Destroying Democracy

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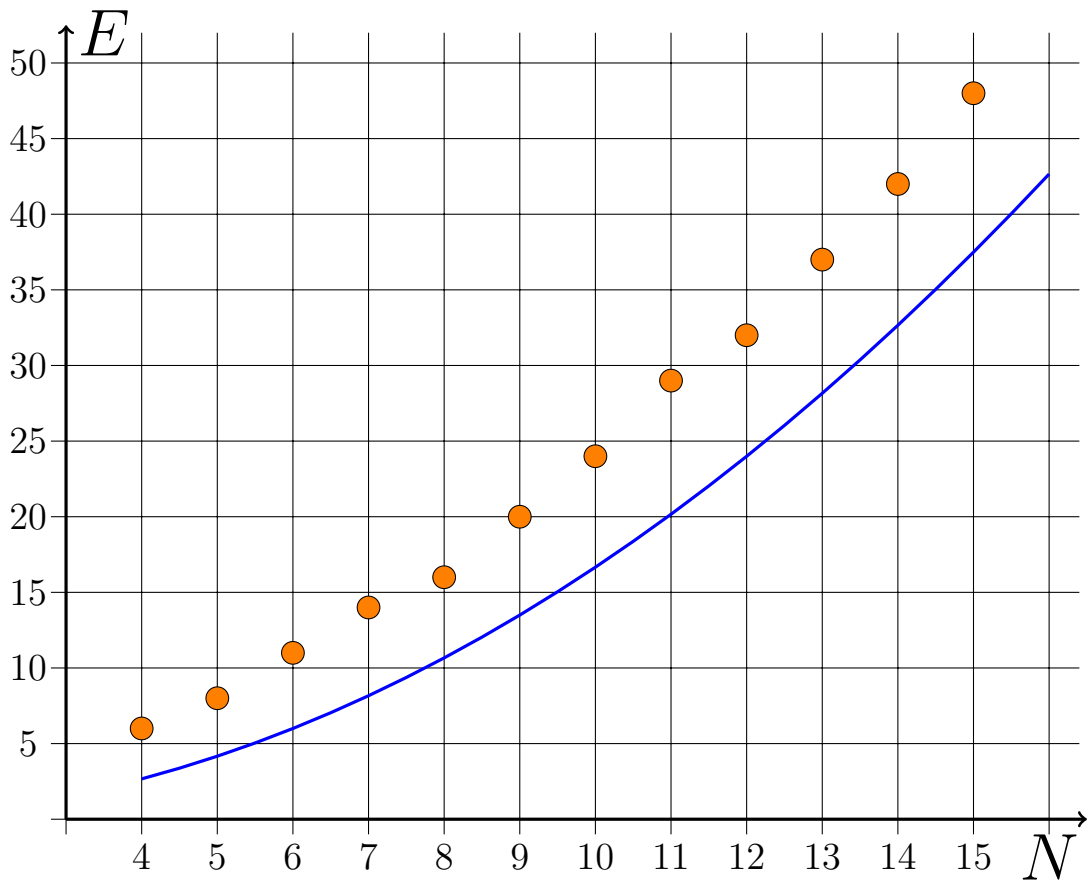
Summary

N	7×7	8×8	9×9	10×10	11×11	12×12	13×13	14×14	15×15
E	14	16	20	24	29	32	37	42	48
G	35	48	61	76	92	112	132	154	177
$\frac{E}{N^2}$	0.29	0.25	0.25	0.24	0.24	0.22	0.22	0.21	0.21

