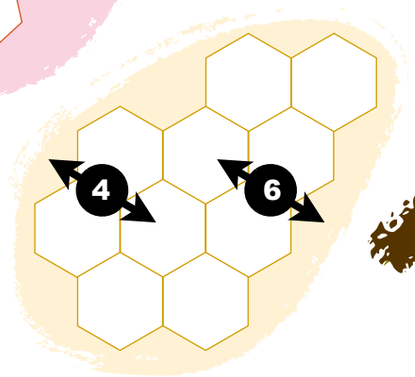
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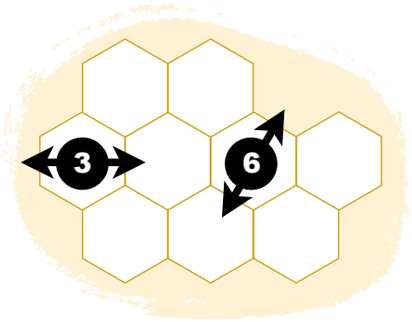
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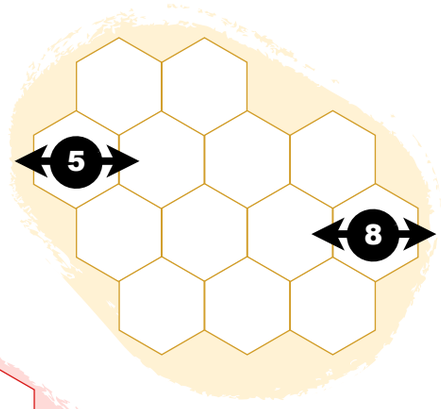
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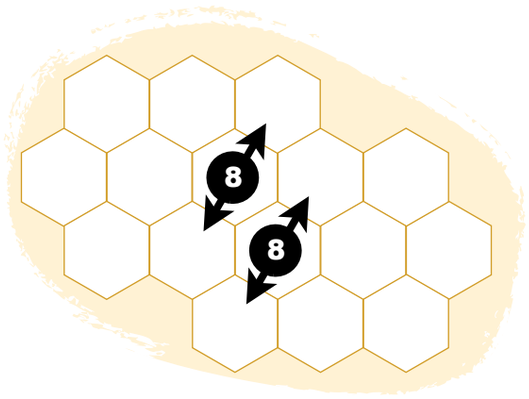
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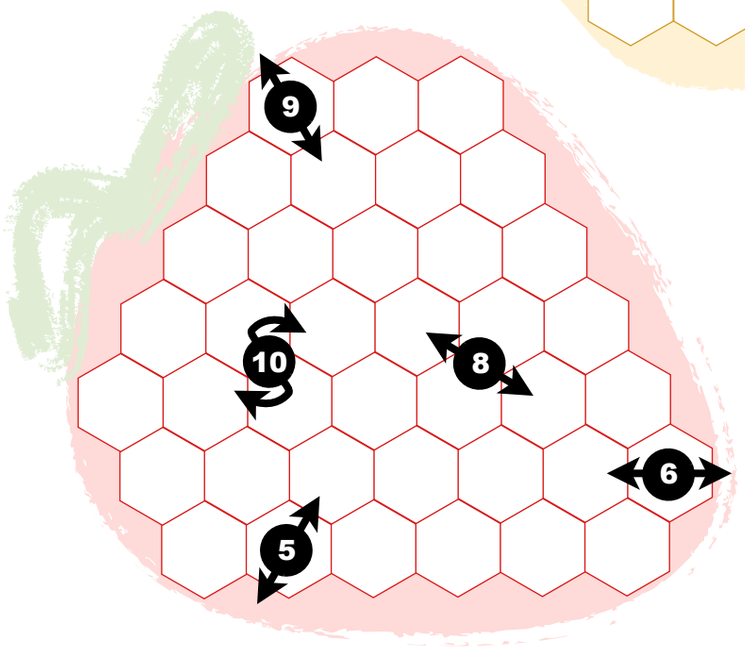
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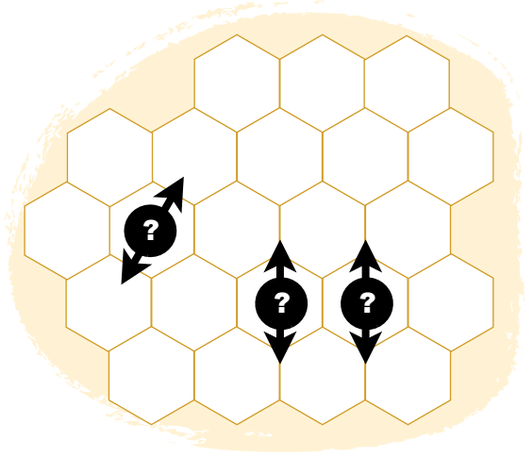
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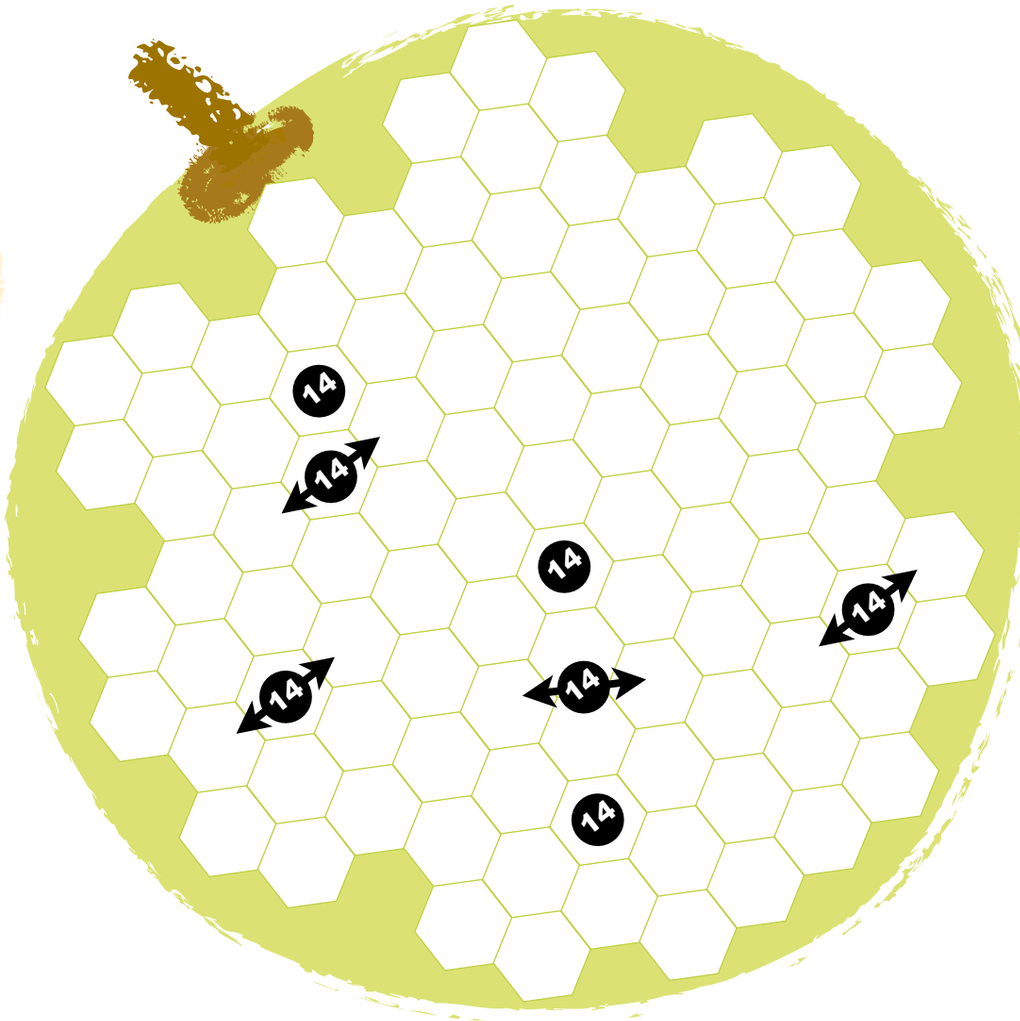
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Put Your Students in a Pickle!