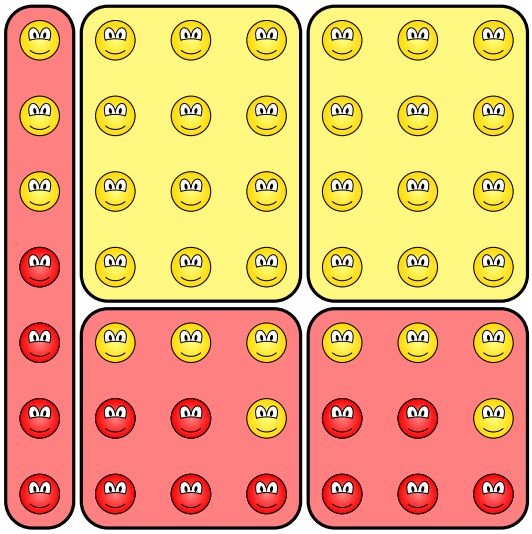


How it is done and the lower bound

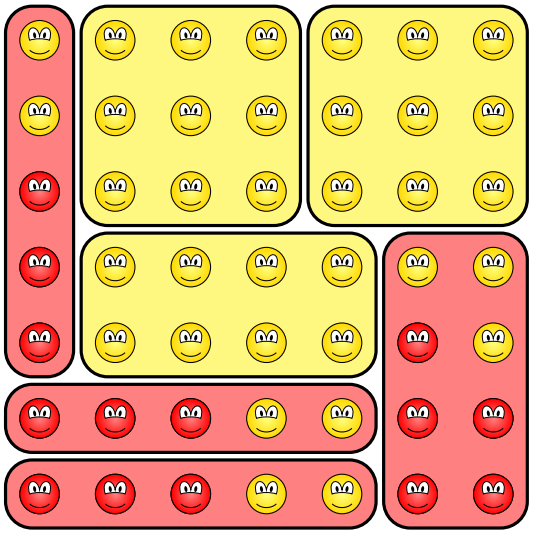
1. Let N^2 be the number of voters on a $N \times N$ grid.
2. Let j be the number of regions.
3. Let n_i be the number of voters in region i with $n_1 \leq n_2 \leq n_3 \cdots \leq n_j$.
4. So
$$\sum_{i=1}^{j} n_i = N^2$$
5. To win region i the number of evil wizards has to be at least
$$E_i = \left\lfloor \frac{1}{2} n_i \right\rfloor + 1$$
.
6. The maximum number of voters in a region is $n_j = 2n_1 - 1$.
7. To win the election the evil wizards have to win regions 1 to $m = \left\lfloor \frac{j}{2} \right\rfloor + 1$.
8. Thus the total number of evil wizards is
$$E = \sum_{i=1}^{m} E_i$$
9. A brute force computer algorithm generates a list of possible $\{n_1, n_2, n_3, \ldots, n_j\}$ and calculates E.
10. For the lowest E it is checked by hand, whether the regions fit on a $N \times N$ grid or not.
11. If it does not fit, the next E is checked.
12. For big n_i and j there is a lower bound for the *evilness:*
$$\frac{E}{N^2} \to \frac{1}{6}$$
13. So $E \leq \frac{N^2}{6}$ (See graph above).

7x7

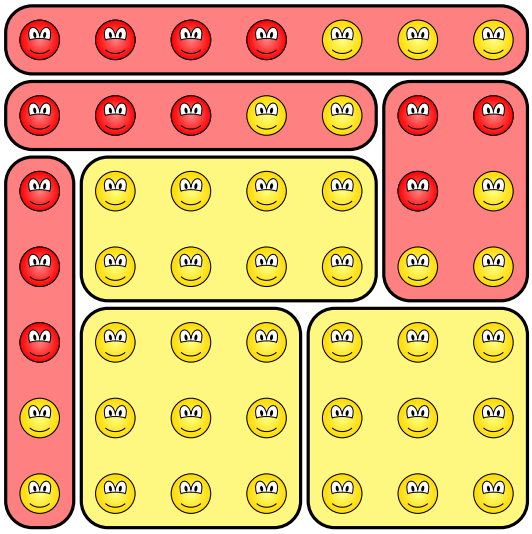
There are 6 optimal solutions with 14 evil and 35 good wizards.



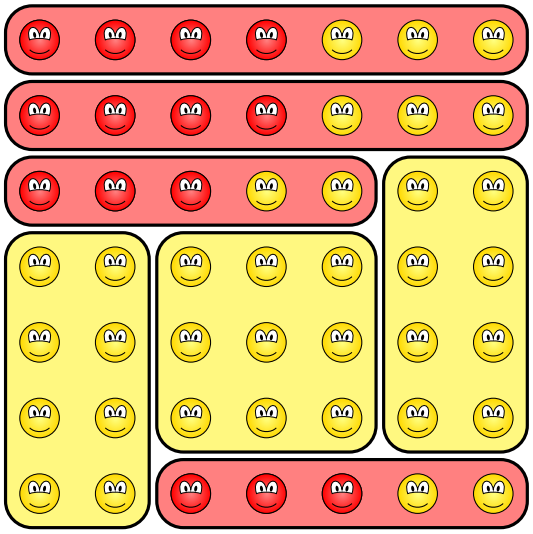
7-9-9-12-12 (3-2 victory)



5-5-5-8-8-9-9 (4-3 victory)



5-5-6-7-8-9-9 (4-3 victory)

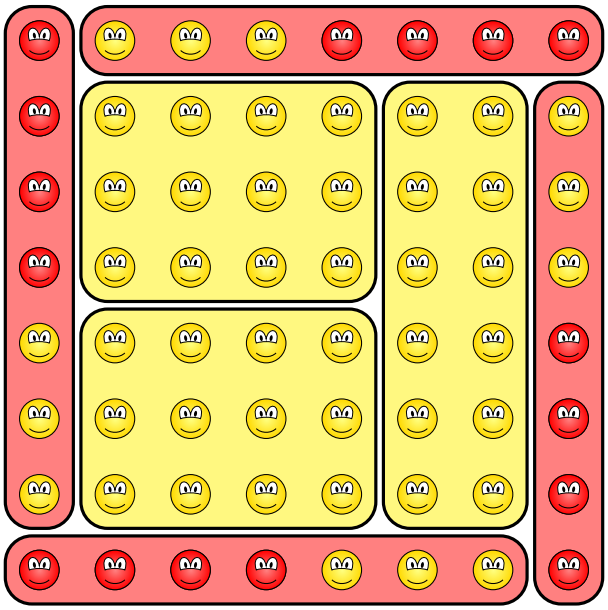


5-5-7-7-8-8-9 (4-3 victory)

Also possible:
3-3-3-3-3-3-3-3-5-5-5-5-5
3-3-3-3-3-3-3-4-4-5-5-5-5 (Mathpickle)

8x8

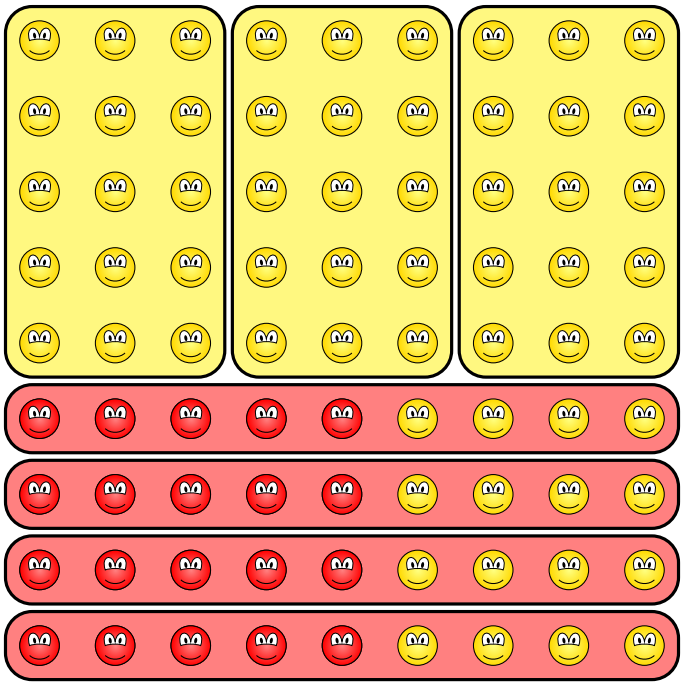
There is only one optimal solution. It is a 4-3 victory with 16 evil and 48 good wizards. There is no solution with 17 evil wizards.



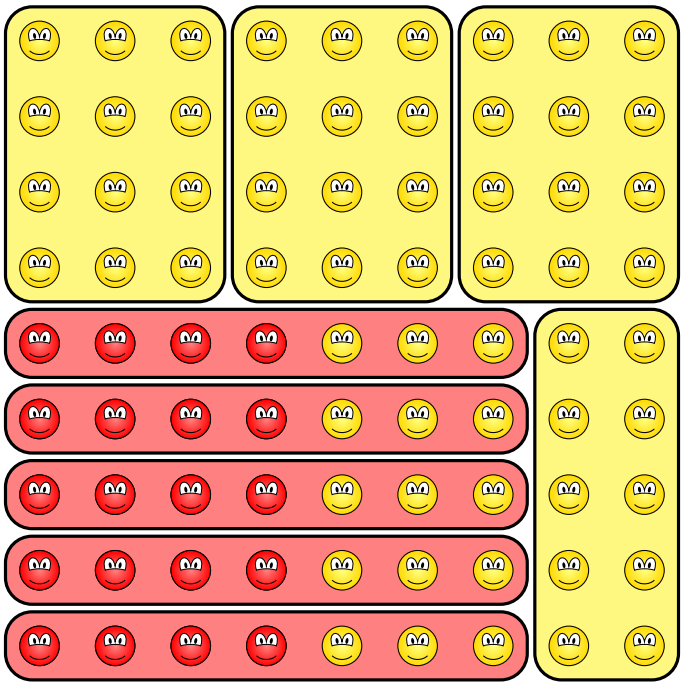
7-7-7-7-12-12-12 (4-3 victory)