I'm a father of two elementary school children, a mathematician, and designer of puzzles and board games. Students call me Dr. Pickle. There is nothing I enjoy more than stumping students and having them stump me.

I founded MathPickle.com in 2010 to inject new ideas into the classroom. MathPickle's primary objective is to get thirteen curricular unsolved problems into classrooms worldwide - one for each grade K-12. A conference in November 2013 established the thirteen unsolved problems. To aid with the dissemination of these awesome problems, MathPickle is looking at setting up a $1,000,000 reward for each - the prize money to be split between the person who solves the problem and their most inspirational K-12 educator.

MathPickle is also developing a range of curricular puzzles like the ones you'll find at TpT. These help teachers them with their number one challenge:

"How to engage the spectrum of student ability?"

Whenever an elementary school teacher wants to teach addition, she will invariably face 20% of students who already know how to add and another 20% who are struggling with last year's curriculum. How can she engage the top students without losing the bottom students? How can she engage the bottom students without boring the top students?

One solution: Parents of top students often ask that their child be allowed to accelerate through the curriculum. This exacerbates the problem for future teachers, and sets up a failure-impoverished education experience for the bright student.

A wiser approach is to use curricular puzzles, games and mini-competitions to simultaneously teach curriculum to the students who need it, and to deflect top students into tough problem solving activities. This is never time wasted, because problem solving is the primary reason we teach mathematics.

The experience of mathematics should be profound and beautiful. Too much of the regular K-12 mathematics experience is trite and true. Children deserve tough, beautiful puzzles.

Gordon Hamilton
MMath, PhD