9x9

There are two optimal solutions. It is a 4-3 and a 5-4 victory with 20 evil and 61 good wizards.



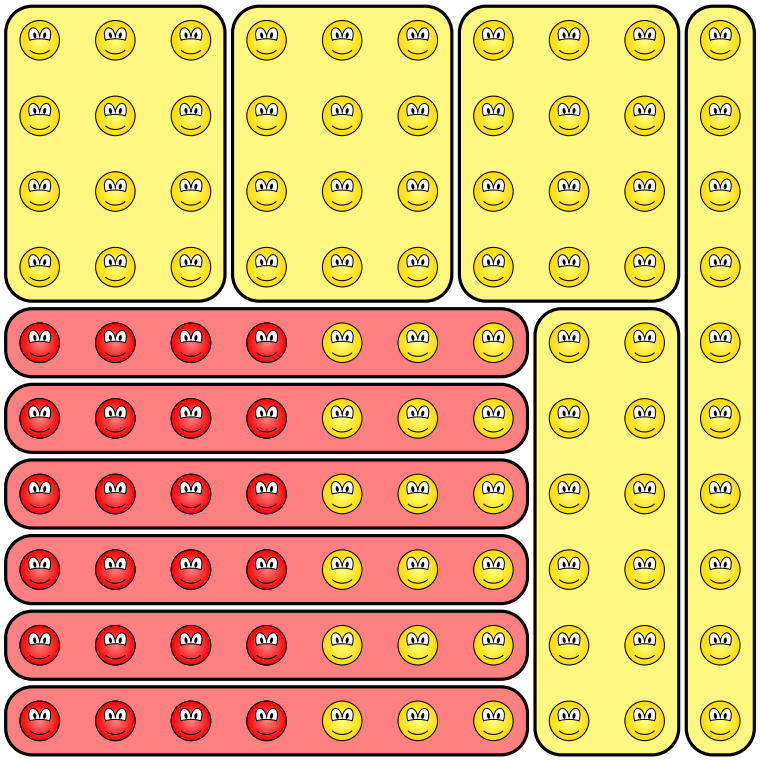
9-9-9-9-15-15-15 (4-3 victory)



7-7-7-7-7-10-12-12-12 (5-4 victory)

10x10

There is at least one optimal solution. It is a 6-5 victory with 24 evil and 76 good wizards.

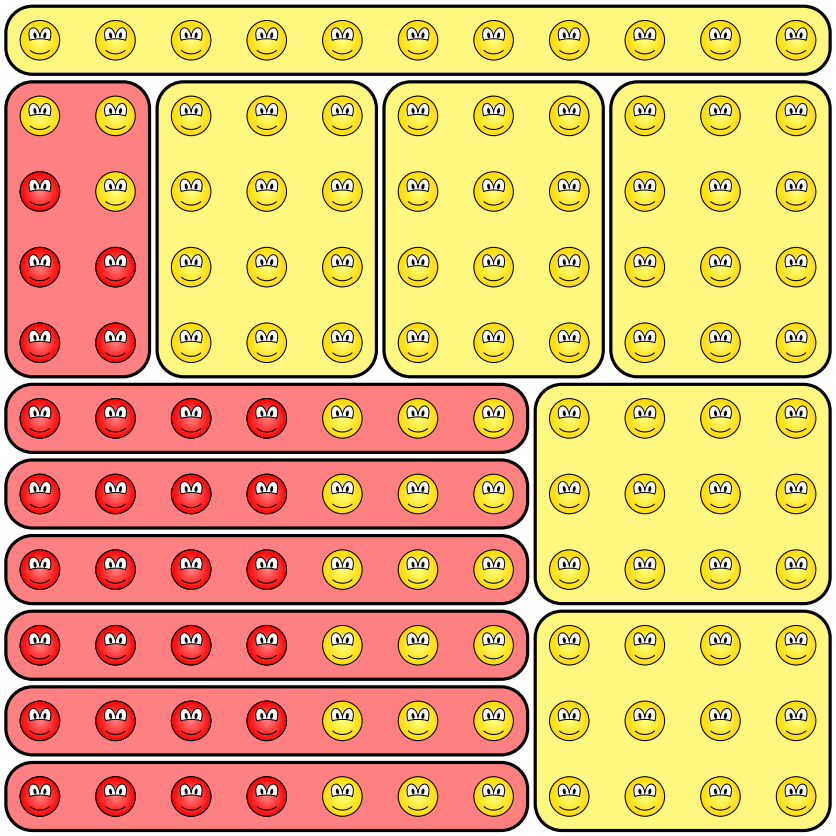


7-7-7-7-7-7-10-12-12-12-12 (6-5 victory)

To be checked:
5-5-5-5-5-5-5-5-6-9-9-9-9-9-9
5-5-5-5-5-5-5-5-7-8-9-9-9-9-9
5-5-5-5-5-5-5-5-8-8-8-9-9-9-9

11x11

There is at least one optimal solution. It is a 7-6 victory with 29 evil and 92 good wizards.

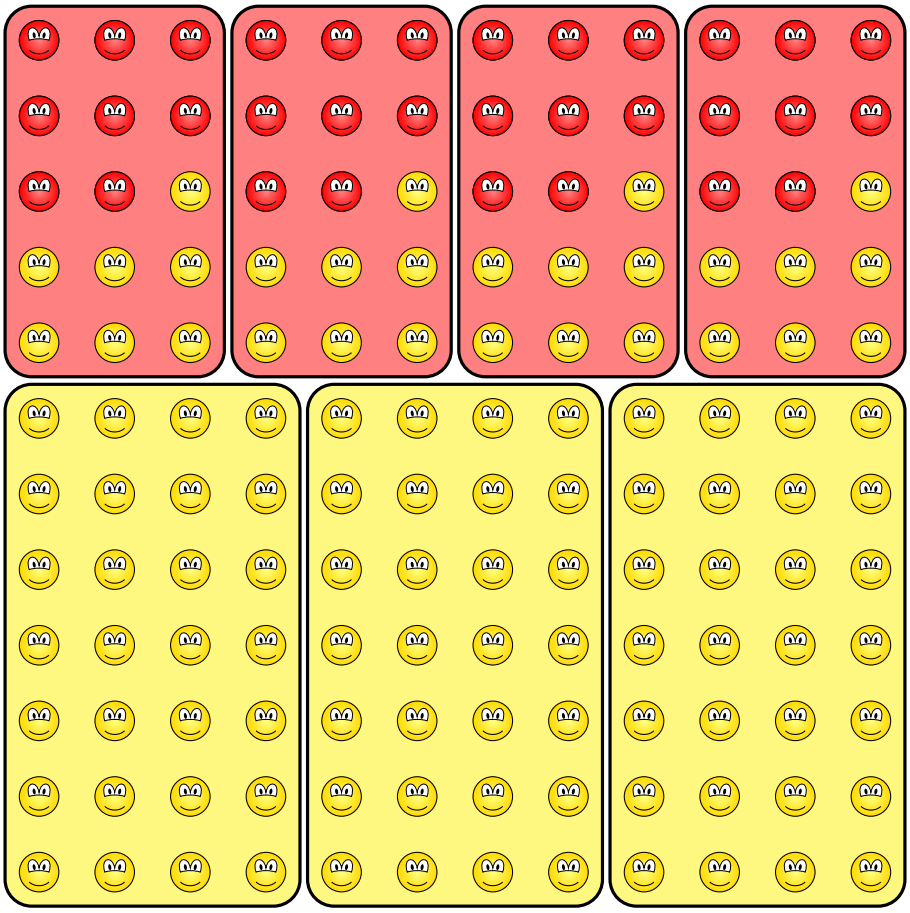


7-7-7-7-7-7-8-11-12-12-12-12-12 (7-6 victory)

To be checked:
7-7-7-7-7-7-9-10-12-12-12-12-12
7-7-7-7-7-7-9-11-11-12-12-12-12
5-5-5-5-5-5-5-5-9-9-9-9-9-9-9-9-9
5-5-5-5-5-5-5-7-7-9-9-9-9-9-9-9-9

12x12

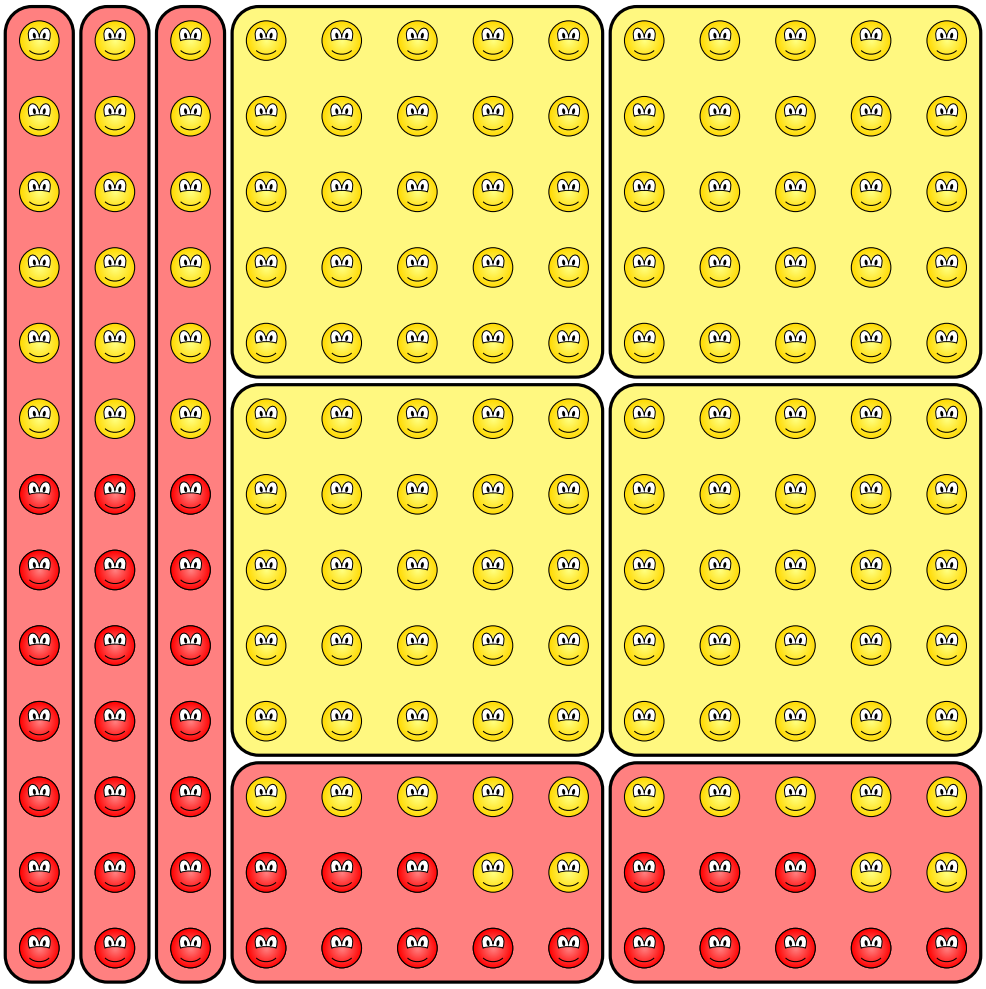
There is one optimal solution. It is a 4-3 victory with 32 evil and 112 good wizards.



15-15-15-15-28-28-28 (4-3 victory)

13x13

There is at least one optimal solution. It is a 5-4 victory with 37 evil and 132 good wizards.

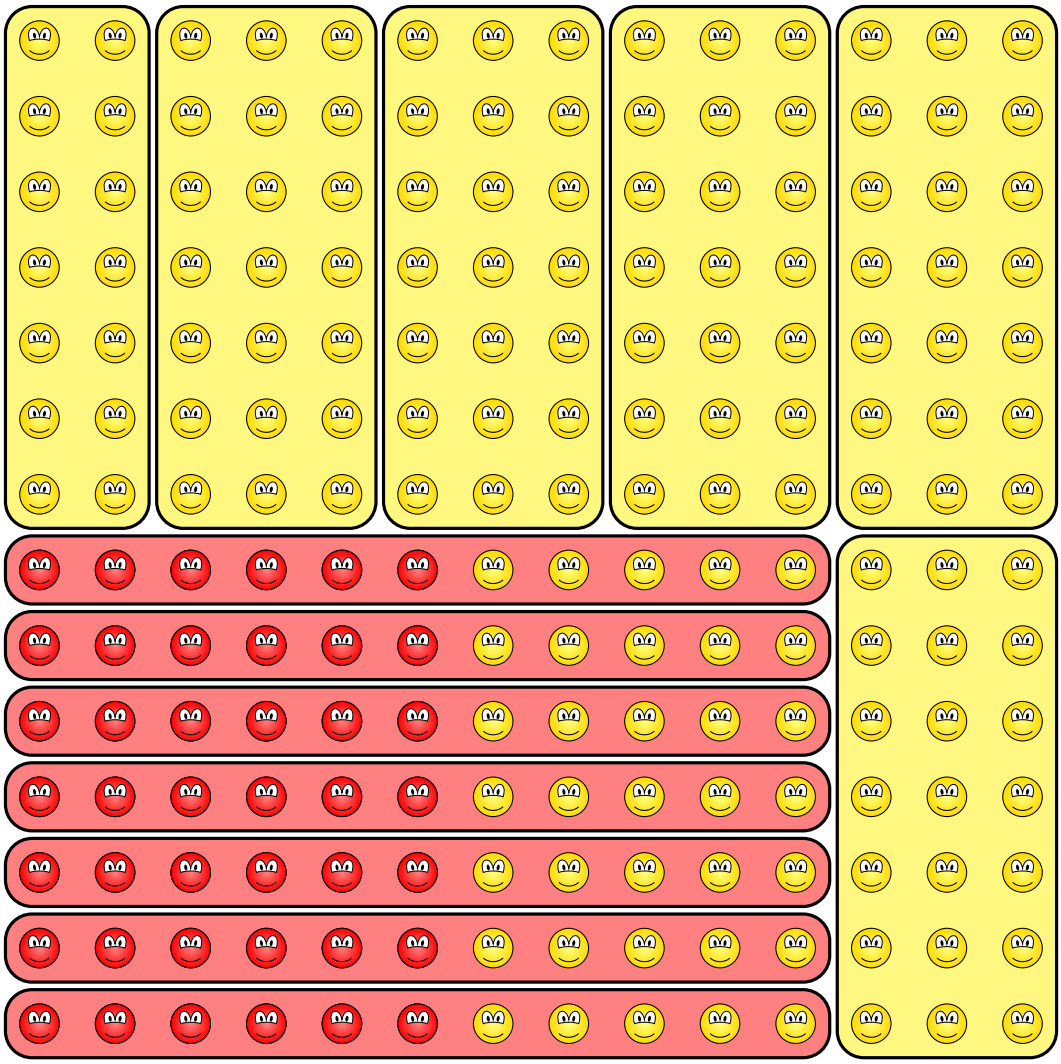


13-13-13-15-15-25-25-25-25 (5-4 victory)

To be checked:
11-11-11-11-11-12-18-21-21-21-21
11-11-11-11-11-12-20-20-20-21-21
11-11-11-11-11-13-18-20-21-21-21
11-11-11-11-11-13-20-20-20-20-21
7-7-7-7-7-7-7-7-9-13-13-13-13-13-13-13-13

14x14

There is at least one optimal solution. It is a 7-6 victory with 42 evil and 154 good wizards.

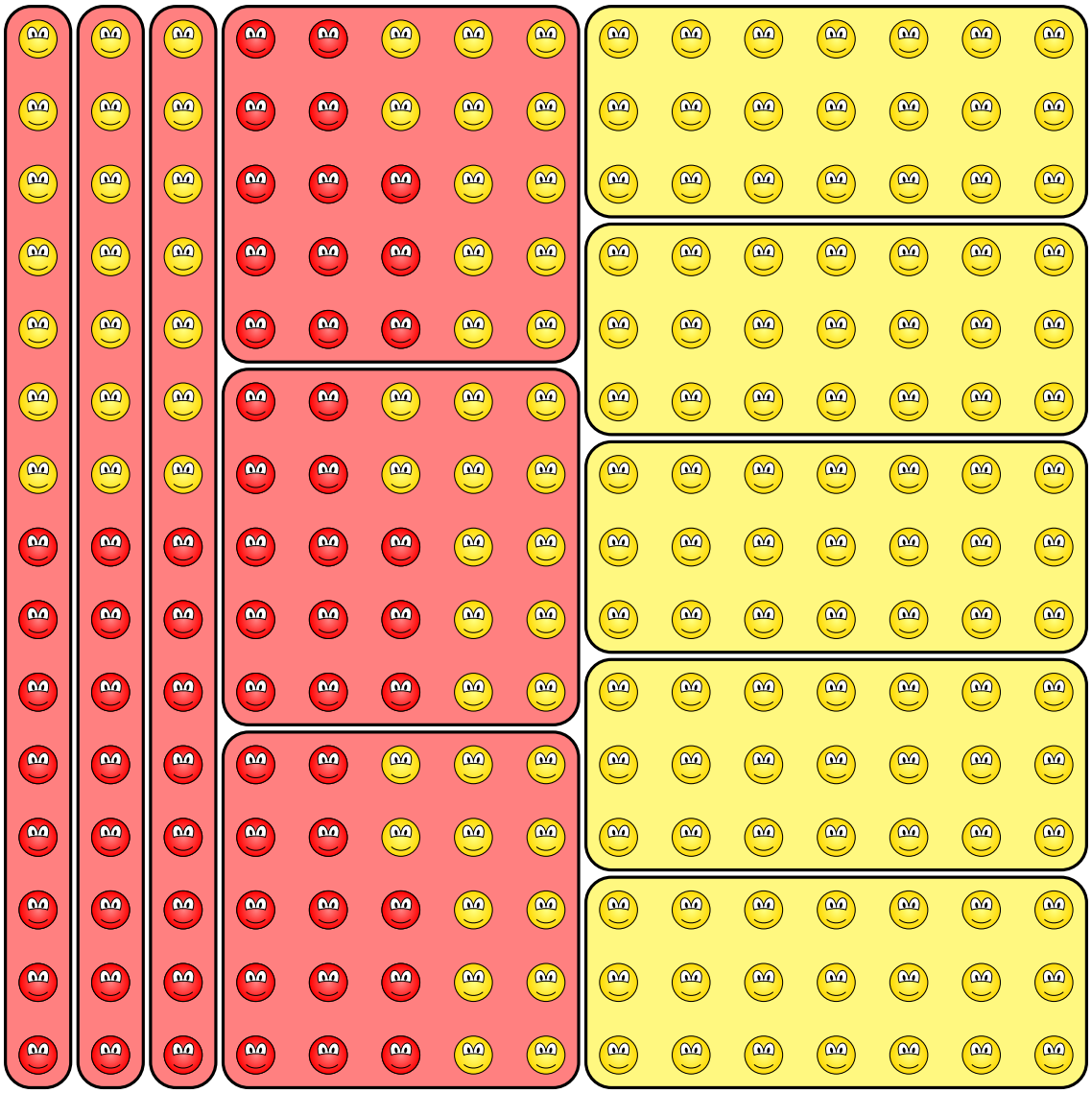


11-11-11-11-11-11-11-14-21-21-21-21-21 (7-6 victory)

To be checked:
11-11-11-11-11-11-11-15-20-21-21-21-21
11-11-11-11-11-11-11-16-20-20-21-21-21
11-11-11-11-11-11-11-18-18-20-21-21-21
11-11-11-11-11-11-11-18-20-20-20-20-21

15x15

There is at least one optimal solution. It is a 7-6 victory with 48 evil and 177 good wizards.



15-15-15-15-15-15-27-27-27-27-27 (6-5 victory)

To be checked:
15-15-15-15-15-15-24-27-28-28-28
15-15-15-15-15-15-25-26-28-28-28
15-15-15-15-15-15-25-27-27-28-28
15-15-15-15-15-15-26-26-27-28-28
15-15-15-15-15-15-26-27-27-27-28
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11-11-11-11-11-11-11-11-12-20-21-21-21-21-21

11-11-11-11-11-11-11-11-11-13-20-20-21-21-21-21
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11-11-11-11-11-11-11-11-11-15-18-20-21-21-21-21
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11-11-11-11-11-11-11-11-11-16-18-20-20-21-21-21
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7-7-7-7-7-7-7-7-7-7-7-7-7-12-12-13-13-13-13-13-13-13-13-13